Making sure your vote does not count: ESG activism, pseudo-green shareholders, and insincere proxy voting

Thomas Noe
Balliol College/SBS

Dunhong Jin
University of Hong Kong

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Abstract

This paper models green activists' proxy campaigns. The activist has the option of investing in a target firm and subsequently developing and making a green proposal. The proposal favours green production but lowers firm value. The firm is controlled by a small number of large diversified institutional investors (universal owners) who vote strategically, have reputation concerns, and may share the pro-green sentiments of the activist. We show that even when the reputation costs of opposing the proposal are small relative to the firm value reduction produced by proposal adoption, as long as the probability that some universal owners have pro-green sentiments is positive, activist proposals succeed with positive probability. However, increasing the probability that some universal owners are pro-green, by triggering more brown universal owner resistance, sometimes lowers the probability of adoption. Moreover, when proposal adoption entails significant reductions in market value, after activists acquire stakes they can reap considerable financial rewards from aborting their own campaigns. Thus, the dynamic consistency conditions required for activism equilibria also constrains the set of viable green proposals.

1 Introduction

The number of ESG-related shareholder proposals has increased significantly over the last two decades. The 2022 U.S. proxy season has yielded another consecutive record high number of shareholder resolutions on ESG issues, a 22% increase from 2021.[1] For example, in May 2021, hedge fund Engine No.1 successfully secured three green board seats via an activist campaign against ExxonMobil, despite owning just 0.02% of ExxonMobil's shares. On the heels of Engine No.1's success, Third Point, another activist hedge fund, submitted a proposal calling for Royal Dutch Shell to separate its oil and gas business from initiatives in renewable energy. A May 2022 vote on a greenhouse gas reduction proposal, submitted by investor representative As You Sow, was supported by 72.18% of Chubb Limited's shareholders despite Chubb board's opposition. Nonetheless, shareholder support for ESG proposals is volatile. Support almost doubled over 2015 to 2020, yet suffered a substantial decline over the last two years.

Who decides whether ESG proposals pass? The largest share blocks in many public companies in the U.S. are

[1] Midyear update from the Proxy Preview Project: https://www.spglobal.com/marketintelligence/en/news-insights/latest-news-headlines/record-number-of-shareholder-esg-proposals-in-2022-defies-gop-political-backlash-71181308.

held by *universal owners*, i.e., large, diversified, institutional investors (Amel-Zadeh et al., 2022).[2] In fact, the three largest institutional investors, BlackRock, Vanguard, and Fidelity, vote 25% of S&P 500 shares (Coffee Jr, 2021). Because these "universal owners" frequently hold the largest share blocks, in many cases, they can determine the outcome of shareholder votes on proxy proposals. Proxy votes are publicly observable. When proxy votes are related to controversial environmental, social, and governance (ESG) issues, votes can trigger adverse reactions to the institutions vote based on social and environmental values. Because universal owners' payoffs are diversified, the effects of these reactions to a universal owner's vote, e.g., withdrawal of funds from the universal owner, sanctions imposed on the universal owner by state governments, can be much larger than the effects produced by the change in corporate value engendered by the success or failure of the proposal.[3] Moreover, institutional investors may themselves value the ESG goals of shareholder proposals and be willing to accept some firm value reductions to further these goals.

ESG proposals require a proposer. This role is frequently assumed by an ESG activist fund. These funds aim to buy shares in firms and use their shareholder rights to offer proposals that further environmental and social objectives. In many ways, the problems faced by ESG activists are similar to the problems faced by non-ESG activists. They need to formulate a viable proposal, acquire shares, and launch a campaign to ensure adoption of their proposal. But, in one important aspect, the ESG fund's problem is quite different from the problem faced by conventional activists: when a conventional activist attempts to purchase shares, current shareholders realize that they can capture the value-add produced by the activist's proposal by holding on to their shares. However, holding shares will not permit existing shareholders to capture the value-add produced by an ESG proposal, because the value-add will not come in the form of increased firm value but rather through reduced carbon output, a more diverse management team, etc.

In this paper, motivated by its real-world importance relative to other ESG objectives, we focus on ESG proposals related to carbon and climate change. In our model, share blocks voted by universal owners determine whether proposals pass. Residual non-institutional share holdings are held by atomistic owners. Some universal owners, *green owners*, share the environmental values of the ESG activist; other universal owners, *brown owners*, do not. Universal owners face *reputation costs*, i.e., costs engendered by voting against ESG proposals. Owners vote strategically. We consider the case where, even accounting for reputation costs, it is in the collective interest of brown owners for the green proposal to fail and in the collective interest of green owners for the proposal to pass. We term the likelihood that a universal owner is green, *green sentiment*.

[2] Amel-Zadeh et al. (2022) term such owners "common owners." We prefer using the term "universal owners" to avoid the miss-impression that this paper relates to the effects of institutional joint ownership on intra-industry competition. The Society of Actuaries defines "universal owners" as "institutional asset owners (pension funds, mutual funds, insurance companies, sovereign wealth funds) that own such a representative slice of the economy as to find it impossible to diversify away from large system-wide risks." See https://www.actuaries.org.uk/learn-and-develop/lifelong-learning/sustainability-and-lifelong-learning/universal-owners.

[3] For an example of state government sanction threats triggered by institutional shareholder proxy votes, see https://www.reuters.com/business/finance/west-virginia-threatens-bar-big-banks-blackrock-over-perceived-fossil-fuel-2022-06-14. For further discussion about the investor catering rationale for pro-ESG voting by institutions, see Wang (2021); Ramelli et al. (2021).

ESG activists, anticipating the costs of an ESG campaign and the likely outcome of the vote, decide whether to attempt an activism campaign. The ESG activist aims to develop a proposal that will substantially reduce the firm's carbon footprint, if the activist can develop a viable proposal and the probability of passing of the proposal is sufficiently large relative to the costs of an activist campaign, the activist buys shares and presents the proposal for shareholder vote.

If the firm has only one universal owner, the proposal fails if and only if the single owner votes against the proposal. Because the cost of reputation is less than the *value reduction* in the owner's share of the firm triggered by the proposal passing, a brown owner will oppose the proposal. By assumption the green benefits of the proposal exceed the value reduction. Thus, the proposal will be supported by the universal owner if the owner is green. Hence, the proposal will pass if and only if the universal owner is green. Hence, increasing green sentiment increases the probability that the proposal will pass.

When there are multiple universal owners, the tension that drives brown owner voting is that each brown owner would like to see the proposal fail but, because of reputation costs, would prefer not to vote against the proposal. A brown owner's vote only affects her welfare when her vote is marginal. So, each brown owner will trade off the benefit of voting yes, avoiding reputation costs, against the cost of voting yes, the value reduction produced by a yes vote when that vote is marginal.

When green sentiment is high, brown owners conjecture correctly that it is likely that many other universal owners are green and thus will support the proposal. Hence their vote is unlikely to be marginal. Thus, brown owners capitulate and insincerely support the green proposal. When green sentiment is moderate, brown owners realize that the proposal can only be defeated if "all hands are on deck," so they opt for extreme voting policies, i.e., they either resist, i.e., all vote no, or capitulate. In both of these cases, increasing green sentiment increases the probability that the proposal will pass.

However, when green sentiment is low, the situation is more complex. Brown owners realize that, if all brown owners resist, it is likely that the proposal will fail by a wide margin, in which case, if all brown universal owners resist, no brown owner is likely to be marginal. Thus, the resistance strategy is not optimal. Instead, brown universal owners adopt partial resistance strategies: some brown owners, those facing the largest reputation costs, insincerely vote for the proposal while others resist. This result provides a theoretical rationale for the strategic blockholder voting on ESG proposals documented in Michaely et al. (2021). In this case, increasing green sentiment can reduce the probability that green proposal succeeds: the increase in green sentiment increases the optimal level of brown resistance, this strategic effect of increased sentiment can dominate the positive mechanical effect of increased green on the probability of proposal success.

These results have implications for the effects of ownership structure on the welfare of brown owners. First note that, in our analysis, each universal owner casts all of her proxy votes either in favor of or against the green proposal.[4] The proposal will fail if a simple majority of proxy votes oppose the proposal. Thus, if the firm is owned by a single brown owner, defeating the proposal requires casting more no votes than is required to defeat the proposal. If there are many owners, as long as the majority of brown owners' proxies oppose the proposal, the proposal will sometimes fail. When green sentiment is sufficiently small, a thin majority of brown owners' proxy votes opposing the proposal is very likely to ensure failure. In this case, somewhat dispersed brown ownership imposes lower reputation costs on brown owners and entails only a negligible probability of incurring the value reduction attendant to proposal success. Hence, unified ownership decreases the welfare of brown owners.

However, there are two costs of divided ownership that become more salient as green sentiment and the number of owners increases. First, dividing ownership creates a *free-rider effect*: brown owners do not internalize the effect of their votes on the welfare of other brown owners. Second, because brown owners do not know which other owners are brown, the effect of a given brown owner's vote on the success of the proposal is uncertain. As green sentiment increases from negligible to appreciable, this *uncertainty effect* reduces the likelihood a given brown owner casts marginal proxy votes and thus attenuates a brown owner's incentive to vote against the proposal. When green sentiment is sufficiently large, the free-rider and uncertainty effects dominate the vote-splitting effect and unified ownership increases the welfare of brown owners.

The voting game determines the probability that the green proposal will pass. However, in order for a green proposal to pass, a green proposal must be submitted. In our analysis, a green proposal is submitted by an ESG activist who controls a financially constrained activist fund. The activist has green preferences, buys a stake in the firm and makes a proposal. The price at which the activist acquires shares is determined by the rational expectations of atomistic investors in a competitive market. In contrast to the standard Grossman and Hart setting, these atomistic investors cannot hold out to obtain the gains from selling to the activist because (a) successful activism reduces firm value and (b) the benefit from the proposal's success captured by green agents, the environmental improvement affected by the adoption of the proposal, is not contingent on whether they retain their shareholding in the firm. Because activists buy shares from a pool of atomistic investors, who by definition, do not believe that their actions affect the success of the proposal, regardless of whether atomistic shareholders are brown or green, their ask price equals the expected monetary share payoff conditioned on the probability of proposal success. For this reason, the activist breaks even when acquiring shares and will buy into the firm whenever the expected green benefit of the campaign exceeds the costs of activism. The nonpecuniary motivations of activists eliminate the Grossman and Hart free-rider problem.

[4]Because voting for the proposal is a weakly dominant strategy if and only if the owner is green, it seems reasonable to assume that any proxy votes opposing proposal engender reputation costs. Vote splitting, i.e., a single fund voting some proxies for and some against a proposal has, to our knowledge, never occurred. In fact, this possibility, to our knowledge, has not even been considered in the legal literature. Fund families can recommend a yes vote to some family members and a no vote to others. However, non-uniform recommendations are purportedly based on the differences between the preferences of the fund's beneficial owners. The typical pattern is for one recommendation to be offered to traditional funds and another to "green" funds.

Related literature

Our paper is closely related to the emergent literature on the effects of corporations' environmental and social policies on shareholder and social welfare. Like many papers in this literature, our green agents have what Gupta et al. (2022a) term "broad green preferences," i.e., part of the utility green investors derive from investing is based on the effect of their investment on environmental outcomes. Thus, in broad green preferences models like ours, green investors, per se, do not increase their utility by divesting from brown assets and green utility is not tied to the number of shares owned by the investor but rather the size of the change in the greenness of output that an investor can affect. In contrast, Goldstein et al. (2022) consider equilibrium security prices when investors have "narrow green preferences," a preference for holding shares of firms producing green output. Most of the broad green preferences literature (e.g., Jagannathan et al., 2022; Gupta et al., 2022a; Broccardo et al., 2022; Albuquerque et al., 2019) models worlds where firms are either green or brown. Green agents affect changes in policy by buying up brown firms. In contrast, we focus on the struggle for control between green and brown shareholders of a given firm. These investors fight for control through proxy voting rather than through acquisition offers.

In this respect our paper is related to Hart and Zingales (2017) who also study corporate policy when firm actions affect owners' utility through channels other than firm value. However, in Hart and Zingales (2017), individual shareholders' green preferences directly affect corporate policies through their voting behavior. In our analysis, decisive votes are cast by universal owners.[5] The green preferences of these institutional investors only indirectly affect the strategic voting of universal owners through the reputational penalties associated with opposing green proposals. Also, unlike Hart and Zingales (2017), which focuses on how firm policy should be determined when shareholders have conflicting objectives, we focus instead on how policy is actually determined when controlling agents are large and strategic.

Our model of ownership structure is to a large extent inspired by the empirical literature documenting the rise of common ownership (Amel-Zadeh et al., 2022) and the legal literature considering the implications of universal/common ownership for securities' regulation (Coffee Jr, 2021). The effect of social pressure on investor, fund, and firm behavior incorporated in our model is motivated by a large empirical literature (e.g., Wang, 2021; Ramelli et al., 2021; Dimson et al., 2015). A prediction of our model, that institutional investors frequently vote strategically, supporting ESG proposals only when their votes are not marginal, is supported by empirical evidence in Michaely et al. (2021).

Our model of activist share acquisition is quite simple and structurally quite similar to the atomistic shareholder model in Grossman and Hart (1980). However, we reach very different conclusions about the ability of share acquirers, corporate raiders in their model, activist investors in our model, to gain from share acquisitions. In

[5]Empirical research (Brav et al., 2022) suggests that our framework matches current institutional practice better than the framework developed by Hart and Zingales (2017). However, institutional practice can change. Devolving index fund proxy voting to the retail investors who own fund shares has been advocated by many legal scholars (e.g., Griffin, 2019). In the US, there are legal barriers to devolution. However, BlackRock UK plans to permit some devolved voting in 2023 (Financial Times, 2022).

Grossman and Hart (1980), acquisitions are motivated by pecuniary gain and current shareholders capture the pecuniary gain generated by acquirers' value-add plans. Thus, when acquirers lack a toe-hold stake, they cannot profit from adding value. In our setting, the small atomistic shareholders who sell to activists also sell their shares at prices that reflect the expected market value effects of activists' interventions. However activists can still gain from intervention because of the non-pecuniary utility they derive from inducing target firms to adopt green policies.

2 Structure of the model

2.1 Precís

We develop a model of activism and shareholder voting for firms controlled by universal owners. Some agents' preferences over actions are completely determined by their monetary payoffs; other agents' preferences also depend on the environmental effects of their actions. An activist fund, henceforth called the *activist*, initiates ESG activism and acquires shares. We focus on *activism equilibria*, equilibria in which activists acquire shares, attempt to identify proposals that, if adopted, will make the firms output greener, and, when such proposals are identified, submit their proposals to shareholders. Shareholders then vote on the proposal. The proxy votes of the universal owners determine whether the proposal succeeds or fails to pass.

2.2 Preferences, agents, and timings

2.2.1 Preferences

All agents are risk neutral and patient. there are two kinds of agent preferences: green and brown. Agents with *brown* preferences simply maximize their expected wealth. Agents with green preferences have, using the terminology in Gupta et al. (2022b), have"wide-green preferences," i.e., they care about the greenness of the world not the greenness of their portfolios. More specifically, let a represent an action that might affect an agent's terminal wealth and the environment; let $\tilde{V}(a)$ represent the agent's random future (date 1) terminal wealth conditioned on a; let $\tilde{G}(a)$ represent the random future greenness of the environment (e.g., some decreasing function of CO_2 ppm) conditioned on a. If the agent has green preferences, the agent's utility is given by

$$\mathbb{E}\big[\tilde{V}(a)\big] + \beta\,\mathbb{E}\big[\tilde{G}(a)\big], \quad \beta > 0. \tag{1}$$

The parameter β measures the extent to which the agent is willing to sacrifice monetary payoffs to increase greenness. We call $\mathbb{E}[\tilde{G}(a)]$ the *green payoff* from action a. Equation (1) implies that an agent with green preferences weakly prefers action a'' to action a' if and only if

$$\mathbb{E}\big[\tilde{V}(a'') - \tilde{V}(a')\big] \geq \beta\,\mathbb{E}\big[\tilde{G}(a'') - \tilde{G}(a')\big]. \tag{2}$$

Green agents tradeoff the effects of their actions on their expected terminal wealth, $\mathbb{E}[\tilde{V}(a)]$, against their expected effects on the environment, $\mathbb{E}[\tilde{G}(a)]$. So, for example, a green agent who owns a firm that has an inherently large carbon footprint (e.g., a coal-fired electricity generator) cannot increase her utility by divesting from the firm through selling out to a brown competitor. If she sold out, her portfolio would be greener but the world would not. In fact, if the brown competitor planned to make the firm's carbon footprint even larger, the green owner would only divest if the brown competitor offered sufficient monetary compensation to offset the environmental effects of the control transfer.

Another obvious implication of equation (2) is that, when two actions, say a' and a'', have the same environmental effects, i.e., $\mathbb{E}[\tilde{G}(a'') - \tilde{G}(a')] = 0$, green agents' and brown agents' preferences coincide; both will choose an action that maximizes their expected terminal wealth, $\mathbb{E}[\tilde{V}(a)]$. For example, suppose an agent is considering whether to buy one share of an oil company's stock. The oil company is controlled by blockholder, whose operating decisions cannot be swayed by small shareholders. Because purchasing the share has no effect on the environment, whether the agent's preferences are brown or green will have no effect on an agent's reservation bid price.

2.2.2 Agents

There are three kinds of agents: universal owners, an activist, and a mass of atomistic small shareholders. The firm has one share outstanding. We refer to the number of shares held by a shareholder before trade as the shareholder's *endowment*. In this section, we outline the qualitative characteristics and the roles of the agents. Specific parameterizations of the agents' payoffs will be provided in the relevant sections of the model.

