Demand-System Asset Pricing: Theoretical Foundations*

Preliminary – comments welcome.

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Abstract

Recent approaches to asset pricing involve the estimation of demand systems for financial securities in which investors are permitted to have non-pecuniary *tastes* over cash flow-irrelevant asset characteristics. We investigate theoretical foundations of demand-system asset pricing using multiple approaches to integrating tastes with portfolio choice. Our analysis raises several conceptual issues, including the appropriate notion of no arbitrage, the pricing of "redundant" assets, and the cardinal interpretation of taste parameters. These issues imply multiple barriers to identifying demand systems for financial securities from observational data, and raise questions about the structural interpretation of financial demand elasticities. We discuss how these issues affect counterfactuals constructed from estimated demand systems.

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1 Introduction

Recent approaches to asset pricing following Koijen and Yogo (2019) involve estimating *demand systems* for financial assets in which investors are permitted to have non-pecuniary tastes (or dogmatic beliefs) regarding the value of certain asset characteristics, such as environmental or social scores. According to this approach, observed portfolio holdings are related to investor preferences and beliefs, and thus carry information absent from prices that can inform researchers about the equilibrium response to a variety of shocks, such as investor tastes or capital reallocation (Koijen, Richmond, and Yogo, 2022).

This approach represents a sharp break from classical asset pricing, which emphasizes the fungibility of securities up to their cash flows and focuses on price data disciplined by no arbitrage. Given the unsatisfactory empirical record of classical approaches, this stark dichotomy may well be an advantage. However, introducing preferences over the *provenance* of cash flows, as opposed to over cash flows alone, also means jettisoning much of the well-established theoretical foundations that underlie classical approaches. Thus, as we move forward in this new direction, it is important to lay down proper foundations for this approach and understand their implications for empirical work.

In this paper, we examine the theoretical foundations of demand-based asset pricing by way of synthesis with canonical models of portfolio choice. In particular, we enrich an endowment economy as in Lucas (1978) with potentially payoff-irrelevant characteristics that affect investors' *tastes* over assets, and derive portfolio demand functions that are sensitive to risk, return and tastes.¹ This allows us to distinguish "hedonic characteristics," such as environmental scores, from "cash flow characteristics" that have been traditionally used to summarize the statistical properties of the cash flow distribution.

The incorporation of hedonic characteristics forces us to revisit some fundamental conceptual issues in theoretical asset pricing, such as the appropriate notion of no arbitrage and the use of cardinal versus ordinal utility indices for tastes. We find that, depending on the asset span, taste-based demand may lead to generic arbitrage opportunities, and that integrating risk and return with tastes *requires* a cardinal interpretation of taste parameters. These results have sharp implications for equilibrium short sales and

¹As will become clear, there is close correspondence between preference-based tastes, and dogmatic beliefs about asset returns. For expositional ease, we focus on tastes, but our results apply to dogmatic beliefs as well.

the pricing of redundant securities, as well as for the sensitivity of demand systems to (arbitrary) changes in the unit of measurement for tastes.

We then draw out implications of these conceptual issues for demand-system identification using observational data on portfolio holdings. We argue that identification of preference parameters is significantly more difficult in financial markets than in canonical consumer good settings, and particularly so if short sales are either prohibited or unobserved. As such, demand systems estimated using state of the art methods may produce valid counterfactuals only for a narrow range of scenarios.

Demand estimation is perhaps the central problem in industrial organization. Accordingly, industrial organization researchers have developed a rich set of tools to estimate preferences parameters from observational data (Berry and Haile, 2021). However, financial assets present a number of unique challenges that differ sharply from the demand for real goods typically studied in industrial organization. Thus, as we discuss, the application of IO techniques is far from straightforward in the case of financial securities.

The first pertains to no arbitrage and the pricing of redundant securities. In contrast to most consumer good settings, traders who find a financial asset too expensive are not forced to exit the market; they can (short) sell the asset. Importantly, they may do so either directly or by trading an alternative portfolio that replicates the asset's cash flows. To discipline equilibrium prices and trading behavior given these considerations, classical asset pricing uses the notions the law of one price and and no arbitrage, which is the idea that investors should not be able to receive "something for nothing."

When investors have tastes over non-pecuniary characteristics, two assets with identical cash flows need no longer have the same price. Thus, we need to modify the definition of the Law of One Price to account for the provenance of the cash flows. Yet, even with this broader understanding of the value of an asset, we find that with sufficiently dissimilar tastes (given a set of allowable trades) in general there would not exist a pricing function (stochastic discount factor) that leaves no arbitrage opportunities. This has direct implications for equilibrium existence, incentives for short sales, and for the use of demand systems for pricing untraded securities. In applied contexts, estimated demand systems may be quite sensitive to misspecified investment universes (i.e., the set of assets investors can trade, and the extent to which investors can engage in short sales).

The second pertains to investor preferences. Models of portfolio choice are based

on theories of choice under uncertainty, such as expected utility theory. As is well known, expected utility theory imposes a *cardinal* interpretation of utility, which is a stronger requirement than the ordinal rankings typically assumed in consumer good demand systems. Under a cardinal interpretation of tastes, identifying financial demand systems requires identification of the *intensity* of tastes *relative* to risk-return considerations, not just their ordinal ranking. In order to integrate non-pecuniary tastes with risk and return, one must therefore admit a cardinal interpretation of taste parameters as well.

This fact has practical relevance. For counterfactuals, preference parameters must be estimated for inframarginal investors as well. Moreover, portfolio choice is generically sensitive to rank-preserving transformations of tastes, such as changes in the units of measurement. Yet, investors may not agree on how to evaluate the "greenness" of an asset. They may also find little consensus on how to *aggregate* multiple hedonic characteristics, such as a firm's environmental social scores, into a single asset-level score. Nevertheless, empirical demand systems may be sensitive to such choices.

Having discussed these conceptual issues, we use a fully specified model of portfolio choice under tastes to examine questions of identification and counterfactuals based on observational data. The model is a variant of the Lucas (1978) endowment economy in which assets ("trees") may be endowed with payoff-irrelevant hedonic characteristics, and investors can differ in their tastes for these characteristics. Addressing the question of cardinal preferences, we work within the expected utility framework. This leads to a framework where investors trade off tastes against canonical risk return considerations, and asset-level taste parameters can be interpreted in marginal utils.

Our model nests two important benchmarks. First, if two assets offer identical cash flows but differ in their hedonic characteristics, then an investor may prefer to buy only the one that aligns with his tastes. In this case, tastes lead to equilibrium sorting. Second, if all investors have the same tastes, then the model is equivalent to the canonical Lucas tree framework that considers only risk and return. When both channels are active, prices are affected by both fundamental cash flows and taste distributions.

This framework has several useful features. Since we fully specify the model, we can explicitly solve for the "correct" counterfactual response to various shocks. Hence, it is a useful laboratory for evaluating identification strategies. In additional, we can explicitly model common approaches to identification that rely on mandates (i.e., exogenous

restrictions on the type of assets an investor can hold) to obtain variation in prices. These are simply constraints on portfolio weights of particular assets. Hence, we use the model to assess identification challenges in financial demand system estimation.

In general, demand system estimation requires price instruments whose variation is sufficient to identify tastes for specific assets. For a general portfolio choice problem, this is exceedingly difficult because the principle of diversification implies that any two assets may exhibit non-linear patterns of complementarity and substitutability that are shaped by the investor's overall portfolio holdings. Hence, in general, even "clean" variation in a single asset price is not enough to identify demand systems because endogenous portfolio changes create demand shocks in other assets (Berry and Haile, 2021).²

Even if price instruments are available, they may not suffice for identifying all taste parameters required to construct counterfactuals based on observational data. Given taste-based demand, an investor may only purchase her most preferred option in equilibrium, and may short sell less-preferred options. If short sales are prohibited, observed portfolio holdings may allow for inference on the *ordinal* ranking of a choice set, but there may not be enough information to estimate cardinal tastes for assets not purchased in equilibrium. When short sales are allowed but unobservable to the econometrician, as they often are in practice, demand systems may be misspecified. As we illustrate using our general equilibrium model, in either case it may be difficult to estimate taste parameters sufficiently accurately to perform counterfactuals. More broadly, our analysis points to a critical question that must be addressed empirically, which is to what extent, if any, derivative securities inherit some of the taste properties of the underlying asset.

To evaluate these concerns, we use our framework to model common identification approaches used in the demand-system asset-pricing literature. For example, Koijen and Yogo (2019) use the sparsity of observed portfolios to construct instruments for asset prices. As the argument goes, sparse portfolios indicate that the investor may have tightly prescribed mandates that make it costly to hold other stocks. If mandates and fund flows are sufficiently exogenous to current investment opportunities, variation in the extent to which a particular stock is present in observed portfolios may be used to

²One way around this problem is to restrict attention to settings where optimal portfolio weights are linear functions of own prices and characteristics (Koijen, Richmond, and Yogo, 2022). In a static setting, this is possible with, e.g., CARA preferences and normally distributed shocks to payoffs. However, even these assumptions would not suffice in a dynamic setting. Given the importance of taste for portfolio choice, prices would be a function of endogenous changes in the wealth distribution.

construct a demand shifter that is independent of prices. We use our model to assess this identification strategy, *taking as given* the exogeneity of mandates and fund flows. An important concern is that mandates and tastes may be observationally equivalent given equilibrium play, even as they have potentially very different implications for counterfactuals. The reason is that mandate investors are insensitive to price changes, whereas taste-based investors are not. As such, misjudging the share of mandate investors can lead to *qualitatively* different counterfactual prices in response to shocks.