Universal owners. There are K universal owners. Each universal owners holds an appreciable share endowment of firm shares. Universal owners do not alter their endowment through buying or selling shares. Universal owners can have either brown or green preference. We refer to universal owners with brown preferences as *brown owners* and refer to universal owners with green preferences as *green owners*. The preferences of universal owners are determined by independent draws from a Bernoulli distribution. With probability γ, the draw results in a assigning green preferences to the universal owner; with probability $1-\gamma$, the universal owner is assigned brown preferences. The assignment is private information of the universal owner receiving the assignment. We refer to γ as *green sentiment* because it measures the extent to which universal owners have an inherent preference for increased greenness. The monetary payoffs to universal owners depend both on the effect that the proposal has on the value of their share endowment and on reputation costs associated with voting in a fashion that indicates that they have brown preferences. Universal owners are decisive in the sense that a proposal succeeds if and only if it supported by the majority of universal owners.

Activists. The activist has green preferences and the activist's preferences are common knowledge. The activist has no endowment of firm shares and acquires shares by trading with the atomistic shareholders. In order to make a proposal at the shareholder meeting, the proposing shareholder must have a sufficient stake in the firm. We capture this restriction by requiring the activist to acquire at least $\underline{n}$ shares.[6]

Atomistic shareholders. Each individual atomistic shareholders is endowed with infinitesimal shareholding. Collectively atomistic shareholder are endowed with $n^{\text{At.}}$ shares. Atomistic shareholders can trade their endowments. For reasons discussed later when we examine the activist's problem, the greenness of atomistic shareholders will have no effect on their behavior. Thus, we impose no restrictions on the portion of activists who have green preferences.

2.2.3 Timing

The sequencing of event in the model is provided below.

Activism phase:

Initiation phase: At date 0, the activist decides whether to initiate activism. If the activist initiates, the activist attempts to acquire shares of the firm and pays an investigation cost, c.

Launch phase: At date 1, if the investigation yields a proposal, the activist decides whether to launch a campaign by submitting a proposal to shareholders; if investigation does not yield a proposal, the activist does not submit a proposal.

Voting phase:

At date 2, if the campaign is launched, shareholders vote on the proposal. If passed, the proposal is implemented.

At date 3, environmental and monetary outcomes are realized.

3 Activism phase

We initiate our analysis by considering activism phase, the problem of an activist deciding whether to initiate and launch an activist campaign. Shareholder voting only affects the activist in so far as it determines the probability that the activist's proposal succeeds, i.e., is approved by shareholders. Therefore, in this section, we assume an

[6] In the US, the ownership threshold is quite modest: owning between \$2,000 and \$25,000 worth of firm shares depending on the length of time the shares have been held. In the UK, the threshold is much higher: a 5% ownership stake is required to compel inclusion of a proposal on the agenda of the annual general meeting.

exogenous probability that the proposal will succeed, denoted by ρ. Intuitively, if ρ is too low, then it would not be worth for the activist to launch the campaign. ρ will be endogenized in Sections 4 and 5.

The activist has wealth $b+c$ and is liquidity constrained. If he initiates activism and attempts to acquire shares, he pays an investigation cost c and invests all of his remaining wealth, b, in the firm.[7] Thus, the activist purchases b/p_0 shares from the atomistic shareholders, where p_0 is the trading price, determined in the equilibrium.

With probability π, investigation yields a proposal. With probability $1-\pi$, investigation fails to yield a proposal. If investigation yields a proposal, the value of the firm, if the proposal is submitted and succeeds, is $V(S)$, and the green payoff is $G(S)$. If the proposal fails, i.e., no proposal is produced by investigation, or a proposal is produced but not submitted. The value of the firm and the green payoff will be $V(F)$ and $G(F)$, respectively. Thus, we can think of (V,G) as representing the value of the firm and green payoff under the firm's status quo policies. In order to avoid considering trivial cases, we assume that (a) there is tradeoff between value maximization and maximizing green payoffs, and (b) despite the value reduction produced by adopting the proposal, its adoption is preferred by green owners, i.e., we assume that

$$\text{(a): } G(S) > G(F) \text{ and } \mathrm{V}(F) > V(S), \tag{3}$$

$$\text{(b): } V(S) + \beta\, G(S) > V(F) + \beta\, G(F). \tag{4}$$

Equation (3) implies that, absent the reputation costs produced by opposing the proposal, brown owners prefer rejection of the proposal. Equation (4) ensures that, if the firm is owned entirely by one green owner, even absent reputational considerations, the owner prefers acceptance of the proposal. Because the green payoff does not vary with the fraction of the firm owned by a green agent, but the value of a green owner's claim on the firm is less than the value of the whole firm, equation (4) ensures that green owners will always support the proposal regardless of the degree to which universal owners' shareholdings are dispersed.

We aim to determine the conditions for the existence of an *activism equilibrium*. In an activism equilibrium, the activist plays the *activism strategy*: the activist initiates activism and, when investigation yields a proposal, launches a campaign. In an activism equilibrium, other agents' beliefs are consistent with the activist following the activism strategy. When other agents conjecture that the activist plays the activism strategy, they estimate that the proposal will be implemented with probability $\pi\rho$ and, with probability $1-\pi\rho$, will not be implemented. When the activist attempts to purchase shares from the atomistic shareholders, these shareholders will post ask prices for their shares. Like the atomistic shareholders in Grossman and Hart (1980), atomistic shareholders do

[7] If we extended the model by dropping the assumption that the greenness of the activist is common knowledge and posited instead that some fraction of activists are fake/pseudo greens, these pseudo greens would have an incentive to initiate campaigns, drive down the stock price, but not follow up by launching, and thereby profit from the increased value of their shareholding. Rational atomistic investors would anticipate this behavior in equilibrium. This would lead to ask prices exceeding the monetary value of shares held by truly green activists. In this setting, increased initiation costs, c, by screening out pseudo-green activists, could favor the viability of activism. We will discuss this scenario more in subsequent drafts of this paper.

not believe that the ask prices they post will have any effect on whether the activist succeeds in purchasing a stake and launching the campaign. Thus, they conjecture that green payoffs will not vary with the ask price they set. This implies, as shown by equation (2), that the ask price set by the atomistic shareholders does not depend on whether their preferences are brown or green. If an atomistic shareholder sells to the activist at ask price p_a, the monetary payoff to the shareholder equals $p_a\,\mathrm{d}n$, where $\mathrm{d}n \simeq 0$ represents the infinitesimal share endowment of an atomistic shareholder. If an atomistic shareholder does not sell, her payoff equals the conjectured value of her share endowment, $\big(\pi\,\rho\,V(S) + (1-\pi\,\rho)\,V(F)\big)\,\mathrm{d}n$. Bertrand competition among atomistic shareholders implies that the activist can purchase shares at the lowest price consistent with selling being a best response for the atomistic shareholders, i.e., the equilibrium ask price p_0, is given by

$$p_0 = \pi\,\rho\,V(S) + (1-\pi\,\rho)\,V(F). \tag{5}$$

Thus in an activism equilibrium, the activist acquires b/p_0 shares. The activist's valuation of the firm is the same as the atomistic shareholder's valuation, namely $\pi\,\rho\,V(S)+(1-\pi\,\rho)\,V(F)$. Thus, equation (5) shows that, if the activist initiates, the expected wealth of the activist equals b. The activist's green payoff equals $\pi\,\rho\,G(S)+(1-\pi\,\rho)\,G(F)$. If the activist does not initiate, his monetary payoff equals $b+c$ and his green payoff equals $G(F)$. Thus, initiation is a best reply for the activist if and only if

$$\pi\,\rho\,\beta\,\big(G(S)-G(F)\big) \geq c. \tag{6}$$

We term this condition the *initiation condition*.

Now consider the question of whether the activist will launch when a proposal has been developed. Conditional on initiating, the payoff to the activist from launching when a proposal has been developed is $\frac{b}{p_0}\big(\rho\,V(S)+(1-\rho)\,V(F)\big)+\beta\,\big(\rho\,G(S)+(1-\rho)\,G(F)\big)$, the payoff from not launching is $\frac{b}{p_0}\,V(F)+\beta\,G(F)$. Hence, using equation (5), we see that the condition for launching the campaign assuming a proposal has been developed, which we term the *launching condition*, is

$$\beta\,\big(G(S)-G(F)\big)-\big(V(F)-V(S)\big)\,\frac{b}{p_0} \geq 0, \text{ where } p_0 = \pi\,\rho\,V(S)+(1-\pi\,\rho)\,V(F). \tag{7}$$

The initiation and launching conditions are two necessary conditions for an activism equilibrium. Another necessary condition is that the share acquisition by the activists is feasible, i.e. activist's demand is less than the potential supply of shares provided by the atomistic shareholders, $n^{\text{At.}}$, and the activist can acquire sufficient shares to qualify for submitting a proposal, $\underline{n}$. These constraints impose another necessary condition for an activism equilibrium:

$$\underline{n} \leq \frac{b}{p_0} \leq n^{\text{At.}}, \quad \text{where } p_0 = \pi\rho\, V(S) + (1 - \pi\rho)\, V(F). \tag{8}$$

The conditions for an activism equilibrium can be simplified by noting that if the ownership condition, (8), is satisfied, the launching condition is redundant. First note that b/p_0 equals the number of shares acquired by the activist, the firm has one share outstanding, so the number of shares acquired by the activist, b/p_0, must be less than one to satisfy the ownership condition, (8). Equation (4) implies that $\beta\,\big(G(S) - G(F)\big) > \big(V(F) - V(S)\big)$. Therefore,

$$\beta\,\big(G(S) - G(F)\big) - \big(V(F) - V(S)\big)\,\frac{b}{p_0} > \big(V(F) - V(S)\big)\big(1 - \frac{b}{p_0}\big) > 0.$$

Next note that the initiation condition, equation (6), can never be satisfied if the probability of success, $\rho = 0$. Thus, if the initiation condition is satisfied, $\rho > 0$. Also note that the activism strategy is the only activist strategy that results in a positive probability of a proposal being adopted. Hence, we have established the following preliminary result.

Lemma 1. *An equilibrium exists in which the green proposal is adopted with positive probability if and only if the initiation condition, equation* (6) *and the ownership condition, equation* (8)*, are satisfied.*

4 Voting phase: Single universal owner

In this section, we consider the case where there is a single universal owner, i.e., $K = 1$, and a proposal has been submitted. Because only the atomistic shareholders trade with the activist, the combined holdings of the activist and the atomistic shareholders will equal the holdings of the atomistic shareholders before trade, $n^{\text{At.}}$. Because one share is outstanding, the universal owner's shareholding, which we represent with N^U, equals $1 - n^{\text{At.}}$. Thus, to ensure the universal owner is decisive, we assume in this section that $n^{\text{At.}} < 1/2$. As we discuss in detail in the following section on voting by multiple universal owners, this assumption is much stronger than required to ensure universal owner control in real-world proxy contests.

The universal owner decides between voting yes, $v = 1$, or no, $v = 0$ on the proposal. The monetary payoff to the universal owner has two components, the value of the universal owner's stake in the firm, $N^U V$, which depends on whether the proposal succeeds, S, or fails, F, and a reputational cost, denoted by $R > 0$. This cost is incurred whenever the owner votes no on the proposal. If the owner is green, the owner's utility also contains a green payoff component, G, which also depends on the success or failure of the proposal. Thus the utilities of a green, U^G, and brown, U^B, single universal owner are given by

$$U^G(v,x) = N^U\, V(x) - R\,\mathbb{1}_{\{v=0\}}(v), \quad U^B(v,x) = N^U\, V(x) - R\,\mathbb{1}_{\{v=0\}}(v), \quad v \in \{1,0\},\; x \in \{S,F\}.$$

Because there is only one universal owner, the universal owner decides the outcome of the vote, i.e., $x = S$ if and only if $v = 1$. Again, to avoid consideration of the trivial case where the proposal is always accepted, we assume that, even factoring in the reputation penalty, a brown owner prefers proposal failure, i.e., $U^B(0,F) > U^B(0,S)$, i.e.,

$$N^U V(F) - R > N^U V(S).$$

Condition (4) ensures that the green owner prefers proposal success. The probability that the universal owner is green is given by $\gamma \in (0,1)$. Thus, when there is a single universal owner, the voting phase is trivial: the proposal succeeds with probability $\rho = \gamma$. These results are obvious but, for the sake of comparison with the multiple universal owner case, we record them below.

Result 1. When there is a single universal owner, the proposal passes if and only if the universal owner is green, which occurs with probability γ. Thus, the probability that a green proposal is adopted equals $\pi\gamma$, and the adoption probability is strictly increasing in green sentiment, γ.

5 Voting phase: Multiple universal owners

5.1 Assumptions: Ownership structure

Assume that there are K universal owners and let $\mathcal{K} := \{0,1,2,\ldots,K\}$. Let n_i^U represent the shareholders of universal owner $i \in \mathcal{K}$. Assume that the universal owner share block are equal sized, i.e., $n_i^U = N^U/K$, $i \in \mathcal{K}$, where, as in the previous section, N^U represents the total shareholdings of the universal owners. The notation n_i^U is thus simplified to n^U henceforth. The assumption that block sizes are exactly equal is not essential for the analysis. However, it does yield a simple necessary and sufficient condition for the number of universal owners voting yes alone determining the effect of universal owner votes on the outcome. Weaker assumptions on universal owners' block sizes can also ensure the number of yes votes alone determines the effect of universal owners on the outcome. However, such conditions are more complex and also impose some homogeneity restrictions, albeit weaker, on the size of blocks. If block sizes varied greatly, then universal owners' effect on the voting outcome would be a function of the subsets of universal owners who vote yes. This would greatly complicate the analysis.

We assume, consistent with actual practice, the universal owners always vote their shares (Brav et al., 2022). We also assume that universal owners are decisive, i.e., whether a proxy proposal passes depends only on the votes of universal owners. Under plurality voting, the standard voting rule in corporate voting, universal owners will be decisive if and only if the number of universal owner yes votes at least equals m, where $m = \lfloor K/2 \rfloor + 1$. This condition ensures that the proposal will pass when supported by the majority of universal owners, and $n^{\text{At.}}/n^U < K - 2\lfloor K/2 \rfloor$. This condition ensures that, regardless of the votes of other shareholders, the proposal will not pass whenever less than m universal owners support the proposal.

Remark 1 (Decisiveness). These conditions will be satisfied if $n^{\text{At.}} < n^U$ and K is odd. We assume that these conditions are satisfied in the subsequent analysis. We also restrict attention to cases where no single universal owner is decisive, i.e., $m > 1$. Collectively these restrictions imply that K is odd, $m \geq 2$, and $K \geq 3$. Next, note that the fact that, K is odd implies that $K-1$ is even, thus $\lfloor K/2 \rfloor = (K-1)/2$, hence the threshold for success, m, equals $(K-1)/2+1$.

Our conditions for decisiveness are probably much stronger than required in real-world corporate voting. They ensure that even if other shareholders block vote against the majority of universal owners, they cannot affect the outcome of corporate votes. In fact, non-institutional investors do not block vote. Moreover, on average, only 30% of non-institutional shares are voted while virtually 100% of institutional shares are voted (Brav et al., 2022). Hence, because non-institutional investors hold approximately 30% of the shares of large U.S. firms, they represent about 9% of the shares voted in corporate proxy contests. Thus, although our implementation of universal owner decisiveness is quite stylized, universal owner decisiveness in proxy contests plausibly approximates many corporate votes. Because the ESG activist is one of the other shareholders, our analysis implicitly assumes that the ESG activist's stake is small and thus the ESG activist cannot affect the outcome of the proxy contest through his proxy votes. In fact, the shareholdings of ESG activist making proxy proposals are frequently quite small (Dimson et al., 2015; Barko et al., 2021; Lopez de Silanes et al., 2022).

To simplify notation, let $w(x)$, $x = S, F$, represent the value of an individual universal owner's stake in the firm conditioned on the success, S, or failure, F, of the proposal, i.e., $w(x) := n^U\, V(x)$, $x = S, F$. Also let $\Delta w := w(F) - w(S)$ represent increase of value accrued by i if the proposal fails and let $\Delta G = G(S) - G(F)$ be increase in the green payoff accrued by i if the proposal passes. Let r_i be the reputation cost incurred by the universal owner i if i votes yes. Let v_i represent the vote of universal owner $i \in \mathcal{K}$, where $v_i = 1$ if the vote is yes, and $v_i = 0$ if the vote is no. Let $\boldsymbol{v} := (v_1, v_2, \ldots v_K)$ represent the vector of universal owner votes. Using these definitions and the decisiveness condition (see Remark 1) we can express the utility of green and brown owners, u_i^G and u_i^B, as follows:

$$u_i^G(\boldsymbol{v}) := w(F) + \beta\, G(F) + (\beta\, \Delta G - \Delta w)\, \mathbb{1}_{\sum_{\mathcal{K}} v_i \geq m} - r_i\, \mathbb{1}_{v_i = 0}, \tag{9}$$

$$u_i^B(\boldsymbol{v}) := w(F) - \Delta w\, \mathbb{1}_{\sum_{\mathcal{K}} v_i \geq m} - r_i\, \mathbb{1}_{v_i = 0}. \tag{10}$$

Condition (2) ensures that $\beta\, \Delta G - \Delta w > 0$. Voting for the proposal weakly increases the probability that the proposal passes and avoids the reputation cost triggered by a yes vote ($v_i = 1$). Thus, equation (9) shows that that voting for the proposal is a strictly dominant strategy when the universal owner is green. For this reason, the voting game is equivalent to a game where all universal owners are brown. For each universal owner, nature makes an independent draw from a Bernoulli distribution, based on this draw, with probability $\gamma \in (0,1)$, nature privately informs the universal owner that nature will vote her shares in favor of the proposal, and with probability $1-\gamma$, privately informs the universal owner that she can decide how to vote her proxies. Thus, we can focus all of our analysis on the voting strategies of brown universal owners.

As in the single universal owner case, we assume that, even net of the reputation penalty incurred by the universal owner, each brown universal owner, if she alone decided the outcome of the vote, would oppose the proposal, i.e., we assume that $r_i < \Delta w$. Also, to further simplify notation, let $y_i = r_i/\Delta w$; y_i represents *normalized reputation cost* of voting no on the proposal incurred by universal owner i. Our assumptions imply that $y_i \in (0,1)$ for all $i \in \mathcal{K}$.

Each universal owner casts a vote, either yes or no on the proposal. Voting yes produces 1 yes vote and voting no produces 0 yes votes. Let σ_i represent the probability that a universal owner, when brown, votes yes. The probability that universal owner i casts a yes vote, which we represent with t_i, is given by $t_i = t(\sigma_i)$, where $t:[0,1]\to[0,1]$ is the function $t(\sigma) = \gamma + (1-\gamma)\sigma$, $\sigma \in [0,1]$. Thus, the vote of each universal owner i is a Bernoulli distributed random variable, $\tilde{B}_i$, equal to 1 with probability $t(\sigma_i)$ and equal to 0 with probability $1-t(\sigma_i)$.

Let $\boldsymbol{t} := (t_1, t_2, \ldots t_K)$ represent the vector of yes-vote probabilities. Since the mixed strategies of the universal owners are jointly independent and independent of nature's brown/green type assignment, the sum of the votes, $\tilde{S}(t)$, is a Poisson-Binomial (PB) random variable, i.e.,

$$\tilde{S}(\boldsymbol{t}) := \sum_{k\in\mathcal{K}} \tilde{B}(t_k). \tag{11}$$

We will denote the sum of yes votes, $\tilde{S}(\boldsymbol{t})$, excluding universal owner i with $\tilde{S}^{-i}(\boldsymbol{t})$, and $\tilde{S}(\boldsymbol{t})$, excluding universal owners i and j, $j \neq i$, by $\tilde{S}^{-ij}(\boldsymbol{t})$, i.e.,

$$\tilde{S}^{-i}(\boldsymbol{t}) := \sum_{k\in\mathcal{K}\setminus\{i\}} \tilde{B}(t_k), \tag{12}$$

$$\tilde{S}^{-ij}(\boldsymbol{t}) := \sum_{k\in\mathcal{K}\setminus\{i,j\}} \tilde{B}(t_k). \tag{13}$$

Note that $\tilde{S}(\boldsymbol{t}) \geq m$ if and only if universal owner i votes yes and at least $m-1$ other universal owners vote yes or universal owner i votes no, and at least m other universal owners vote yes. Hence,

$$\mathbb{P}[\tilde{S}(\boldsymbol{t}) \geq m] = t_i\,\mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) \geq m-1] + (1-t_i)\,\mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) \geq m] = \mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) \geq m] + t_i\,\mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) = m-1]. \tag{14}$$

Using equation (14), it is apparent that

Lemma 2. *If $\tilde{S}(\boldsymbol{t})$ is PB$(t_1, t_2, \ldots, t_K)$ distributed, then*

$$\frac{\partial}{\partial t_i}\mathbb{P}[\tilde{S}(\boldsymbol{t}) \geq m] = \mathbb{P}[\tilde{S}^{-i}(\boldsymbol{t}) = m-1],$$
$$\frac{\partial^2}{\partial t_i \partial t_j}\mathbb{P}[\tilde{S}(\boldsymbol{t}) \geq m] = \mathbb{P}[\tilde{S}^{-ij}(\boldsymbol{t}) = m-2] - \mathbb{P}[\tilde{S}^{-ij}(\boldsymbol{t}) = m-1], \quad \text{if } i \neq j,$$
$$\frac{\partial^2}{\partial t_i^2}\mathbb{P}[\tilde{S}(\boldsymbol{t}) \geq m] = 0.$$

5.2 Nash equilibria

The "greenness" of other universal owners is private information. Thus, a brown universal owner does not know which other universal owners are green. Having rational expectations, she conjectures each of the other universal owners is green with probability γ. Suppose that the candidate equilibrium strategy is $\boldsymbol{\sigma}$. Let $\tau : [0,1]^K \to [0,1]^K$ be the map defined by

$$\tau(\boldsymbol{\sigma}) := \big(t(\sigma_1), t(\sigma_2), \ldots, t(\sigma_K)\big).$$

The distribution of yes votes under strategy vector $\boldsymbol{\sigma}$ is Poisson-Binomial (PB) where the yes vote probability for each Bernoulli random variable is given by $t_i = t(\sigma_i)$. Hence the distribution of yes votes is $\mathrm{PB}\big(t(\sigma_1), t(\sigma_2), \ldots, t(\sigma_K)\big) = \mathrm{PB}(\tau(\boldsymbol{\sigma}))$. Let u_i represent the payoff to universal owner i when i is brown in the mixed strategy extension of the brown owners payoff function defined by equation (10). The linearity of payoffs in mixed strategies implies that the payoff to a brown universal owner if the brown common who plays σ_i, given that other universal owners play $\boldsymbol{\sigma}$, is given by

$$u_i(\sigma_i|\boldsymbol{\sigma}^{-i}) = u_i(0|\boldsymbol{\sigma}^{-i}) + \sigma_i\left(u_i(1|\boldsymbol{\sigma}^{-i}) - u_i(0|\boldsymbol{\sigma}^{-i})\right).$$

The first term in this expression represents a brown owner's payoff from voting no. The second term represents the difference between a brown owner's payoff when she votes yes and votes no. The difference between the yes and no payoffs results from two effects: (a) voting yes avoids the reputation cost but (b) increases the probability that the proposal will pass, which reduces the brown universal owners' payoff by Δw. The proposal will pass with i's support but not without i's support if and only if $m-1$ other universal owners vote for the proposal. Thus observations verify that

$$u_i(0|\boldsymbol{\sigma}^{-i}) = w_F - \Delta w\,\mathbb{P}[\tilde{S}^{-i}(\tau(\boldsymbol{\sigma})) \geq m] - r_i,$$
$$u_i(1|\boldsymbol{\sigma}^{-i}) - u_i(0|\boldsymbol{\sigma}^{-i}) = r_i - \Delta w\,\mathbb{P}[\tilde{S}^{-i}(\tau(\boldsymbol{\sigma})) = m].$$

Thus, expressed in terms of normalized reputation costs, y_i, the payoff to i from strategy σ_i given that the other brown owners play $\boldsymbol{\sigma}$ is given by

$$\begin{aligned} u_i(\sigma_i|\boldsymbol{\sigma}^{-i}) = u_i(0|\boldsymbol{\sigma}^{-i}) + \sigma_i \left(u_i(1|\boldsymbol{\sigma}^{-i}) - u_i(0|\boldsymbol{\sigma}^{-i})\right) = \\ u_i(0|\boldsymbol{\sigma}^{-i}) + \sigma_i \Delta w \left(y_i - \mathbb{P}[\tilde{S}^{-i}(\tau(\boldsymbol{\sigma})) = m-1]\right). \end{aligned} \tag{15}$$

Next note that $u(0|\boldsymbol{\sigma}^{-i})$ is constant in σ_i as is the term in parenthesis on the right-hand-side of the last line of equation (15). Thus, the set of best responses of i to $\boldsymbol{\sigma}$, which we represent by BR_i is given by

$$\mathrm{BR}_i(\boldsymbol{\sigma}) = \begin{cases} \{1\} & y_i - \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1] > 0, \\ [0,1] & y_i - \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1] = 0, \\ \{0\} & y_i - \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1] < 0. \end{cases} \tag{16}$$

The best response correspondence, BR, for the game is given by

$$\mathrm{BR}(\boldsymbol{\sigma}) := \left(\mathrm{BR}_1(\boldsymbol{\sigma}), \mathrm{BR}_2(\boldsymbol{\sigma}), \ldots, \mathrm{BR}_K(\boldsymbol{\sigma})\right). \tag{17}$$

A *Nash equilibrium of the voting game*, is a strategy vector, $\boldsymbol{\sigma}^*$, satisfying $\boldsymbol{\sigma}^* \in \mathrm{BR}(\boldsymbol{\sigma}^*)$.

5.3 The potential for the game and its properties

Define the function $\Pi : [0,1]^K \to \mathbb{R}$ by

$$\Pi(\boldsymbol{\sigma}) = \Delta w \sum_{k \in \mathcal{K}} \sigma_k \, y_k - \frac{\mathbb{P}[S(\tau(\boldsymbol{\sigma})) \geq m]}{1-\gamma}. \tag{18}$$

Noting that $\frac{\partial}{\partial \sigma_i} t(\sigma_i) = 1-\gamma$ and $\frac{\partial}{\partial \sigma_i} t(\sigma_j) = 0,\ j \neq i$, we see the composition rule for differentiation and Lemma 2 imply that

$$\frac{\partial}{\partial \sigma_i} \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) \geq m] = (1-\gamma)\, \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1].$$

Thus, using the definition of Π (equation (18)) we see that

$$\frac{\partial}{\partial \sigma_i} \Pi(\boldsymbol{\sigma}) = \Delta w \left(y_i - \mathbb{P}[S^{-i}(\tau(\boldsymbol{\sigma})) = m-1]\right). \tag{19}$$

Differentiation of equation (15) and inspection of (19) imply that

$$\frac{\partial}{\partial \sigma_i} u_i(\sigma_i|\boldsymbol{\sigma}^{-i}) = \frac{\partial}{\partial \sigma_i} \Pi(\boldsymbol{\sigma}). \tag{20}$$

Thus, Π is an exact potential for the voting game which implies that voting game is an exact potential game (Monderer and Shapley, 1996). Potential games are games in which all agents' gain from changing strategies, in our case, from voting no to voting yes is determined by a single function, the potential of the game, in our case Π. Brown universal owners act as if they control one component of a single function, Π, and use their control to select strategies that maximize Π. The potential function Π is not a welfare function. Although the potential captures each brown universal owner's effect on the incentives of other brown universal owners, the potential does not capture "pure externalities," i.e., the effects a common owner i's vote on the welfare of universal owner $j \neq i$ that accrue to j regardless of her vote. These pure externalities do not factor into whether a strategy vector is a Nash equilibrium or a potential maximizer but would factor into a social welfare function. Also, the potential is not unique. Adding any function that is independent of the strategic decisions of the players to a potential function yields another potential function.

Potential maximizers are always Nash equilibria. To see this, note that first-order necessary conditions for $\boldsymbol{\sigma}^*$ being a local maximizer of Π, i.e.,

$$\frac{\partial}{\partial \sigma_i}\Pi(\boldsymbol{\sigma}^*) > 0 \Longrightarrow \sigma_i = 1,$$
$$\frac{\partial}{\partial \sigma_i}\Pi(\boldsymbol{\sigma}^*) < 0 \Longrightarrow \sigma_i = 0,$$
$$\frac{\partial}{\partial \sigma_i}\Pi(\boldsymbol{\sigma}^*) = 0 \Longrightarrow \sigma_i \in [0,1],$$

are identical to the best response conditions for a Nash equilibrium (see equation (16)). Thus, any strategy vector, $\boldsymbol{\sigma}$, that is a local maximizer of the potential function is a Nash equilibrium strategy vector. However, because the first-order conditions are not sufficient to ensure that a strategy vector is a local maximizer of the potential, Nash equilibria need not be potential local maximizers, and *a fortiori*, Nash equilibria need not be potential maximizers. So the set of potential maximizers is subset of the set of Nash equilibria, and thus potential maximization can be viewed as a Nash equilibrium refinement (Monderer and Shapley, 1996).

In potential games, such as coordination games (Chen and Chen, 2011), congestion games (Sandholm, 2002), voting games (Bouton et al., 2021), potential maximization is commonly used to refine the set of Nash equilibria. In potential games, potential maximizers have many "nice properties" with respect to learning dynamics, stability, and robustness to perturbations of the information environment. Young (1993, 2020) shows in a noisy learning setting where agents have a vanishingly small probability of making errors, agents' strategy vectors converge to potential maximizing strategies. Carbonell-Nicolau and McLean (2014) show that the set of potential maximizers contains a strategically stable set of pure strategy equilibria and that, in generic potential games, potential maximizers are perfect and essential Nash equilibria. Ui (2001) shows that potential maximizers are robust to the introduction of incomplete information.

There are many Nash equilibria of the voting game, We focus our attention on potential maximizers, i.e., strategy vectors that maximize the potential function. It is well known that, in potential games, pure strategy potential maximizers always exist. In this voting game, we can make a somewhat stronger assertion, mixed strategy equilibria are almost never potential maximizers.

Lemma 3. *A pure strategy vector (i.e. $\sigma_i \in \{0,1\}$) that maximizes the potential function, Π, always exists. Let $\bar{\boldsymbol{\sigma}}$ be a strategy vector that maximizes the potential, Π. Let $\mathcal{R}$ be the set of brown universal owners who randomize, i.e., $\mathcal{R} := \{i \in \mathcal{K} : \bar{\sigma}_i \neq 0 \text{ or } 1\}$. Then, for all $i, j \in \mathcal{R}$, $i \neq j$, $y_i = y_j$ and, if the number of randomizing universal owners, $\#\mathcal{R}$, is greater than one, then*

$$\gamma \in \left\{ \frac{m-1-j}{K-1-j} : j = 0, 1 \ldots m-2 \right\}.$$

The set of $\gamma \in [0,1]$ and $\mathbf{y} \in [0,1]^K$ such that $\boldsymbol{\sigma}$ is a potential maximizer and any universal owner plays a mixed strategy has measure 0.

Thus, mixed strategy vectors that maximize the potential rarely exist and strategy vectors where two or more brown owners randomize are extremely rare and are only possible when the exogenous "greenness" parameter, γ, takes one of its $m-1$ possible values on the unit interval continuum. The intuition for the Lemma is illustrated by Figure 1. The example is symmetric strategy vector, $\boldsymbol{\sigma}^*$, in a symmetric parametrization of the game, i.e., $y_i = y$ for all $i \in \mathcal{K}$. In this equilibrium, all brown universal owners vote yes with probability $\sigma_i^* = \sigma^* = 3/25$. The graph plots the value of the potential function (on the z-axis) when σ_1 and σ_2 are allowed to vary (on the x and y-axes), holding the other brown universal owners strategies at their equilibrium values. The fact that σ^* is a best response for brown universal owners 1 and 2 is verified by the fact that moving along the red (blue) line, which leaves the strategy of the other brown owner fixed, does not increase the potential function. By definition, the derivative of the potential function with respect to σ_i equals the derivative of a brown owner's payoff with respect to σ_i. So, neither i nor j can gain by unilaterally deviating from $\boldsymbol{\sigma}^*$. Because the game is symmetric, unilateral deviations will also not increase the other brown owners' payoffs. Hence, $\boldsymbol{\sigma}^*$ is a Nash equilibrium. However, moving along the black line, i.e., in the a direction that increases (reduces) σ_1 and, at the same time reduces (increases) σ_2 by an equal amount, increases the potential function. This symmetric Nash equilibrium is thus a saddle point of the potential function.

What is the intuition behind the example? First note that the probability of any given universal owner voting yes equals $t(\sigma^*) = \gamma + (1-\gamma)\,\sigma^* = 1/11 + (10/11) \times (3/25) = 1/5$. So, the expected number of yes votes equals $K\,t(\sigma^*) = 1$. Despite the expected number of yes votes being small relative to the passing threshold, $m = 3$, because of brown owner randomization, there is an appreciable probability the proposal will pass. The probability that the proposal passes can be reduced by reducing the dispersion of the yes-vote distribution, by increasing

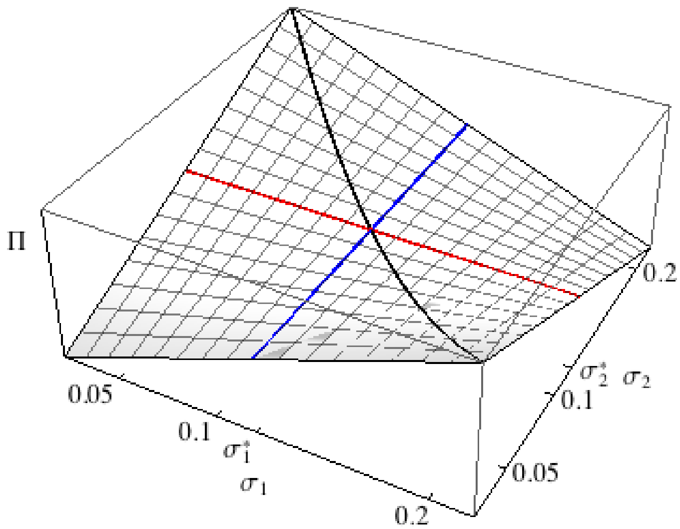


Figure 1: Mixed strategy Nash equilibria are not potential maximizing. The figure presents the value of the potential function when brown universal owners 3, 4, ... K's strategies are fixed at $\sigma_i = 3/25$ and brown universal owners 1 and 2's strategies, σ_1 and σ_2, are allowed to vary around 3/25. The parameters of game are $\gamma = 1/11$, $K = 5$, $m = 3$, and $y_i = 96/625$ for all $i \in \mathcal{K}$.

one brown universal owner's probability of voting yes and reducing another brown universal owner's probability of voting yes by an equal amount. Thus shift does not affect, $y \sum_k \sigma_k$, and thus will increase the potential, Π. Because a pure strategy potential maximizer always exists and, generically, mixed strategy maximizers do not, in the subsequent analysis, we consider only pure strategy vectors.

5.4 Characterizations of potential maximizers

5.4.1 Preliminary results

Again, we need some facts, this time about the binomial distribution. Let $b(n;N,t)$ represent the probability of exactly n success realizations of a Binomial distribution with N trials and success probability t:

$$b(n;N,t) = \mathbb{P}[X = n] = \begin{cases} 0 & n > N \\ t^n(1-t)^{N-n}\binom{N}{n} & 0 \le n \le N \\ 1 & n < 0. \end{cases} \tag{21}$$

Let $\hat{B}$ represent the probability that a binomially distributed random variable X is greater than or equal to n,

$n = 0, 1, \ldots N$.[8] That is, define $\hat{B}$ as follows: For an integer n, $N \in \{0, 1, 2, 3, \ldots\}$, and $t \in [0, 1]$,

$$\hat{B}(n;N,t) = \mathbb{P}[X \geq n] := \begin{cases} 0 & n > N \\ \sum_{k=n}^{N} b(k;N,t) & 0 \leq n \leq N \\ 1 & n < 0, \end{cases} \tag{22}$$

The key result about the Binomial distribution that we will use in the sequel is presented below.