These concerns are naturally entangled with wealth effects. While price changes always induce income and substitution effects, in most consumer good settings wealth effects are likely to be negligible and are thus frequently ignored when modeling demand. In contrast, financial assets are investment goods, and so wealth changes may have first order effects in portfolio choice. In particular, an investor's measured elasticity for two otherwise identical assets may be very different depending on if they already hold the asset in their portfolio or not.³ Hence, it is important to control for the evolution of portfolios when estimating demand elasticities.

We end by discussing the structural interpretation of demand elasticities. First, we discuss how the structural interpretation of demand elasticities depends critically on the investment universe. In particular, when there are close substitutes available, *asset-level* demand elasticities may be very high even when *consumption-level* are low. Second we show that even in classic general equilibrium frameworks such as Lucas (1978), demand elasticities can range from near zero to near infinite, depending on whether the driving shocks are common to all investors are merely introduce reallocative tades between investors. For these reasons, it is difficult to discriminate between models of asset pricing based on estimated demand elasticities alone.

The rest of the paper is structured as follows. The rest of this section discusses related literature. Section 2 discusses conceptual issues related to incorporating non-pecuniary tastes in an asset pricing framework. Section 3 provides our framework. Section 4 studies the implications on identification and counterfactuals. Section 5 discusses the structural interpretability of asset demand elasticities. Section 6 concludes.

³This concern is amplified when interpreting investors as financial intermediaries, since capital flows from households can induce wealth changes even when intermediary constraints and preferences are fixed.

Related Literature

This paper studies the theoretical foundations of two closely-related literatures: demandbased asset pricing that tries to model equilibrium returns using estimated portfolio choice models as in Koijen and Yogo (2019), and models in which investors may hold certain financial securities because of non-pecuniary values associated with them (Starks, 2023).

Demand-based approaches have been used to address a number of substantive questions. These include computing counterfactuals for price informativeness and sustainable investing in response to changing non-pecuniary tastes or changes in the size distribution of institutional investors (Koijen, Richmond, and Yogo, 2022), global imbalances and currency returns (Jiang, Richmond, and Zhang, 2023) or corporate bond returns (Bretscher, Schmid, Sen, and Sharma, 2022).

The fact that estimated demand systems appear to reveal that financial institutions exhibit rather low demand elasticities has also been used to argue that financial markets as a whole are inelastic, with implications for the equity premium (Gabaix and Koijen, 2020). We show that such elasticities may not always be interpretable as deep parameters, and may differ by the level of aggregation and the nature of the shock. Inelastic trading patterns are also often attributed to mandates, and these are used as instruments to identify demand parameters. Our analysis suggests that it is difficult to empirically distinguish between mandates and "tastes," and this matters for counterfactuals.

Valued-based approaches have been used to study investment in so-called "green assets," such as stocks or bonds associated with sustainable, environmentally-friendly firms or government expenditures. Pastor, Stambaugh, and Taylor (2021) provide an equilibrium model of such sustainable investment, whereby firms differ in their "green scores," but the set of marketable securities consists only of firm shares and a risk-free asset. We address implications and micro-foundations of such sustainable investing for financial market equilibrium with redundant securities under various forms of tastes, and ask how such tastes might be identified in equilibrium.

Pastor, Stambaugh, and Taylor (2022) provide early evidence that such tastes may be reflected in yield differences between otherwise identical German government bonds, while D'Amico, Klausmann, and Pancost (2023) show that the underlying "greenium" appears to have shrunk over time. Our analysis shows that, in the presence of short selling by at least some investors, premiums extracted from marginal prices (i.e., prevailing market prices) may not be informative about infra-marginal preferences, and thus may not be sufficient to infer counterfactuals. More generally, we argue that counterfactuals are sensitive to the precise modeling of environmental concerns, including to simple monotone transformations of taste parameters. As such, one conclusion of our paper is that there is research in modeling the precise foundations of tasted-based investment.

2 Incorporating Tastes into an Asset Pricing Framework

Demand-based asset pricing allows investors to have preferences over asset characteristics that may or may not be directly related to cash flows. Incorporating tastes within canonical asset pricing frameworks requires making a number of important conceptual decisions that can substantially alter some of the key theoretical underpinnings of asset pricing theory. This is because asset pricing requires a theory of choice under uncertainty, and this places relatively stringent constraints on the way we model investor preferences.