Fact 1. For integers, K, n such that $K \geq n \geq 1$ and $t \in (0, 1)$,

$$\frac{d}{dt}\hat{B}(n;N,t) = N\, b(n-1;N-1,t).$$

5.4.2 o-strategies and Π_o functions

Given Lemma 3, we can concentrate on pure strategies when finding potential maximizing Nash equilibria. Because universal owners have only two pure strategies: vote yes, $\sigma = 1$, or vote no, $\sigma = 0$, determining the set of universal owners who vote yes determines the effect of brown owners on the probability that the proposal passes. Thus, the effect of each universal owner vote is the same. However the reputation cost saving, r_i, resulting from a yes vote varies across universal owners. Inspection of the potential function shows that its maximization requires that the set of universal owners who vote yes when brown to contain the universal owners with the largest reputation costs. Thus, without loss of generality, and with a great deal of notational simplification, assume henceforth that reputation costs are weakly decreasing in the index of the universal owner, i.e.,

$$\Delta w \geq r_1 \geq r_2 \geq r_3 \ldots r_{K-1} \geq r_K > 0.$$

Thus assumption implies that normalized reputation costs, y, are also weakly decreasing in the index of the universal owner.

For each $o \in \mathcal{K}$, define an o-strategy as follows:

$$o\text{-strategy} : \begin{cases} \text{if } i \in \{1, 2, \ldots, o\} & \text{universal owner } i \text{ votes yes, i.e., } \sigma_i = 1, \text{ when } i \text{ is brown,} \\ \text{if } i \in \{o+1, o+2, \ldots, K\} & \text{universal owner } i \text{ votes no, i.e., } \sigma_i = 0, \text{ when } i \text{ is brown.} \end{cases} \tag{23}$$

These arguments show that one of these o-strategies is a potential maximizer. Next note when o is greater than

[8] $\hat{B}$ is not equal to the survival function (i.e., complementary distribution function) of a Binomially distributed random variable. The survival function of an (N, p) binomial distribution represents $\mathbb{P}[X > n] = \mathbb{P}[X \geq n-1]$. So if we used the survival function, the threshold for success would be $m-1$, which might be confusing.

$m-1$ but less than K, then an o-strategy is not a potential maximizer. Under such strategies, the probability that the proposal passes equals 1 yet some brown owners vote no, and thus incur a reputation penalty without affecting the outcome. Thus, when identifying the o-strategies that maximize the potential, we need not consider o-strategies where $o \in \{m+1, m+2, \ldots, K-1\}$. Hence, the set of candidate pure strategy potential maximizers is given by o strategies where $o \in \mathcal{O} =: \{0, 1, 2, \ldots m-1, K\}$.

Henceforth, an o-strategy refers to o-strategy in which $o \in \mathcal{O}$. We will term the $o = K$-strategy the *capitulation strategy*, where brown owners vote for the proposal even though each brown owner is better off if the proposal fails. We call all o-strategies such that $o \neq K$, *non-capitulation strategies*. We term a non-capitulation strategy where $o \neq 0$ a *partial resistance strategy*, and term the $o = 0$ strategy the *complete resistance strategy* and the $o = m-1$ the *minimal resistance strategy*.

Under an o-strategy, the distribution of votes has the following properties, for $i \in \mathcal{O}$, universal owner i votes yes when brown. Because green universal owners always vote yes, the universal owners in $\{1, 2, \ldots o\}$ will always cast o yes votes. The $K-o$ universal owners in $\{o+1, o+2, \ldots, K\}$ will vote no ($\sigma_i = 0$) if they are brown and vote yes ($\sigma_i = 1$) if they are green. Thus, the sum of the universal owners' yes votes from universal owners in $\mathcal{K} \backslash \mathcal{O}$ is a Binomially distributed random variable with $N = K-o$ and success probability $t = \gamma$. Let Z_o represent this random variable. Hence, the proposal will pass if and only if $o + Z_o \geq m$, or equivalently, $Z_o \geq m-o$. Hence, probability that the proposal will pass is thus given by

$$\mathbb{P}[Z_o \geq m-o] = \hat{B}(m-o; K-o, \gamma).$$

Consequently, the value of the potential if brown universal owners play strategy $o \in \mathcal{O}$, which we represent by Π_o, is given by

$$\Pi_o = \Delta w \left(\Sigma_1^o - \frac{\hat{B}(m-o, K-o, \gamma)}{1-\gamma} \right), \quad \text{where } \Sigma_1^o := \sum_{i=1}^{o} y_i. \tag{24}$$

The arguments developed thus far establish our first basic characterization of potential maximizers.

Proposition 1. *There exists an o-strategy, $o \in \mathcal{O}$, such that o maximized the potential, Π, i.e.,*

$$\max_{\sigma \in [0,1]^K} \Pi(\boldsymbol{\sigma}) = \Pi_o.$$

Let Π^* represent the maximum value of the potential under the o-strategies, i.e.,

$$\Pi^* := \max_{o \in \mathcal{O}} \Pi_o, \tag{25}$$

and let o^*equal the argmax of Π^*, i.e., the set of o-policies that attain the maximum payoff,

$$o^* := \{o \in \mathcal{O} : \Pi_o = \Pi^*\}. \tag{26}$$

Proposition 1 shows that Π^* is the maximum value for the potential and that any strategy $o \in o^*$ is a maximizer for the potential.

5.5 Properties of o-strategies

5.5.1 General properties

A few properties are presented in the following lemmas. We first consider non-capitulation strategies.

Lemma 4. *The Π_o functions of non-capitulation strategies, $o \in \mathcal{O}\backslash\{K\}$, have the following properties:*
(a) Π_o, $o \in \mathcal{O}\backslash\{K\}$, *is strictly decreasing in* γ.
(b) If $\gamma = 0$ *or* 1, *then* $\Pi_{m-1} > \Pi_{m-2} > \ldots > \Pi_1 > \Pi_0$.
(c) For $o \leq m-2$,

$$\Delta\Pi_o := \Pi_{o+1} - \Pi_o = \Delta w\,(y_{o+1} - b(m-o-1; K-o-1, \gamma)).$$

(d) For $o \leq m-3$,

$$\Delta^2\Pi_o := \Delta\Pi_{o+1} - \Delta\Pi_o =$$
$$\Delta w\big(y_{o+2} - y_{o+1}\big) + \Delta w\left((1-\gamma)^{K-m}\gamma^{m-o-2}\binom{K-o-1}{m-o-1}\left(\gamma - \frac{m-o-1}{K-o-1}\right)\right).$$

Proof. See appendix. □

Part (a) of Lemma 4 simply shows that the potential is decreasing in green sentiment γ. This result is expected. The potential measures the effect of other brown owners' actions on each others' payoffs. As γ increases, this effect diminishes. Part (b) is more interesting. It shows that, relative to other non-capitulation strategies, the minimal resistance strategy, $o = m-1$, is attractive both when green sentiment, γ, is very high and very low. However, in these two cases, minimal resistance is attractive for different reasons. When $\gamma = 0$, brown owners are sure that the proposal will succeed only when at least m brown universal owners vote yes. Because of reputation costs, the potential is maximized over non-capitulation strategies by minimizing the number of brown owners who vote no subject to the constraint that the proposal fails. Thus, having the $m-1$ brown owners with the largest reputation costs vote yes and the remaining $K-m$ brown owners vote no, ensures the proposal will fail at minimum reputation cost. When $\gamma = 1$, the proposal will pass with certainty and so potential is only affected by reputation costs, because $m-1$-strategy features the most yes votes of any non-capitulation strategy, it is the optimal non-capitulation strategy. This result suggests that payoffs under the o-strategies will not satisfy in single crossing property with respect to green sentiment, γ.

This suggestion is confirmed by part (c). Because, $o \leq m-1$, $m-o-1 \geq 1$, thus, the map $\gamma \to b(m-o-1; K-o-1, \gamma)$ is inverse-U shaped ($\nearrow\searrow$). Part (c) of the lemma shows that the crossings of the potential under two adjacent o-strategies, o and $o+1$ as γ varies is determined by $b(m-o-1; K-o-1, \gamma)$. Hence, it implies that the potentials under the two strategies will either cross (i.e., transversally intersect) twice or not at all. This mathematical result reflects the fact that one brown owner voting yes does not uniformly either increase or decrease the gain to other brown owners voting yes. When the number of yes votes is small (large), one brown owner voting yes increases (decreases) the probability that other brown owners will be marginal and thus increases (decreases) the pressure on other brown owners to vote yes. Hence, although the voting game is a potential game, it is not a coordination game or an anti-coordination game, the two most commonly analyzed types of potential games analyzed in the literature.

In contrast, as shown by part (d), second differences between adjacent o-polices do have the single crossing property with respect to γ. As we will show later, this single crossing property permits determinant characterizations of effect of γ on the optimality of non-capitulation polices. However these predictions do not relate to the intensity of resistance but rather to whether intermediate partial resistance policies, i.e., $o \in \{1, 2, \ldots, m-2\}$, are preferred to the extreme non-capitulation policies.

For a non-capitulation o-strategy to be optimal, the value of the potential under o must at least equal the value of potential under the capitulation, Π_K. Thus, a non-capitulation strategy can only maximize the potential when $\Pi_o - \Pi_K \geq 0$. Some of the properties of the difference, $\Pi_o - \Pi_K$, are provided by the following Lemma.

Lemma 5. *The differences between the non-capitulation strategies, $o \in \mathcal{O} \backslash \{K\}$, and the capitulation strategy, K, $\Pi_o - \Pi_K$, have the following properties:*

(a) When $\gamma = 0$, $\operatorname{sgn}[\Pi_o - \Pi_K] = \operatorname{sgn}[1 - \Sigma_{o+1}^K]$. When γ is sufficiently close to 1, $\Pi_K > \Pi_o$.

(b) If $o = m-1$, then $\Pi_o - \Pi_K$ is decreasing ($\searrow$) in γ.

(c) If $o < m-1$, then $\Pi_o - \Pi_K$ is inverse U-shaped ($\nearrow\searrow$) in γ.

Part (a) of Lemma c establishes the fairly obvious result that, when green pressure is sufficiently severe, the potential is maximized by brown capitulation and that, even in the absence of green sentiment, non-capitulation is only optimal when the normalized reputation cost faced by the no-voting brown owners under the non-capitulation strategy, Σ_{o+1}^K, is less than 1, the normalized effect of the proposal passing on each brown owner's wealth.

Parts (b) and (c) show that with the exception of the $m-1$ strategy of minimal resistance, the advantage of each non-capitulation strategy over capitulation is not monotonically decreasing in green sentiment, γ. However, if $\Sigma^K_{o+1} < 1$, part (a) and the ($\nearrow\searrow$) relationship between the advantage of non capitulation over capitulation, $\Pi_o - \Pi_K$, reported in part (c) show that $\Pi_o - \Pi_K$ crosses 0 from above as γ increases. So the region where non capitulation is optimal is an interval with lower end point 0. Combining this result with part (c) shows that increased green sentiment has two effects on potential maximizing policies. First, as shown by this lemma, if green sentiment is sufficiently great after the increase, increased sentiment leads to capitulation. However, if even after the increase in sentiment, capitulation is not optimal, the increased sentiment may increase resistance by brown owners, i.e., reduce o^*.

The next result uses Lemmas 4 and Lemma 5 to identify a simple necessary and sufficient condition for brown resistance to be a viable strategy at some level of green sentiment, γ. The proposition shows that if the sum of normalized resistance costs entailed by the minimum resistance strategy, Σ^K_m, at least equals 1, brown owners will always capitulate, if $\Sigma^K_m < 1$, brown owners will sometimes resist.

Proposition 2.

(a) If $\Sigma^K_m \geq 1$, then $\Pi_o - \Pi_K < 0$, for all $\gamma \in (0,1)$. Hence, the unique potential maximizing strategy is capitulation, $o^* = K$.

(b) If $\Sigma^K_m < 1$, then for $\gamma > 0$ but sufficiently small, the unique potential maximizing policy is the minimum resistance policy, i.e., $o^* = m - 1$.

5.5.2 Optimality conditions for non-capitulation strategies

Proposition 2 provides conditions for the viability of non-capitulation o-strategies. However, except for situations where green sentiment is negligible it did not provide any insight into *which* o-strategy is optimal. Because determining the optimality of any given o-strategy requires comparisons between the values of high-order polynomials, it is not possible to provide simple closed-form expressions that characterize the optimality of specific o-strategies. However, as the results in this section show, it is possible to characterize the convexity/concavity of the relationship between o-strategies and the value of the potential.

Remark 2 (Sequential convexity/concavity). We will say that the Π_o, is *concave (convex)* at o', if $\Delta^2\Pi_{o'-1} \leq (\geq) 0$. If the inequalities are strictly satisfied we will say that o' is strictly concave (convex). Unravelling the definition of Δ^2 shows that $\Delta^2\Pi_{o'-1} \leq (\geq) 0$ if and only if $(\Pi_{o'+1} + \Pi_{o'-1})/2 \leq (\geq) \Pi_{o'}$. Thus, the notion of convexity/concavity of o-sequence above corresponds to standard notions of convexity for functions defined on intervals. If for all $o \in \{1,2,\ldots,m-2\}$, Π_o is (strictly) concave at o, we will say that the map from non-capitulation strategies to the value of the potential under these strategies, $o \to \Pi_o$, is (strictly) concave. If for all $o \in \{1,2,\ldots,m-2\}$, Π_o is (strictly) convex at o, we will say that the map $o \to \Pi_o$ is (strictly) convex.

Proposition 3. *For $o \in \{1,2,\ldots,m-2\}$ and $m \geq 3$,*[9]

[9] The excluded case $K = 3$ and $m = 2$ is excluded simply because, in this case, there are only two non-capitulation strategies, $o = 0$ and $o = 1$ so convexity/concavity of the sequence of o-strategies cannot be meaningfully defined.

(a) Low green sentiment: $\gamma < 4/(K+3)$ *is a sufficient condition for the map* $o \to \Pi_o$ *being concave. If all universal owners* $m, m+1, \ldots, K$ *have the same reputation costs, i.e.,* $\Delta y_i := y_{i+1} - y_i = 0$*, for all* $i \in \{m, m+1, \ldots, K-1\}$ *this condition is also a necessary.*

(b) Intermediate green sentiment: If $\gamma \in [4/K+3, 1/2]$ *and the differences between the reputation costs of the brown owners are constant, i.e., for some constant* $c \le 0$, $\Delta y_i = y_{i+1} - y_i = c$*, for all* $i \in \{m, m+1, \ldots, K-1\}$*, the map* $o \to \Pi_o$ *is initially concave and ultimately convex, i.e., there exists no* $o_1, o_2 \in \mathcal{O} \backslash \{K\}$ *such that* $o_1 < o_2$ *and* Π *is strictly convex at* o_1 *and strictly concave at* o_2*.*

(c) High green sentiment: $\gamma > 1/2$ *is a necessary condition for the map* $o \to \Pi_o$ *being convex. If all universal owners have the same reputation cost, this condition is also sufficient.*

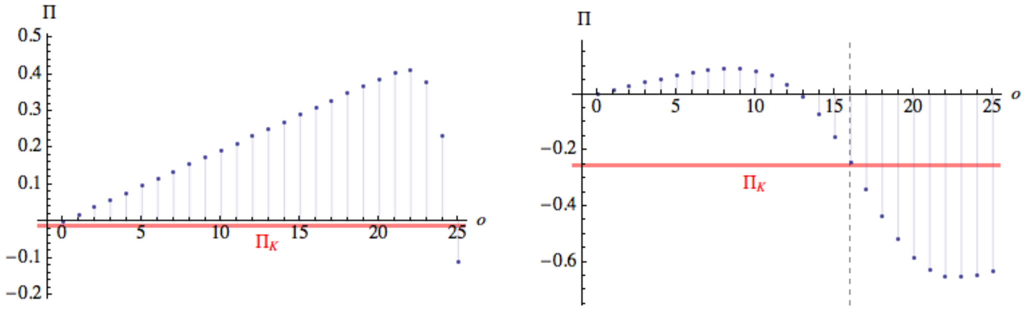
The intuition for this proposition is that convexity/concavity of $o \to \Pi$ depends on the second differences, $\Delta^2\Pi$. As shown in part (d) of Lemma 4, these differences depend on two effects. The first effect is the differences between reputation costs Δy_i (which are second differences of Σ^K_{o+1}) amongst the $K - o$ no-voting brown universal owners. Because $i \to y_i$ is decreasing, $\Delta y_i \le 0$. This effect thus favors concavity. The second effect arises from the second differences in the effect of the probability of success. This effect has, as shown in Lemma 4.(d), the same sign as $\gamma - (m-o-1)/(K-o-1)$ and thus favors concavity when γ is low and opposes concavity when γ is high. The map $o \to (m-o-1)/(K-o-1)$ is decreasing and thus, this effect favors concavity at $o-1$. It will favor concavity at o, either for all non-capitulation strategies, $o \in \{1, 2, \ldots, m-1\}$, $\gamma - (m-o-1)/(K-o-1) < 0$ (part (a)), or for some o-non-capitulation strategies $\gamma - (m-o-1)/(K-o-1) \le 0$ or $\gamma - (m-o-1)/(K-o-1) \ge 0$ (part (b)), or for all non-capitulation strategies $\gamma - (m-o-1)/(K-o-1) > 0$ (part (c)). Because the differences effect always favors concavity, the condition in part (a) is a sufficient condition and the condition in part (c) is a necessary condition. These conditions become necessary and sufficient when the reputation cost differences effect is turned off. As long as the differences effect is constant the set of non-capitulation strategies under which Π_o is concave in o is always a lower segment (possibly empty and possibly the entire set of non-capitulation strategies) of set of non-capitulation strategies. Note that the results in this proposition are not affected by level translations of reputation costs. Such translations obviously affect the optimality of non-capitulation strategies relative to the capitulation strategy but they do not affect the shape of the $o \to \Pi_o$ map.

Proposition 3 shows that when green sentiment, γ, is low, $o \to \Pi_o$ is strictly concave. The optimal capitulation strategy is unique and interior whenever $\Delta\Pi_0 > 0$ and $\Delta\Pi_{m-2} < 0$. This case is illustrated in Panel A of Figure 2. In this illustration, and in the next figure, we turn off the effect of the reputation cost differences by considering cases where all universal owners have the same reputation costs.[10]

When γ is small, brown owners' votes are very likely to be decisive. By completely resisting the proposal ($o = 0$), they can with very high probability, prevent it from passing. However, complete resistance entails significant

[10]Note also that, because we aim to make concavity/convexity visually obvious, in this figure and the following figure, the number of universal owners is unrealistically large and thus per universal owner reputation costs are also quite small.

reputation costs and only a slightly larger probability of success than partial resistance strategies. When green sentiment is intermediate, as illustrated in Panel B, the map $o \to \Pi_o$ is concave-convex. Thus, the only low partial resistance (high o) strategy that can maximize the potential over non-capitulation strategies, is the minimal resistance strategy $o = m - 1$. Otherwise, as in the case illustrated in the figure, a partial resistance strategy is optimal. However, when green sentiment is greater (Panel B), resistance is stronger (i.e., o is smaller) than in the low green sentiment case illustrated in Panel A.



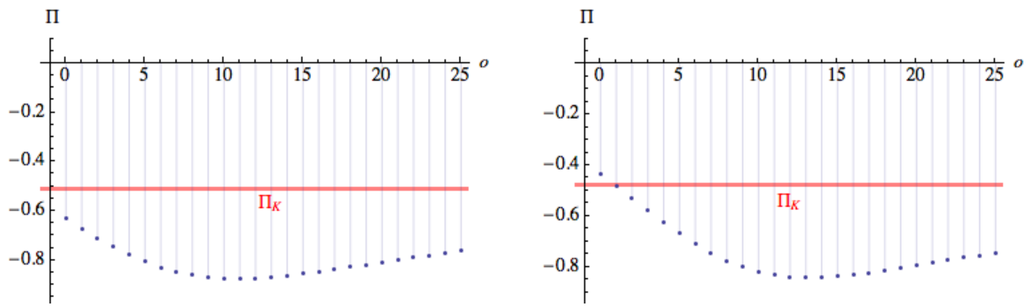
A. Low green sentiment ($\gamma = 0.03$) B. Intermediate green sentiment ($\gamma = 0.27$)

Figure 2: *Low to moderate green sentiment: partial resistance.* In the figure, the number of green universal owners supporting the proposal for different non-capitulation o-strategies is plotted on the horizontal axis. The value of the potential under these strategies is plotted on the vertical axis. The value of the potential under of the capitulation strategy, $o = K$, is represented by the red horizontal line. In both panels, $K = 51$, $m = 26$ and $\boldsymbol{y} = (0.02, 0.02, \ldots, 0.02)$.

When green sentiment is high, the map $o \to \Pi_o$ is convex. Only minimal resistance or complete resistance can be optimal non-capitulation strategies. This case is illustrated in Figure 3. In this case, unless all brown owners oppose the proposal, it is likely to pass and in many states of nature the proposal will pass even if all brown owners oppose. The map $o \to \Pi_o$ is convex and non-extreme resistance strategies are not potential maximizers. In Panel 3A of Figure 3 the optimal resistance strategy, complete resistance, $o = 0$, is dominated by capitulation. In Panel 3B complete resistance dominates capitulation. Note that this is the only case in the four panels of Figures 2 and 3 where brown owners vote sincerely based on their own economic interests and lack of green preferences.

6 Comparative statics

In this section, we consider the effects of normalized reputation costs, $\boldsymbol{y}$, green sentiment, γ, and ownership dispersion, on brown resistance to green proposals. We first consider the level of normalized reputation costs, then consider green sentiment and the heterogeneity of normalized reputation costs. Lastly, we discuss the effect of ownership dispersion.



A. High green sentiment ($\gamma = 0.48$), capitulation B. High green sentiment ($\gamma = 0.52$), complete resistance

Figure 3: *High green sentiment: capitulation vs. complete resistance.* In the figure, the number of green universal owners supporting the proposal for different non-capitulation o-strategies is plotted on the horizontal axis. The value of the potential under these strategies is plotted on the vertical axis. The value of the potential under of the capitulation strategy, $o = K$, is represented by the red horizontal line. In both panels, $K = 51$, $m = 26$. In Panel A, $\boldsymbol{y} = (0.02, 0.02, \ldots, 0.02)$ and in Panel B, $\boldsymbol{y} = (0.01, 0.01, \ldots, 0.01)$.

6.1 Level of reputation costs

Normalized reputation costs increase when the reputation cost of voting yes, r, increases or the value difference between the green proposal and the brown status quo, Δw, decreases. Increasing normalized reputation costs, increases the gain to brown owners, per unit of value difference, from avoiding the reputation penalties that result from opposing green proposals. Thus, not surprisingly, increasing normalized reputation costs reduces brown resistance, and thereby increases the probability that green proposals pass.

Lemma 6. *Suppose that $\mathbf{y}^1$ and $\mathbf{y}^2$ are two vectors representing normalized reputation costs. Then for any fixed $\gamma \in (0,1)$, if $\mathbf{y}^2 \geq \mathbf{y}^1$, then $o^*(\mathbf{y}^2) \geq o^*(\mathbf{y}^1)$.[11] Hence, increasing reputation costs increases the probability that green proposals pass.*

6.2 Green sentiment

In contrast to the effect of increasing normalized reputation costs, increasing green sentiment can actually reduce the probability that green proposals pass. This observation is formalized by the following lemma.

Lemma 7. *Holding normalized reputation costs fixed, if (a) $\Sigma_m^K < 1$ and (b) there exists $\tilde{\gamma} \in (0,1)$, such that (i) for all $\gamma \in (0, \tilde{\gamma})$, $K \notin o^*(\gamma)$ and (ii) $o^*(\tilde{\gamma}) \neq m - 1$, then the probability of success is not monotonic in γ, i.e., increased green sentiment can reduce the probability that green proposals pass.*

The intuition for this result is fairly simple: Inspecting equation (24) shows that the potential's value under a given o strategy has two components: (a) a reputation cost component representing the reduction in reputation costs associated by o universal owners voting yes even when brown, and (b) the vote outcome component

[11] In non-generic cases where either $o^*(\boldsymbol{y}^1)$ or $o^*(\boldsymbol{y}^2)$ is not singleton set, "$o^*(\boldsymbol{y}^2) \geq o^*(\boldsymbol{y}^1)$" should be interpreted as $\max o^*(\boldsymbol{r}^2) \geq \max o^*(\boldsymbol{r}^1)$ and $\min o^*(\boldsymbol{r}^2) \geq \min o^*(\boldsymbol{r}^1)$.

representing the effect of o universal owners voting yes on the outcome.

$$\Pi_o = \Delta w \left(\overbrace{\Sigma_1^o}^{\text{Rep. Cost}} - \overbrace{\frac{\hat{B}(m-o, K-o, \gamma)}{1-\gamma}}^{\text{Vote Outcome}} \right).$$

When a change in γ induces a change in the o-strategy that maximizes the potential, the change in the strategy causes the reputation cost component to make a discrete jump upward or downward. Because the function mapping γ into the maximized potential function, $\gamma \to \Pi^*(\gamma)$, is the maximum of a finite number of continuous functions of γ , namely the Π_o functions, the potential's value under the optimal policy is continuous function of γ. Hence, when a change in γ induces a change in the optimal o-strategy, to maintain the continuity of Π^*, the jump in the reputation cost component must be compensated by an equal jump in the same direction in the vote outcome component. The vote outcome component is proportional to the probability that the proposal passes at γ, $\hat{B}(m-o, k-o, \gamma)$.

When normalized reputation costs, $\boldsymbol{y}$, and green sentiment, γ, are sufficiently small, the potential-maximizing policy is the minimal resistance policy, $o = m-1$. As γ increases, the optimal non-capitulation strategy shifts to a stiffer resistance policy ($o < m-1$) with a smaller reputation cost component. If the non-capitulation policy dominates capitulation and thus is an optimal policy, at the point that the optimal o-strategy switches, reputation cost jumps down, and thus, to maintain the continuity of Π^* in γ, the probability the proposal passes must also jump down.

The non-monotone relation between green sentiment, γ, and the probability that proposal passes is illustrated in Figure 4. In the figure, when green sentiment is very low, the potential is maximized by the minimal resistance policy, $o = 2$; as green sentiment increases, resistance stiffens and the optimal resistance policy shifts from $o = 2$ to $o = 1$ and then to complete resistance, $o = 0$. Finally, green sentiment becomes so large that the proposal will pass regardless of brown opposition, at which point, the optimal strategy shifts to capitulation $o = 5$. Each time increased green sentiment causes the optimal non-capitulation strategy to switch to a higher level of resistance, the probability that the proposal passes jumps down. When brown owners abandon resistance and switch to capitulation, the probability that the proposal passes jumps up. In between jump points, increased green sentiment increases the probability that the proposal passes through its mechanical effect: increasing γ increases the probability that universal owners are green, and thus increases the expected number of yes votes.

6.3 Heterogeneous reputation costs

Because we allow reputation costs to differ across universal owners, we can also examine the effect of the dispersion of reputation costs on the passing probability. Holding total reputation costs fixed, increasing the dispersion of

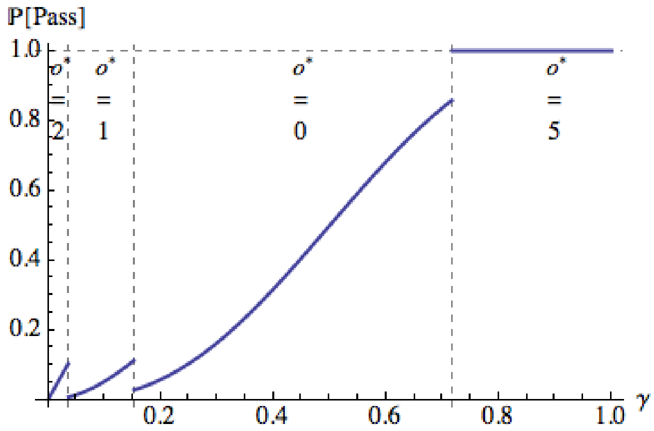


Figure 4: In the figure, $y_k = 0.10$ for all $k \in \mathcal{K}$, $K = 5$, and $m = 3$.

reputation costs can either increase or decrease o^*. Correspondingly, the level of dispersion of reputation costs can be either positively or negatively correlated with the probability of the proposal passing. We provide a numerical example here to illustrate these effects.

Assume the total number of universal owners $K = 7$. In this case, the threshold is $m = 4$. When normalized reputation costs are symmetric and given by $\boldsymbol{y} = (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1)$, we can find a feasible level of γ (relatively small), at which the potential maximizing strategy is $o^* = 2$. For example, as can be seen from Figure 5, at $\gamma = 0.1$, $o^* = 2$. In this benchmark case, the passing probability is $\rho = 8.15\%$.

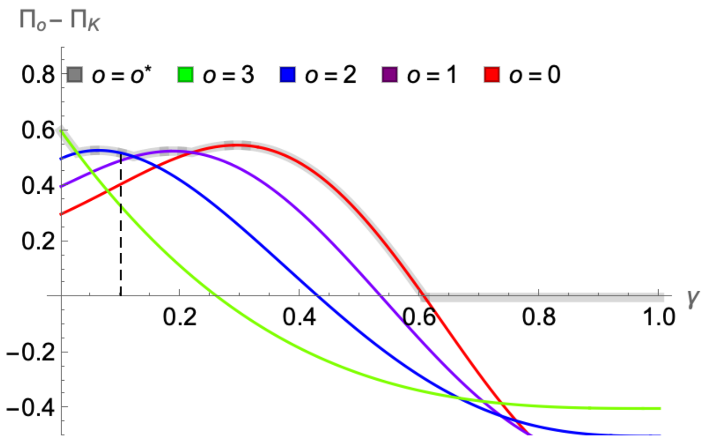


Figure 5: In the figure, $y_k = 0.1$ for all $k \in \mathcal{K}$, $K = 7$, and $m = 4$.

Using the standard measure of dispersion, majorization (see Marshall et al., 2011), consider first the more dispersed normalized reputation vector produced by transferring all reputation costs to owner 1, i.e., $\boldsymbol{y} = (0.7, 0, 0, 0, 0, 0, 0)$. It is obvious that the potential maximizing solution is $o^* = 1$, i.e., only owner 1, the only owner with reputation costs, votes yes when brown and the other owners, if brown, vote no. In this case, the probability of passing is $\rho = 1.59\%$, less than the benchmark homogeneous reputation cost case.

In contrast, if we consider another normalized reputation cost vector more dispersed than the benchmark vector, one that transfers the reputation costs of owners 4, 5, 6, and 7 more or less uniformly to owners 1, 2, and 3, $\boldsymbol{y} = (0.3, 0.2, 0.2, 0, 0, 0, 0)$, we find that the potential maximizing solution is $o^* = 3$. Because three owners are now voting yes when brown, compared to the benchmark case where only two owners, owners 1 and 2, vote yes when brown, the proposal is more likely to pass ($\rho = 34.39\%$).

6.4 Universal ownership dispersion

In Section 4, we analyzed voting when control rests in the hands of a single universal owner. In Section 5 we considered voting when control is dispersed across many universal owners. This analysis raises a question: how does the dispersion of universal owners' shareholdings impact the effectiveness of brown-owner resistance to ESG activism?

In this section we answer this question. We fix total firm value effects of proposals and the total reputation cost of resistance. Ownership dispersion is increased by increasing the number of universal owners. It is obvious that, if the division of ownership is accompanied by an extremely asymmetric division of reputation costs, increasing the number of owners can significantly reduce the probability of green proposals passing. For example, if the ownership stake of a single universal owner is divided and assigned to a large number of universal owners, and all reputation costs are assigned to one of these owners, then, for all o-strategies except $o = 0$, resistance to green proposals will be costless. In the limit, as the number of universal owners increases without bound, brown owners will block all green proposals without incurring any reputation costs.

To avoid considering trivial cases like this, we assume that total reputation costs, like total universal owner shareholdings, are divided symmetrically. Thus, we consider parameterizations of the model where

$$\forall i \in \mathcal{K},\ r_i = r := \frac{R}{K}, \quad \Delta w = \frac{\Delta W}{K}, \quad \forall i \in \mathcal{K}, \quad y_i = y := \frac{r}{\Delta w} = \frac{R}{\Delta W}. \tag{27}$$

When considering shifts in the number of owners it is convenient to parametrize the model using the threshold required for passage, m, rather than the total number of universal owners, K. Recalling that m and K are related by $K = 2m - 1$ (see Remark 1), we see that we can express the potential function as follows:

$$\Pi_o^m = \left(\frac{\Delta W}{2m-1} \right) \left(o\,y - \frac{\hat{B}(m-o, 2m-1-o, \gamma)}{1-\gamma} \right). \tag{28}$$

The superscript m explicitly represents the dependence of the potential on m. Note that the case of $m = 1$ and thus $K = 1$ represents the single universal owner case.

We consider two measures of the relationship between ownership dispersion and the effectiveness of brown opposition to green proposals: the *pass probability*, i.e., the probability that proposals pass. and the *monetary payoff*, i.e., total expected monetary payoff received by brown universal owners. We show that, despite the rather obvious free-rider problem produced by ownership dispersion, for some configurations of the model parameters, ownership dispersion reduces the pass probability and increases the monetary payoff.

6.4.1 The pass probability

The effect of increasing the dispersion, i.e., increasing m, on the pass probability depends on both (a) the effect of dispersion on the willingness of brown owners to resist green proposals and (b) the impact of dispersion on the effectiveness of resistance. Because, potential maximizing strategies are Nash equilibrium strategies, the increase in other brown owners' welfare engendered by a given brown owner's opposition to a green proposal does not affect the potential solution. This "free-rider problem" militates in favor of dispersion increasing the pass probability.

However, dispersion also impacts the effectiveness of resistance. Under the complete resistance strategy, $o = 0$, it is very easy to characterize this effect: suppose we increase dispersion by increasing the passing threshold, m, by one unit and thus increase the number of owners by two. If the two new universal owners turn out to be green, the pass probability increases, if both turn out to be brown, the pass probability decreases, and if one is brown and one is green the pass probability is not changed. Thus, the effect of increased dispersion on the pass probability will be positive (negative) if universal owners are more (less) likely to be green than brown. We record this simple result below.

Result 2. Under the complete resistance strategy $o = 0$, reducing concentration by incrementing m to $m+1$ (or equivalently K to $K+2$) strictly increases (reduces) the pass probability if $\gamma > \frac{1}{2}$ ($\gamma < \frac{1}{2}$).

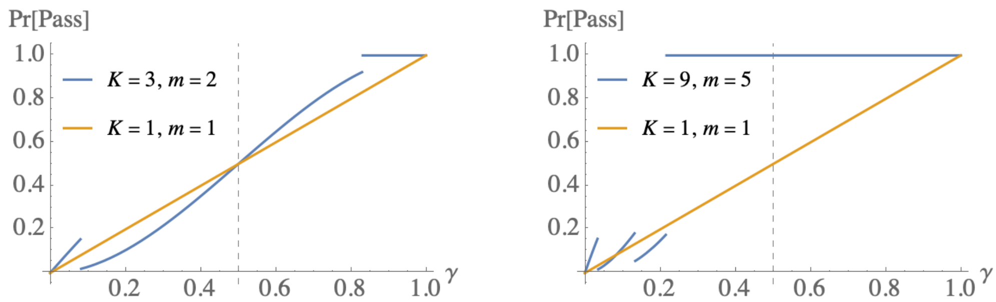
High green sentiment, $\gamma > \frac{1}{2}$ Result 2 has important consequences when $\gamma > \frac{1}{2}$. As shown in Proposition 3.c, when $\gamma > \frac{1}{2}$, the potential is a convex function of o for $o < K$. Thus, the potential maximizing resistance o-strategy is extreme, either maximal resistance, $o = 0$, or minimal resistance, $o = m-1$. When, as we assume in this section, normalized reputation costs are the same for all universal owners, this implies that, when $\gamma > \frac{1}{2}$, the optimal resistance strategy is maximum resistance, $o = 0$, and this strategy is optimal if its payoff is no less than the payoff of the capitulation strategy, $o = K = 2m-1$. This result is recorded below.

Result 3. If $\gamma > \frac{1}{2}$, the potential maximizing o-strategy is either complete resistance, $o = 0$, or capitulation, $o = 2m-1$.

When $\gamma > \frac{1}{2}$, Result 3 shows that brown universal owners will either completely resist or capitulate. Result 2 shows the effectiveness of complete resistance is reduced by increased ownership dispersion. Thus, increased dispersion decreases the attractiveness of complete resistance relative to capitulation and also increases the pass probability even when brown owners adopt the complete resistance strategy. Thus, when $\gamma > \frac{1}{2}$, increased dispersion increases the pass probability.

Result 4. If $\gamma > \frac{1}{2}$, increasing the number of universal owners weakly increases the pass probability.

Low to moderate green sentiment, $\gamma < \frac{1}{2}$ When $\gamma < \frac{1}{2}$, the relationship between dispersion and the pass probability is much more subtle. Result 2 shows that, in this case, the effectiveness of complete resistance strategies is increased by dispersion. This effect favors increased dispersion reducing the pass probability. However, when $\gamma < \frac{1}{2}$, the relationship between the resistance o-strategies and the value of the potential is generally not convex (See Proposition 3). Thus intermediate partial resistance o-strategies with $0 < o < m-1$ can maximize the potential. Dispersion increases the attractiveness partial resistance and capitulation relative to complete resistance. Thus, developing a simple general comparative static in this case is not possible. However, as illustrated in the following figures, it is easy to provide examples with plausible numbers of universal owners under which multiple universal owners more effectively block proposal passage than a single universal owner. Such cases are illustrated in Figure 6.



A. $K = 3$ B. $K = 9$

Figure 6: *Ownership concentration and the pass probability.* In both panels, the horizontal axis represents the level of green sentiment amongst universal owners, γ, and the vertical axis represents the pass probability, Pr[Pass]. The reduction in the value of the universal owners' shares produced by the proposal passing is $\Delta W = 1$. The reputational cost incurred by the universal owners if they all oppose the proposal is $\Delta R = 0.15$. For the sake of comparison, the relationship between the pass probability and green sentiment when there is a single universal owner, $K = 1$, is represented by the orange line. The blue line represents the relationship between green sentiment and the pass probability when there are K universal owners. In Panel A, $K = 3$ and in Panel B, $K = 9$.

6.4.2 Monetary payoffs

We now consider the effect of universal ownership dispersion on universal owners' monetary payoff. The monetary payoff has two components: one that captures the effect of proposal passage on the expected value of the universal owners' stake and one that captures the expected reputation costs imposed by opposing the proposal:

$$\text{Exp. Univ. Owner Value:} \quad W(F) - \Delta W \, \overbrace{\hat{B}(m-o, 2m-1-o, \gamma)}^{\text{Pr[Pass]}} \tag{29}$$

$$\text{Exp. Reputation Costs:} \quad \underbrace{(1-\gamma)(2m-1-o)}_{\text{exp. \# resisting owners}} \underbrace{\left(\frac{R}{2m-1}\right)}_{\text{Rep. cost per owner}} \tag{30}$$

Subtracting reputation (equation (30)) from value effects (equation (29)) and simplifying yields the following expression for the monetary payoff, M-payoff:

$$\text{M-Payoff}^m_o := W(F) - (1-\gamma)R + \left((1-\gamma)\,\frac{R}{2m-1}o - \Delta W\,\hat{B}(m-o, 2m-1-o, \gamma)\right). \tag{31}$$

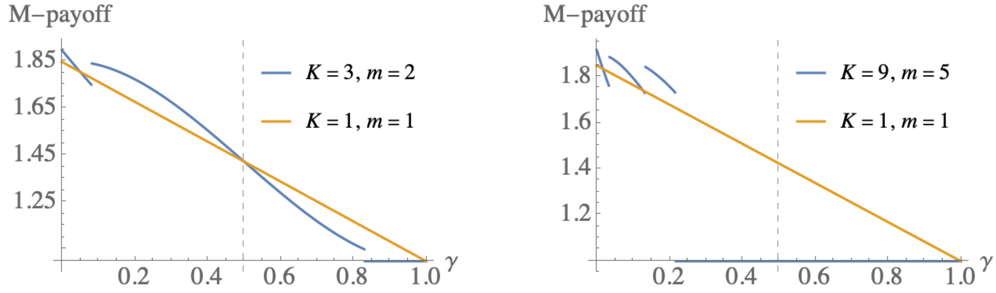
In order to facilitate comparison with the potential function, Π^m_o, we can rewrite the expression for M-payoff as follows:

$$\text{M-Payoff}^m_o := W(F) - (1-\gamma)R + (1-\gamma)\,\frac{\Delta W}{2m-1}\left(y\,o - (2m-1)\,\frac{\hat{B}(m-o, 2m-1-o, \gamma)}{1-\gamma}\right). \tag{32}$$

This formulation highlights the difference between the potential function, Π^m_o, defined by equation (28), and the monetary payoff function, M-Payoff^m_o, defined by equation (32). First, note that, for a fixed number of universal owners, the o-strategy maximizing Π^m_o and the o-strategy maximizing M-Payoff^m_o depend only on the parts of these expression enclosed in the large parenthesis on the right hand side of their defining equations. The only difference between the expressions for Π^m_o and M-Payoff^m_o within these parentheses is that M-Payoff^m_o multiplies the probability of passage by $K = 2m-1$. Thus, when $m > 1$ and thus $K > 1$, the monetary payoff factors in the effect of proposal passage on *all* universal owners while the potential only factors in the effect on an individual universal owner. The gap between the monetary payoff and the potential (the function which determines the actual strategy played by brown owners) increases with the number of universal owners. This gap militates in favor of dispersion reducing the monetary payoff. However, there are two countervailing forces: the increased efficiency of resistance engendered by ownership dispersion when $\gamma < \frac{1}{2}$ (see Result 2) and a new force: the reputation cost savings from strategic voting. When green sentiment is sufficiently low, even if some brown owners insincerely vote in favor the green proposal (and thus avoid incurring reputation costs), the proposal is still very likely to fail. Thus, when green sentiment is sufficiently low, strategic insincere voting can appreciably reduce total expected reputation costs while only negligibly increasing the pass probability. In this case, dispersion also increases the monetary payoff. These observations are recorded in the following result.

Result 5. (i) If $\gamma > \frac{1}{2}$, the monetary payoff is lower if ownership is divided amongst multiple owners rather than concentrated in the hands of a single universal owner, (ii) When $\gamma < \frac{1}{2}$ then, whenever (a) green sentiment $\gamma > 0$ is sufficiently small or (b) under divided ownership, the potential maximizing o-strategy is complete resistance, the monetary payoff is higher under divided ownership.

We illustrate these results in Figure 7. Note that when the number of universal owners is greater than 1 but small, $K = 3$ in Panel A, the increased resistance efficiency and reputation cost minimization effects ensure that, despite the free-riding incentives engendered by divided ownership, as long as $\gamma < \frac{1}{2}$ and reputation costs are small relative to value effects, divided universal ownership generally produces higher monetary payoffs than unified ownership. In contrast, when the number of universal owners is large, $K = 9$ in Panel B, divided ownership only produces higher monetary payoffs when green sentiment is very low.



A. $K = 3$ B. $K = 9$

Figure 7: *Ownership concentration and the monetary payoff.* In both panels, the horizontal axis represents the level of green sentiment amongst universal owners, γ, and the vertical axis represents the monetary payoff, M-Payoff. The reduction in the value of the universal owners' shares produced by the proposal passing is $\Delta W = 1$. The reputational cost incurred by the universal owners if they all oppose the proposal is $\Delta R = 0.15$. For the sake of comparison, the relationship between the monetary payoff and green sentiment when there is a single universal owner, $K = 1$, is represented by the orange line. The blue line represents the relationship when there are K universal owners. In Panel A, $K = 3$ and in Panel B, $K = 9$.

7 Conclusion

In this paper, we modeled the ESG activists' green campaigns and universal owners' voting in proxy contests. First we showed that, in contrast to the case of activist aiming to increase firm value, green activism is not constrained by the hold-up problem first modeled in Grossman and Hart (1980). As in Grossman and Hart (1980), the price at which the activist purchases shares fully impounds the effects of intervention on firm value. However, the ESG activist has another source of gains from share acquisition that is not appropriable by selling shareholdings, the change in the environment produced by adoption of the activist's proposal. In equilibrium, the ESG activist's payoff captures the entire "environmental return" from activism. As long as this return exceeds the cost of activism, launching a campaign is a viable strategy for the ESG activist. Thus, in a world where a subset of investors have strong pro-environment preferences, activism campaigns are not very costly relative to the potential environmental benefits of changing corporate policies, and ESG proposals have some chance of being adopted, many activist campaigns will be launched.

When voting on proposals by ESG activists, universal owners face the trade-off between reputation costs and financial value reduction. This leads to strategic voting. Brown owners tend to vote insincerely in favor of a proposal when their no votes are not likely to be required to defeat the proposal or when the proposal is likely to pass by a wide margin regardless of their vote. We find that increasing the reputation cost of no votes on green proposals always increases the probability that green proposals will pass. However, a higher likelihood that universal owners have pro-environment, “green,” preference, does not always increase the probability that green proposals pass. More green sentiment can trigger more resistance from the remaining brown shareholders, making the proposal less likely to pass.

When there are multiple universal owners, the presence of reputation costs ensures that, even when green sentiment and public pressure are low, and, net of reputation costs, brown universal owners are better of when green proposals fail, there is always some, albeit small, probability that green proposals pass. Thus, if the cost of activism are small, many green proposals will be advanced and few will pass. In this case, the passing probability of green proposals is very sensitive to the firm value reduction required to affect the environmental improvement. In periods of public “moral panic” about the environment, even if green sentiment of brown owners remains quite low and reputation penalties are less than the value loss from adopting green proposals, brown universal owners capitulate. As a consequence, aggressive proposals that require considerable sacrifices of firm value to achieve environmental objectives have a significant probability of passing.

A Appendix

Proof of Lemma 2. To prove (a), first note that

$$\mathbb{P}[S(t) \geq m] = t_i \mathbb{P}[S^{-i}(t) \geq m-1] + (1-t_i)\mathbb{P}[S^{-i}(t) \geq m].$$

Thus,

$$\frac{\partial}{\partial t_i}\mathbb{P}[S(t) \geq m] = \mathbb{P}[S^{-i}(t) \geq m-1] - \mathbb{P}[S^{-i}(t) \geq m] = \mathbb{P}[S^{-i}(t) = m-1].$$

To prove (b), note that (a) implies that

$$\frac{\partial}{\partial t_i}\mathbb{P}[S(t) \geq m] = \frac{\partial}{\partial t_j}\left(\frac{\partial}{\partial t_i}\mathbb{P}[S(t) \geq m]\right) = \frac{\partial}{\partial t_j}\mathbb{P}[S^{-i}(t) = m-1] \tag{A-1}$$

and that

$$\mathbb{P}[S^{-i}(t) = m-1] = t_j \mathbb{P}[S^{-ij}(t) = m-2] + (1-t_j)\mathbb{P}[S^{-ij}(t) = m-1].$$

Thus,

$$\frac{\partial}{\partial t_j}\mathbb{P}[S^{-i}(t) = m-1] = \mathbb{P}[S^{-ij}(t) = m-2] - \mathbb{P}[S^{-ij}(t) = m-1]. \tag{A-2}$$

and (b) follows from (A-1) and (A-2). □

Proof of Lemma 3 This proof is established through the following Lemmas.

Lemma A.1. *The exists a pure strategy (i. e., for all $i \in \mathcal{K}$, $\sigma_i \in \{0,1\}$) maximizer of the potential.*

Proof. First note that the domain of Π is the compact set $[0,1]^K$ and Π is continuous, so a maximizer, perhaps mixed, exists. Next note the Π is multilinear so suppose that $\bar{\boldsymbol{\sigma}}$ maximizes Π and $\bar{\sigma}_i \in (0,1)$. Define the function $\nu : [0,1] \to \mathbb{R}$ by $\nu(\sigma_i) = \Pi(\sigma^i|\bar{\boldsymbol{\sigma}}^{-i})$. Since $\bar{\boldsymbol{\sigma}}$ maximizes Π, $\bar{\sigma}_i$ maximizes ν. Because, Π is affine, this implies that $\sigma_i = 0$ and $\sigma_i = 1$ also maximize ν. Hence, $(0|\bar{\boldsymbol{\sigma}}^{-i})$ and $(1|\bar{\boldsymbol{\sigma}}^{-i})$ also maximize Π. If, in fact $\bar{\boldsymbol{\sigma}}$ maximizes Π, we can continue in like fashion, replace all mixed components in $\bar{\boldsymbol{\sigma}}$ with 0 and 1 without affecting the value of Π. □

Lemma A.2. *If strategy vector $\boldsymbol{\sigma}$ in which at least two brown universal owners randomize, is a potential maximizer then $y_i = y_j$ for all i, j such that $i, j \notin \{0,1\}$.*

Proof. Consider a vector $\bar{\boldsymbol{\sigma}}$ in which at least two brown universal owners, say i and j randomize, i.e., $\sigma_i \in (0,1)$ and $\sigma_j \in (0,1)$. If, in fact $\bar{\boldsymbol{\sigma}}$ maximizes the potential function, then

$$\text{Max}\{\Pi(\sigma_i, \sigma_j|\bar{\boldsymbol{\sigma}}^{-ij}) : (\boldsymbol{\sigma}^i, \sigma_j) \in [0,1]^2\} = \Pi(\bar{\boldsymbol{\sigma}}).$$

First note that

$$\mathbb{P}[S(\tau(\boldsymbol{\sigma})) \geq m] = \mathbb{P}[S^{-ij}(\tau(\boldsymbol{\sigma})) \geq m] + \Big(t(\sigma_i) + t(\sigma_j) - t(\sigma_i)t(\sigma_j)\Big)\mathbb{P}[S^{-ij}(\tau(\boldsymbol{\sigma})) = m-1] + t(\sigma_i)t(\sigma_j))\mathbb{P}[S^{-ij}(\tau(\boldsymbol{\sigma})) = m-2], \quad \text{(A-3)}$$

and that the distribution of S^{-ij} is not affected by σ_i or σ_j. So, to reduce our notational burden somewhat define.

$$\bar{s}_0 = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) \geq m], \quad \bar{e}_1 = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) = m-1], \quad \bar{e}_2 = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) = m-2], \quad \text{(A-4)}$$

and define the function, $\pi : [0,1]^2 \to \mathbb{R}$ by

$$\psi(\boldsymbol{\sigma}_{ij}) := \Pi(\sigma_i, \sigma_j | \bar{\boldsymbol{\sigma}}^{-ij}), \text{ where } \boldsymbol{\sigma}_{ij} := (\sigma_1, \sigma_2)$$

Not that equation (A-3) and the definition of the potential function show that ψ can be expressed as follows:

$$\psi(\boldsymbol{\sigma}_{ij}) = \sum_{K\setminus\{i,j\}} \bar{\sigma}_k y_k + \sigma_i y_i + \sigma_j y_j - \frac{\bar{s}_0 + \Big(t(\sigma_i) + t(\sigma_j) - t(\sigma_i)t(\sigma_j)\Big)\bar{e}_1 + t(\sigma_i)t(\sigma_j)\bar{e}_2}{1-\gamma} \quad \text{(A-5)}$$

A straightforward computation, shows that the second derivative (i.e., the Hessian of π), $D^2\pi$, is given by

$$D^2\psi(\boldsymbol{\sigma}_{ij}) = -(1-\gamma)\Delta w H, \text{ where } H = \begin{pmatrix} 0 & \bar{e}_2 - \bar{e}_1 \\ \bar{e}_2 - \bar{e}_1 & 0 \end{pmatrix}. \quad \text{(A-6)}$$

Because the first derivative of ψ (i.e. the gradient) must vanish by the first-order condition and all derivative forms higher than two vanish because D^2 is constant, the multivariate version of Taylor's Theorem shows that

$$\psi(\boldsymbol{\sigma}_{ij}) = \pi(\bar{\boldsymbol{\sigma}}_{ij}) - \frac{1}{2}(1-\gamma)\Delta w \Big(\boldsymbol{\sigma}_{ij} - \bar{\boldsymbol{\sigma}}_{ij})^T H (\boldsymbol{\sigma}_{ij} - \bar{\boldsymbol{\sigma}}_{ij})\Big).$$

First, consider the case where $\bar{e}_2 - \bar{e}_1 \neq 0$. In this case, we see that H has two non-zero eigenvalues with opposite signs, $\bar{e}_2 - \bar{e}_1$ and $-(\bar{e}_2 - \bar{e}_1)$. Thus, inspecting equation (A-6), shows that $\mathcal{H}$ is not positive semi-definite and $(\bar{\sigma}_i, \bar{\sigma}_j)$ cannot maximize π and hence $\bar{\boldsymbol{\sigma}}$ cannot maximize the potential, Π. In fact, the eigenvectors of H are $(1,1)$ and $(1,-1)$ and thus in this case, $\bar{\boldsymbol{\sigma}}$ is a saddle point, and not a local maximizer of π.

No suppose that $\bar{e}_2 - \bar{e}_1 = 0$. If $\bar{e}_2 - \bar{e}_1 = 0$ then equation (A-5) reduces to a linear function of $\boldsymbol{\sigma}_{ij}$, i.e.,

$$\psi(\boldsymbol{\sigma}_{ij}) = \sigma_i y_i + \sigma_j y_j + \sum_{K\setminus\{i,j\}} \bar{\sigma}_k y_k - \frac{\bar{s}_0 + \Big(t(\sigma_i) + t(\sigma_j)\Big)\bar{e}_1}{1-\gamma}$$

So, $\bar{\boldsymbol{\sigma}}_{ij}$ maximizes ψ if and only if $(y_i - \bar{e}_1, y_j - \bar{e}_1) = (0,0)$. In which case all $\boldsymbol{\sigma}_{ij} \in [0,1]^2$ also maximize ψ. Thus if $\bar{\sigma}$ maximizes Π then all vectors of the form$(\boldsymbol{\sigma}_{ij}|\bar{\boldsymbol{\sigma}}^{-ij})$ also maximize the potential. Moreover, $y_i = y_j = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) = m-1] = \mathbb{P}[S^{-ij}(\tau(\bar{\boldsymbol{\sigma}})) = m-2]$. □

Lemma A.3. *If the strategy vector, $\bar{\boldsymbol{\sigma}}$ contains two or more mixed components, say $\bar{\sigma}_i \in (0,1)$ and $\bar{\sigma}_j \in (0,1)$, then σ can only be a potential maximizer when $\gamma = (m-1-j)/(K-1-j)$, where $j \in \{0,1,2,\dots m-2\}$.*

Proof. Let

$$\mathcal{O}(\boldsymbol{\sigma}) := \{i \in \mathcal{K} : \sigma_i = 1\}, \quad \mathcal{R}(\boldsymbol{\sigma}) := \{i \in \mathcal{K} : \sigma_i \in (0,1)\}, \text{ and } \mathcal{Z}(\boldsymbol{\sigma}) := \{i \in \mathcal{K} : \sigma_i = 0\}.$$

By assumption, $\#\mathcal{R} \geq 2$; so select two members of this set, which without loss of generality, we assume includes $i = 1,2$. Define three new strategy vectors, $\boldsymbol{\sigma}^\ell$, $\ell = 0,1,2$ as follows:

$$\sigma_i^0 = \begin{cases} 0 & 0 \in \mathcal{Z}(\bar{\boldsymbol{\sigma}}) \cup \mathcal{R}(\bar{\boldsymbol{\sigma}}) \\ 1 & i \in \mathcal{O}(\bar{\boldsymbol{\sigma}}) \end{cases}, \sigma_i^1 = \begin{cases} 0 & i \in \mathcal{Z}(\bar{\boldsymbol{\sigma}}) \cup (\mathcal{R}(\bar{\boldsymbol{\sigma}}) \backslash \{1\}) \\ 1 & i \in \mathcal{O}(\bar{\boldsymbol{\sigma}}) \cup \{1\} \end{cases}, \sigma_i^2 = \begin{cases} 0 & i \in \mathcal{Z}(\bar{\boldsymbol{\sigma}}) \cup (\mathcal{R}(\bar{\boldsymbol{\sigma}}) \backslash \{1,2\}) \\ 1 & i \in \mathcal{O}(\bar{\boldsymbol{\sigma}}) \cup \{1,2\} \end{cases}$$

Next note that, by hypothesis $\bar{\boldsymbol{\sigma}}$ maximizes the potential and is not a pure strategy vector, implies that $\#\mathcal{O}(\bar{\boldsymbol{\sigma}}) \leq m-1$. Otherwise the proposal would pass with certainty and, in this case, the unique optimal strategy is for all brown owners to vote yes, $\sigma_i = 1$ for all $i \in \mathcal{K}$, and this strategy vector is pure. Next note that Π being multilinear and the hypotheses that $\bar{\boldsymbol{\sigma}}$ is a potential maximizer implies that

$$\Pi(\bar{\boldsymbol{\sigma}}) = \max_{\boldsymbol{\sigma} \in [0,1]^K} \Pi(\boldsymbol{\sigma}), \quad \text{and} \quad \Pi(\bar{\boldsymbol{\sigma}}) = \Pi(\boldsymbol{\sigma}^0) = \Pi(\boldsymbol{\sigma}^1) = \Pi(\boldsymbol{\sigma}^2).$$

Note that $\#\mathcal{O}(\boldsymbol{\sigma}^\ell) < K$, for $\ell = 0,1,2$. Thus follows because, as argued above $\#\mathcal{O}(\bar{\boldsymbol{\sigma}}) \leq m-1$, and the cardinality of $\mathcal{O}(\boldsymbol{\sigma}^\ell)$ exceeds the cardinality of $\mathcal{O}(\bar{\boldsymbol{\sigma}})$ by at most two. So, $\#\mathcal{O}(\boldsymbol{\sigma}^\ell) \leq m-1+2 = m+1$. By model assumptions, $m+1$ is less than K. Thus, if it were the case that $\#\mathcal{O}(\boldsymbol{\sigma}^\ell) > m-1$ then $\#\mathcal{O}(\boldsymbol{\sigma}^\ell) \in \{m, m+1, \dots, K-1\}$. In which case $\boldsymbol{\sigma}^\ell$ would not maximize the potential because the proposal would be passing with certainty and, for some i, $\boldsymbol{\sigma}^i \neq 1$. But this contradicts equation A.

Note that the $\boldsymbol{\sigma}^1$ and $\boldsymbol{\sigma}^0$ differ only in their strategy assignment to brown owner 1: under $\boldsymbol{\sigma}^0$, brown owner 1 votes against the proposal, $\sigma_1^0 = 0$ and, under σ^1, brown owner 1 votes for the proposal, $\sigma_1^1 = 1$. Note also that because both 1 and 2 randomize under $\bar{\boldsymbol{\sigma}}$, Lemma A.2 implies that $y_1 = y_2$. Let y_o represent their common value, and let $\bar{o} = \#\mathcal{O}(\boldsymbol{\sigma}^0)$ Inspection of the definition of the potential function shows that

$$\Pi(\boldsymbol{\sigma}^1) - \Pi(\boldsymbol{\sigma}^0) = 0 \Longleftrightarrow y_o - \left(\frac{\hat{B}(m-(\bar{o}+1); K-(\bar{o}+1), \gamma)}{1-\gamma} - \frac{\hat{B}(m-\bar{o}; K-\bar{o}, \gamma)}{1-\gamma} \right) = 0. \tag{A-7}$$

The argument used in the proof of part (c) of Lemma 4, shows that

$$\frac{\hat{B}(m-\bar{o};K-(\bar{o}+1),\gamma)}{1-\gamma}-\frac{\hat{B}(m-\bar{o};K-\bar{o},\gamma)}{1-\gamma}=b(m-(\bar{o}+1);K-(\bar{o}+1),\gamma) \tag{A-8}$$

From equations (A-7) and (A-8) we see that

$$\Pi(\boldsymbol{\sigma}^1)-\Pi(\boldsymbol{\sigma}^0)=0 \Longleftrightarrow y_o=b(m-(\bar{o}+1);K-(\bar{o}+1),\gamma). \tag{A-9}$$

An identical argument shows that

$$\Pi(\boldsymbol{\sigma}^2)-\Pi(\boldsymbol{\sigma}^1)=0 \Longleftrightarrow y_o=b(m-(\bar{o}+2);K-(\bar{o}+2),\gamma). \tag{A-10}$$

Hence,

$$b(m-(\bar{o}+1);K-(\bar{o}+1),\gamma)=b(m-(\bar{o}+2);K-(\bar{o}+2),\gamma)$$

Algebraic simplification shows that

$$b(m-(\bar{o}+1);K-(\bar{o}+1),\gamma)=b(m-(\bar{o}+2);K-(\bar{o}+2),\gamma) \Longleftrightarrow \gamma=\frac{m-1-\bar{o}}{K-1-\bar{o}}.$$

Thus, if at least two shareholders play mixed strategies, it must be the case that γ satisfies

$$\gamma=\frac{m-1-\bar{o}}{K-1-\bar{o}}, \text{ where } \bar{o}\in\{0,1,\dots m-2\}.$$

□

Lemma A.4. *The set of $\gamma\in[0,1]$ and $\boldsymbol{y}\in[0,1]^K$ such that $\boldsymbol{\sigma}$ is a potential maximizer and any universal owner plays a mixed strategy has measure 0.*

Proof. We have seen (Lemma A.3) that the set of γ that support an equilibrium in which two brown owners randomize is finite and thus clearly measure 0 in $[0,1]\times[0,1]^K$. Now consider the set of $(\gamma,\boldsymbol{y})\in[0,1]\times[0,1]^K$ such that one universal owner randomizes. For a strategy vector featuring randomization by one owner, say owner j, to maximize the potential when other owner's play pure strategies, it must be the case that at least two pure strategies for the other owners, say $\boldsymbol{\sigma}_1^{-j}$ and $\boldsymbol{\sigma}_2^{-j}$ produce the same payoff for all $\sigma_j\in[0,1]$. For any fixed $\boldsymbol{y}$, these pure strategies (i.e. $\sigma_i\in\{0,1\}$) are polynomials in γ. Because they are polynomials, the polynomial that represents their difference has only a finite number of zeros, and thus, for any fixed $\boldsymbol{y}$, the measure of $\gamma\in[0,1]$ such that the two strategies have the same payoff equals 0. Using Fubini's Theorem to integrate these zero measure sets over $[0,1]^K$, shows that the measure of the set $(\gamma,\boldsymbol{y})\in[0,1]\times[0,1]^K$ such that a potential maximizer features one owner randomizing has measure 0. □

Lemmas A, A.2, A.3, and A.4 establish Lemma 3. □

Proof of Lemma 4. (a) When $\gamma = 0$, the green proposal will pass if and only if it is supported by at least m brown universal owners. Under all the $o \in \mathcal{O}$ polices, less than m owners vote yes. Hence, the proposal fails, i.e. $\hat{B} = 0$. Because, $\boldsymbol{y} > 0$, the result is apparent. When $\gamma = 1$, the proposal will pass regardless of the votes of the brown universal owners, so $\hat{B} = 1$, and the result is then apparent from inspecting the definitions.

(b) Differentiation shows that

$$\frac{d}{d\gamma}\Pi_o = -\Delta w((1-\gamma)^{-2})\Big((K-o)(1-\gamma)b(m-o-1;K-o-1,\gamma) + \hat{B}(m-o-1;K-o-1,\gamma)\Big).$$

The terms in the parentheses are all positive for all $\gamma \in (0,1)$, so the right-hand side of the equation is negative.

(c) First note that the definition of the Π_o functions implies that

$$\Pi_{o+1}(\gamma,y) - \Pi_o(\gamma,y) = \Delta w \left(y_{o+1} - \left(\frac{\hat{B}(m-o-1;K-o-1,\gamma) - \hat{B}(m-o;K-o,\gamma)}{1-\gamma} \right) \right).$$

Noting that

$$\hat{B}(m-o,K-o,\gamma) = \hat{B}(m-o-1,K-o-1,\gamma)\,\gamma + \hat{B}(m-o,K-o-1,\gamma)(1-\gamma).$$

we see that

$$\frac{\hat{B}(m-o-1;K-o-1,\gamma) - \hat{B}(m-o;K-o,\gamma)}{1-\gamma} = \hat{B}(m-o-1;K-o-1,\gamma) - \hat{B}(m-o;K-o-1,\gamma) = b(m-o-1;K-o-1,\gamma),$$

and the result follows.

(d) Part (d). Using part (c) we see that

$$\Delta^2\Pi_o := \Delta\Pi_{o+1} - \Delta\Pi_o = \Big(y_{o+2} - y_{o+1}\Big) + \Big(b(m-(o+1)-1;K-(o+1)-1,\gamma) - b(m-o-1;K-o-1,\gamma)\Big).$$

Part (d) then follows by algebraic simplification and rearrangement of the second term in parentheses on the right hand side of the equation above.

□

Proof of Lemma 5. (a) Part (a). First note that the definition of the Π_o functions (Equation (24)) shows that

$$\Pi_o - \Pi_K = \frac{1-\hat{B}(m-o;K-o,\gamma)}{1-\gamma} - \Sigma_{o+1}^K. \tag{A-11}$$

When $\gamma = 0$, $1-\hat{B}(m-o;K-o,\gamma)/(1-\gamma) = 1$ and application of LH́opital's rule shows that $\lim_{\gamma\to 1} 1 - \hat{B}(m-o;K-o,\gamma)/(1-\gamma) = 0$. Thus, the assertions in this part follow from inspection of equation (A-11) and the continuity of the Π_o functions.

(b) Part (b). When $o = m-1$, $m-o = 1$. This observation and equation A-11 shows that

$$\Pi_{m-1} - \Pi_K = (1-\gamma)^{K-m} - \Sigma_m^K,$$

which is evidently strictly decreasing in γ.

(c) Part (c). This is the only part of the lemma that is somewhat difficult to establish. We will used the general form of the what is frequently termed the monotone LH́opital rule.

Using equation (A-11) we can express $\Pi_o - \Pi_K$ for $o > m-1$ as follows:

$$\Pi_o - \Pi_K = \frac{1-\hat{B}(m-o;K-o,\gamma) - (1-\gamma)\,\Sigma_{o+1}^K}{1-\gamma} := \frac{N(\gamma)}{D(\gamma)}. \tag{A-12}$$

Let $\rho = N'/D'$ and let $\tilde{\rho} = (D\rho - N)\ \mathrm{sgn}[D']$. Inspection shows that

$$\lim_{\gamma\to 1} N(\gamma) = 0 \text{ and } \lim_{\gamma\to 1} D(\gamma) = 0. \tag{A-13}$$

Equation A-12 and Fact 1 show that

$$\rho(\gamma) = (K-o)\,b(m-o-1;K-o-1,\gamma) - \Sigma_{o+1}^K, \tag{A-14}$$

$$\tilde{\rho}(\gamma) = -\Bigg((1-\gamma)\left((K-o)\,b(m-o-1;K-o-1,\gamma) - \Sigma_{o+1}^K\right) - \left(1-\hat{B}(m-o;K-o,\gamma) - (1-\gamma)\,\Sigma_{o+1}^K\right)\Bigg). \tag{A-15}$$

Because $0 \le o < m-1$, $0 < m-o-1 < K-o$, Because $m-o-1$ lies between 0 and $K-o$, the two extreme realizations of the Binomial$(\cdot;K-o,\gamma)$ distribution, the probability of $m-o-1$ first increases and then decreases in γ, i.e, $\gamma \to b(m-o-1;K-o-1,\gamma)$ is $\nearrow\searrow$ in γ; thus, inspecting equation (A-14) shows that

$$\rho \text{ is } \nearrow\searrow. \tag{A-16}$$

Because $o < m-1$, $b(m-o-1; K-o-1, \gamma) \to 0$ and (the probability the proposal fails) $1 - \hat{B}(m-o; K-o, \gamma) \to 1$ as $\gamma \to 0$. These observations applied to equation (A-15) show that

$$\lim_{\gamma \to 0} \tilde{\rho}(\gamma) = 1 > 0. \tag{A-17}$$

Proposition 4.4 in Pinelis (2002) shows that equations (A-13), (A-16), and (A-17) are sufficient N/D to be $\nearrow\searrow$. Because $N/D = \Pi_o - \Pi_K$ (see equation (A-12)), part (c) is established.

□

Proof of Proposition 2

Proof of part (a). First consider the case where $o = m-1$, When $\gamma = 0$, $\Pi_{m-1} - \Pi_K = 1 - \Sigma_m^K$ Because the normalized reputation costs are decreasing in the index, if $\Sigma_m^K > 1$ then at $\gamma = 0$, $\Pi_{m-1} - \Pi_K \le 0$. Lemma 5.(b) shows that, when $o = m-1$, $\Pi_o - \Pi_K$ is strictly decreasing in γ. So, for all $\gamma \in (0,1)$, $\Pi_{m-1} - \Pi_K < 0$.

Now consider the more challenging case, $o < m-1$. In this case, $\Pi_o - \Pi_K$ is $\nearrow\searrow$ in γ; so the fact that, at $\gamma = 0$, $\Pi_o - \Pi_K < 0$ does not imply that, for all $\gamma \in [0,1]$, $\Pi_o - \Pi_K < 0$.

We start by defining

$$\bar{y} := \frac{1}{K-m+1}. \tag{A-18}$$

Because the normalized reputation costs are decreasing in the index, if $\Sigma_m^K \ge 1$ then $\Sigma_{o+1}^K \ge (K-o)\,\bar{y}$. Hence $\Pi_o - \Pi_K \le g_o(\gamma)/(1-\gamma)$, where

$$g_o(\gamma) := (1 - \hat{B}(m-o; K-o, \gamma)) - (1-\gamma)\,\bar{y}\,(K-o). \tag{A-19}$$

Thus, to establish the proposition for $o < m-1$ we need only show that

$$\gamma \in (0,1) \text{ and } o \in \mathcal{O} \backslash \{K, m-1\} \Longrightarrow g_o(\gamma) < 0.$$

Using equation (A-19) we compute

$$g_o'(\gamma) = \bar{y} - (K-o)\,(1-\gamma)^{K-m}\,\gamma^{m-o-1}\binom{K-o-1}{m-o-1}, \tag{A-20}$$

$$g_o''(\gamma) = (K-o-1)\,(K-o)\,(1-\gamma)^{K-m-1}\,\gamma^{m-o-2}\binom{K-o-1}{m-o-1}(\gamma - \gamma_o), \text{ where} \tag{A-21}$$

$$\gamma_o := \frac{m-o-1}{K-o-1}. \tag{A-22}$$

Equations (A-19), (A-20), and (A-21) imply that

$$g_o(0) = 1 - (K - o)\bar{y} < 0, \tag{A-23}$$

$$g_o'(0) = \bar{y}, \tag{A-24}$$

$$\operatorname{sgn}[g_o''(\gamma)] = \operatorname{sgn}[\gamma - \gamma_o]. \tag{A-25}$$

Equation (A-25) shows that g_o is concave on the interval $[0, \gamma_o]$ and is convex on the interval $[\gamma_o, 1]$.

First, we show that $\max\{g_o(\gamma) : \gamma \in [0, \gamma_o]\} < 0$. To show this, note that when $\gamma \in [0, \gamma_o]$, g_o is concave (equation A-25) and thus bounded from above by its support lines and, in particular, by its support line at 0, i.e.,

$$g_o(\gamma) \leq g_o(0) + \gamma g_o'(0), \quad \gamma \in [0, \gamma_o].$$

Equation (A-24) shows that $g_o'(0) = \bar{y} > 0$. Hence,

$$g_o(\gamma) \leq g_o(0) + \gamma_o g_o'(0).$$

Substituting in the values of $\bar{y}$, γ_o, $g_o(0)$, and $g_o'(0)$, provided by equations (A-18), (A-22), (A-23), and (A-24), we see that

$$g_o(0) + \gamma_o g_o'(0) = -\frac{2K - (m + o + 1)}{(K - m - 1)(K - o - 1)} < 0.$$

Thus, over the interval $[0, \gamma_o]$, $g_o < 0$.

Because g_o is strictly convex over $[\gamma_o, 1]$, it attains its maximum over this interval only at the extreme points of this interval, 1 and γ_o. The definition of g_o (equation (A-19)) shows that $g_o(1) = 0$ and we have just shown that $g_o(\gamma_o) < 0$. Hence, $g_o(\gamma) < 0$ on $(\gamma_o, 1)$. Combining the concave and convex cases, shows that $g_o \leq 0$ over $[0, 1]$ and the result is established.

Proof of part (b). This result follows directly from Lemma 4.(b), Lemma 5.(a), and the continuity of the Π_o, $o \in \mathcal{O}$, functions.

□

Proof of Proposition 3. We consider parts (a) and (c) as the arguments supporting these parts of the lemma are interconnected.

Proof of parts (a) and (c). By definition $o \to \Pi_o$ is concave (convex) at o if $\Delta^2\Pi_{o-1} \leq (\geq) 0$ (see Remark 2).

Lemma 4.(d) shows that

$$y_{o+1} - y_o = 0 \Longrightarrow \operatorname{sgn}[\Delta^2\Pi_{o-1}] = \operatorname{sgn}\left[\gamma - \frac{m - o}{K - o}\right]. \tag{A-26}$$

This establishes necessity condition in part (a) and the sufficiency condition in part (c).

Because of the decreasing arrangement of owners by reputation costs, it is always the case that $y_{o+1} - y_o \leq 0$. Thus, we now need only consider the $y_{o+1} - y_o < 0$ case. Lemma 4.(d) shows

$$\gamma - \frac{m-o}{K-o} < 0 \Longrightarrow \Delta^2 \Pi_{o-1} < 0, \tag{A-27}$$

i.e., $o \to \Pi_o$ is strictly concave at o.

Again, because $y_{o+1} - y_o \leq 0$, Lemma 4.(d) shows that, if i.e., $o \to \Pi_o$ is strictly convex at o, i.e.,

$$\Delta^2 \Pi_{o-1} > 0 \Longrightarrow \gamma - \frac{m-o}{K-o} > 0. \tag{A-28}$$

Thus, equation (A-27) implies that if

$$\gamma < \min\left\{\frac{m-o}{K-o} : o \in \{1,2,\ldots,m-2\}\right\} = \frac{2}{K-m+2}.$$

then $o \to \Pi_o$ is strictly concave. Part (a) follows by noting (Remark 1) that $m = 1 + (K-1)/2$.

Equation (A-28) implies that if the map $o \to \Pi_o$ is strictly convex, it must be the case that

$$\gamma > \max\left\{\frac{m-o}{K-o} : o \in \{1,2,\ldots,m-2\}\right\} = \frac{m-1}{K-1}.$$

Part (c) follows by noting (Remark 1) that $m = 1 + (K-1)/2$.

Proof of part (b) This part follows directly from noting that the hypotheses of the this part of the lemma, and the fact that $o \to (m-o)/(K-o)$ is decreasing; these facts imply that the map $o \to \operatorname{sgn}[\Delta^2 \Pi_{o-1}]$, $o \in \{1,2,\ldots,m-2\}$ is (weakly) increasing.

□

Proof of Lemma 6. Lemma 4.(c) shows that $\boldsymbol{y}^2 \geq \boldsymbol{y}^1$ implies $\Delta\Pi_o(\boldsymbol{y}^2) \geq \Delta\Pi_o(\boldsymbol{y}^1)$, for all $o \in \{0,1,\ldots,m-2\}$. If we compare any two non-capitulation strategies, o' and o'' such that $o' < o''$, we see that for an $\boldsymbol{y} \in [0,1]^K$

$$\Pi_{o''}(\boldsymbol{y}) = \Pi_{o'}(\boldsymbol{y}) + \sum_{o=o'}^{o''-1} \Delta\Pi_o(\boldsymbol{y}).$$

So, for any non-capitulation strategy

$$o'' > o' \Longrightarrow \Pi_{o''}(\boldsymbol{y}^2) - \Pi_{o'}(\boldsymbol{y}^2) \geq \Pi_{o''}(\boldsymbol{y}^1) - \Pi_{o'}(\boldsymbol{y}^1).$$

Similarly, if $o \in \mathcal{O} \backslash \{K\}$

$$\Pi_K(\boldsymbol{y}^2) - \Pi_o(\boldsymbol{y}^2) \geq \Pi_K(\boldsymbol{y}^1) - \Pi_o(\boldsymbol{y}^1).$$

So for all $o \in \mathcal{O}$, if $o'' > o'$

$$\Pi_{o''}(\boldsymbol{y}^1) - \Pi_{o'}(\boldsymbol{y}^1) \geq 0 \Longrightarrow \Pi_{o''}(\boldsymbol{y}^2) \geq \Pi_{o'}(\boldsymbol{y}^2).$$

□

Proof of Lemma 7. Part (a.i) of the hypothesis, Lemma 4.(b) and the fact that the functionsΠ_o, are continuous in $\gamma \in (0,1)$ implies that for all γ in some neighbourhood of 0 $o^*(\gamma) = m-1$. Now let $\bar{\gamma} := \sup\{\gamma' \in (0,\tilde{\gamma}) : \forall \gamma \in (0,\gamma'), o^*(\gamma) = m-1$.

Hypothesis, (b.i) implies that $\bar{\gamma} < \tilde{\gamma}$ and (b.ii) implies that $o^*(\gamma) \neq m-1$. Because $\gamma \to \Pi_o(\gamma)$ is a polynomial (and thus continuous) in γ and no two Π_o functions are identical, the set γ values at which any two of the $\gamma \to \Pi_o(\gamma)$ functions have the same values is discrete. Thus, there exists some interval $(\bar{\gamma}, \bar{\gamma}+\varepsilon)$, $\varepsilon > 0$, such that for all γ in this interval, $\Pi^*(\gamma) = \Pi_{\bar{o}}(\gamma)$, where $\bar{o} \neq m-1$ or K, i.e.,

$$\text{for all } \gamma \in (\bar{\gamma}, \bar{\gamma}+\varepsilon), \quad \begin{aligned} \Pi^*(\gamma) &= \Delta w \left(\Sigma_1^{o^*(\gamma)} - \frac{\hat{B}(m-o^*(\gamma); K-o^*(\gamma),\gamma)}{1-\gamma} \right) \\ &= \Delta w \left(\Sigma_1^{\bar{o}} - \frac{\hat{B}(m-\bar{o}; K-\bar{o},\gamma)}{1-\gamma} \right) = \Pi_{\bar{o}}(\gamma). \end{aligned} \tag{A-29}$$

Similarly, the definition of $\bar{\gamma}$ there exists an interval $(\bar{\gamma}-\varepsilon, \bar{\gamma})$, such that $\Pi^*(\gamma) = \Pi_{m-1}(\bar{\gamma})$, i.e.

$$\text{for all } \gamma \in (\bar{\gamma}-\varepsilon, \bar{\gamma}), \quad \begin{aligned} \Pi^*(\gamma) &= \Delta w \left(\Sigma_1^{o^*(\gamma)} - \frac{\hat{B}(m-o^*(\gamma); K-o^*(\gamma),\gamma)}{1-\gamma} \right), \\ &= \Delta w \left(\Sigma_1^{m-1} - \frac{\hat{B}(1; K-(m-1),\gamma)}{1-\gamma} \right) = \Pi_{m-1}(\gamma). \end{aligned} \tag{A-30}$$

$$\begin{aligned} \lim_{\gamma \uparrow \bar{\gamma}} \Pi^*(\gamma) = \Pi_{m-1}(\bar{\gamma}) &= \Delta w \left(\Sigma_1^{m-1} - \lim_{\gamma \uparrow \bar{\gamma}} \frac{\hat{B}(1; K-(m-1),\gamma)}{1-\gamma} \right), \\ \lim_{\gamma \downarrow \bar{\gamma}} \Pi^*(\gamma) = \Pi_{\bar{o}}(\bar{\gamma}) &= \Delta w \left(\Sigma_1^{\bar{o}} - \lim_{\gamma \downarrow \bar{\gamma}} \frac{\hat{B}(m-\bar{o}; K-\bar{o},\gamma)}{1-\gamma} \right). \end{aligned} \tag{A-31}$$

The continuity of Π^*, Π_{m-1} and $\Pi_{\bar{o}}$, in γ, and equation (A-31) imply that

$$\lim_{\gamma \uparrow \bar{\gamma}} \Pi^*(\gamma) = \lim_{\gamma \uparrow \bar{\gamma}} \Pi_{m-1}(\gamma) = \Pi_{m-1}(\bar{\gamma}) = \Pi_{\bar{o}}(\bar{\gamma}) = \lim_{\gamma \downarrow \bar{\gamma}} \Pi_{\bar{o}}(\gamma) = \lim_{\gamma \downarrow \bar{\gamma}} \Pi^*(\gamma). \tag{A-32}$$

Equations (A-31) and (A-32) imply that

$$\Sigma_1^{m-1} - \lim_{\gamma \uparrow \bar{\gamma}} \frac{\hat{B}(1; K-(m-1),\gamma)}{1-\gamma} = \Sigma_1^{\bar{o}} - \lim_{\gamma \downarrow \bar{\gamma}} \frac{\hat{B}(m-\bar{o}; K-\bar{o},\gamma)}{1-\gamma}. \tag{A-33}$$

Because, $\bar{o} < m-1$, $\Sigma_1^{m-1} > \Sigma_1^{\bar{o}}$ equation (A-33) implies that

$$\lim_{\gamma \downarrow \bar{\gamma}} \hat{B}(m-\bar{o}; K-\bar{o}, \gamma) < \lim_{\gamma \uparrow \bar{\gamma}} \hat{B}(1; K-(m-1), \gamma). \tag{A-34}$$

Equations (A-29), (A-30), (A-32), and (A-34), imply that, at $\bar{\gamma}$, the probability that the green proposal passes, $\hat{B}(m-o^*(\gamma); K-o^*(\gamma), \gamma)$ jumps down. Hence the probability the probability that the green proposal passes is not monotonic is green sentiment, γ. □

Proof of Result 2. Given that the complete resistance, $o=0$, strategy is adopted at both m and $m+1$. The probability that the proposal passes at m is $\hat{B}(m, 2m-1, \gamma)$; the probability that the proposal passes at $m+1$ is $\hat{B}(m+1, 2m+1, \gamma)$. Simple algebra shows that

$$\hat{B}(m+1, 2m+1, \gamma) - \hat{B}(m, 2m-1, \gamma) = (2\gamma - 1)\binom{2m-1}{m} \gamma^m (1-\gamma)^m.$$

□

Proof of Result 3. As shown in Proposition Xrefprop:CvCx.Xrefenum:CvCxHigh, when $m > 2$, the map $o \to \Pi_o$ is convex for $o < K$. So the only candidate optimal resistance strategies are the two extreme strategies, $o = 0$ or $o = m-1$. When $m = 2$, there are only two resistance strategies, $o = m-1 = 1$ and $o = 0$. So to show that the optimal resistance strategy is $o = 0$ we need only show that $o = m-1$ is not optimal. To show this note that if $o = m-1$ is optimal than this strategy must produce a value for the potential function at least as high as the value produced by $o = 0$ and $o = m-2$. Thus it must be the case that

$$\Pi_{m-1}^m \geq \Pi_K^m, \tag{A-35}$$

$$\Pi_{m-1} - \Pi_{m-2} = \Delta\Pi_{m-2} > 0. \tag{A-36}$$

The definition of the potential function and Lemma Xreflem:PropertiesDeltaPi.enum:DeltaPiOSgn show that these two conditions will only be satisfied when

$$my - (1-\gamma)^{m-1} \leq 0, \tag{A-37}$$

$$y - b(1; m, \gamma) \geq 0. \tag{A-38}$$

Noting that $b(1; m, \gamma) = m\gamma(1-\gamma)^{m-1}$, we see that, because $m \geq 2$ there exists no $\gamma > \frac{1}{2}$ that can satisfy both equation (A-37) and (A-38). Thus the if resistance is optimal, the complete resistance strategy, $o = 0$, is the optimal resistance strategy. □

Proof of Lemma 4. This result is a direct consequence of Result 2 and Result 3 and the following result.

Result 6. If $\gamma > \frac{1}{2}$, and capitulation, $(o = 2m-1)$ is optimal at m, then capitulation is strictly optimal at $m+1$

Proof of Result 6. For the sake of readability define, for this proof only, the probabilities of the proposal passing when $o = 0$, given passing threshold m and thus $2m-1$ universal owners.

$$p^m := \hat{B}(m, 2m-1, \gamma).$$

By hypothesis, $\Pi_K^m \geq \Pi_0^m$ We need to show that this hypothesis implies that $\Pi_K^{m+1} \geq \Pi_0^{m+1}$. To see this note that

$$\Pi_K^m \geq \Pi_0^m \Longleftrightarrow \Big((2m-1)\,y - \frac{1}{1-\gamma}\Big) - \Big(0\,y - \frac{p^m}{1-\gamma}\Big) \geq 0, \tag{A-39}$$

$$\Pi_K^{m+1} \geq \Pi_0^{m+1} \Longleftrightarrow \Big((2m+1)\,y - \frac{1}{1-\gamma}\Big) - \Big(0\,y - \frac{p^{m+1}}{1-\gamma}\Big) \geq 0, \tag{A-40}$$

Result 2 shows that, when $\gamma > \frac{1}{2}$, $p^{m+1} > p^m$ and $2m+1 > 2m-1$. Hence, we see that satisfaction of (A-39) implies the satisfaction of (A-40). □

Result 3 shows that, at m, the potential maximizing o-strategy is either complete resistance,$o = 0$, or capitulation, $o = K$. If capitulation is optimal, then Result 6 shows that the potential maximizing strategy at $o = m+1$ is also capitulation. So, at both m and $m+1$, the probability the proposal passes equals one.

If complete resistance maximizes the potential at m, then, Result 3 shows that, at $m+1$, the potential maximizing strategy is either capitulation or complete resistance. Clearly if the potential maximizing strategy is capitulation, the probability of proposal success is higher at $m+1$. If at $m+1$ the potential maximizing strategy is also complete resistance, $o = 0$, then Result 2 shows that the probability of success is higher at $m+1$. □

Proof of Result 5. Most of this proof is supplied by earlier results. To prove (i) note that the Result 2 shows that, when $\gamma > \frac{1}{2}$, increasing ownership dispersion increases the probability of passage under the complete resistance, $o = 0$ strategy. when there is one universal owner capitulation is never optimal (by our assumption that $R < \Delta W$) and resistance is always complete resistance. Thus, if the potential maximizing policy under dispersed ownership is complete resistance, the monetary payoff of the universal owners is higher under unified ownership. If, under dispersed ownership, the potential maximizing o-strategy is capitulation, the monetary payoff equals $W(F) - \Delta W$. Under unified ownership, the single owner resists and thus the monetary payoffs equals $W(F) - \Delta W + (1-\gamma)(\Delta W - R)$. Hence, the monetary payoff is larger under unified ownership. Result 3 shows that, when $\gamma > \frac{1}{2}$, the only candidate optimal o-strategies are complete resistance or capitulation.

To prove (ii.a) note that, as shown by Xreflem:PropertiesDeltaPi, for γ sufficiently small, $o^* = m-1$ and the probability of the proposal passing is thus, $1-(1-\gamma)^m$. Thus, under dispersed ownership the monetary payoff equals

$$W(F) - (1-\gamma)R + \left((1-\gamma)\left(\frac{m-1}{2m-1}\right)R - \Delta W\left(1-(1-\gamma)^m\right)\right).$$

Under unified ownership, the monetary payoff equals

$$W(F) - (1-\gamma)R - \Delta W\,\gamma.$$

So, we see that, for γ sufficiently small, the monetary payoff is larger under dispersed ownership.

To prove (ii.b), Note that because under unified ownership complete resistance is the optimal strategy, if complete resistance is also the potential maximizing strategy under dispersed ownership, expected reputation costs are identical under unified and dispersed ownership. Because, by assumption, $\gamma < \frac{1}{2}$, Result 2 shows that the probability of success is less under dispersed ownership. Thus, the monetary payoff is larger under dispersed ownership. □

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