The standard approach is von Neumann-Morgenstern expected utility theory, which defines preferences over lotteries (distributions over payoffs). This approach is axiomatic, and thus satisfies a number of desirable properties. However, it requires a *cardinal* interpretation of utility functions, whereby only positive linear transformations of the utility function preserve the underling preferences. We will show that this leads to two main concerns. The first is that predictions for equilibrium portfolio choices under tastes will be sensitive to the precise formulation of taste parameters (or taste functions). The second is that identification of demand system requires the identification of taste *intensities*, not just their ordinal ranking.

We consider two main formulations of tastes, and variants thereof. The first is consumption-augmenting tastes, by which we mean that depending on their provenance the investors derive additional "consumption-equivalent" value from asset cash flows. This captures the notion that an investor may, ceteris paribus, value cash flows produced by environmentally-friendly firms more than cash flows produced by other firms.⁴ The second is consumption-separable tastes, by which we mean that the investor obtains some additional value (or disutility) from holding certain assets. This captures the idea that an investor may earn a "warm glow" from its holding in addition to potential cash flows.

⁴This is similar to the idea that some consumers are willing to pay more for fair trade certified foods which are otherwise identical.

However, it will turn out that both approaches lead to similar conceptual issues.

Consumption-Augmenting Tastes. Consider a one-shot portfolio choice problem in which an investor can choose to consume at date 0 and/or at date 1. A random state of the world $z \in \mathcal{Z} := \{1, ..., Z\}$ is realized at date 1, and the probability of state z is $\pi_z \in (0, 1)$. The set of assets is \mathcal{J} . Asset $j \in \mathcal{J}$ offers state-contingent cash flows $y_j(z)$ in state z. There is a set of investors indexed by i. Investor i has a von Neumann-Morgenstern utility function defined over lotteries. She evaluates her payoffs from holding a certain portfolio of assets $(a_j^i)_{j \in \mathcal{J}}$ by the cash flows it generates, and by her *tastes* $(\theta_j^i)_{j \in \mathcal{J}}$ over assets, where $\theta_j^i > 0$. To integrate tastes with portfolio choice, we define the *effective units of consumption* delivered by a portfolio (a_i^i) for consumer i in state z to be

$$\tilde{c}_1^i(z) = \sum_j \theta_j^i y_j(z) a_j^i + w_1^i(z),$$

where $w_1^i(z)$ captures a non-marketable endowment that may be zero.⁵ Tastes differentiate effective consumption from pure consumption, which is defined as

$$c_1^i(z) = \sum_j y_j(z) a_j^i + w_1^i(z).$$

Under consumption-augmenting tastes, investor *i*'s portfolio maximization problem is to maximize expected utility over effective consumption subject to budget balance,

$$\begin{aligned} \max_{\substack{(a_j^i)_{j\in\mathcal{J}}}} & (1-\beta)u^i(c_0^i) + \beta \sum_{z\in\mathcal{Z}} \pi_z u^i(\tilde{c}_1^i(z)) \end{aligned} \tag{P-CA} \\ \text{s.t.} & c_0^i = w_0^i - \sum_{j\in\mathcal{J}} p_j(a_j^i - e_j^i) \\ & \tilde{c}_1^i(z) = \sum_{j\in\mathcal{J}} \theta_j^i y_j(z) a_j^i + w_1^i(z), \end{aligned}$$

where β is the discount factor, p_j is the price of asset j, w_0^i is initial wealth, and e_j^i is investor i's endowment of asset j. It is clear that tastes can lead to heterogeneous portfolios even

⁵Another possibility is to define consumption-augmenting tastes in an additive manner, $\tilde{c}^i(z) = \sum_j (\theta_j^i + y_j(z))a_j^i + w_1^i(z)$. The main difference is that tastes operate like a "risk-free" component of returns for every asset, with obvious implications for portfolio choice. Overall, however, the main conclusions are unchanged.

when investors share beliefs about the cash flows generated by all assets and do not differ in their non-marketable endowment.

Additive Separable Tastes. In the previous approach, tastes directly modified an investor's evaluation of her consumption process. An alternative specification is to model tastes as affecting investor utility additive-separably from consumption according to function $G^i\left((a_j^i)_{j\in\mathcal{J}}\right)$ that maps the investor's portfolio into utils. The decision problem under separable tastes can then be written as:

$$\max_{\substack{(a_{j}^{i})_{j \in \mathcal{J}} \\ \text{s.t.}}} (1 - \beta) u^{i}(c_{0}^{i}) + \beta \sum_{z \in \mathcal{Z}} \pi_{z} u^{i}(c_{1}^{i}(z)) + G^{i}\left((a_{j}^{i})_{j \in \mathcal{J}}\right)$$
(P-AS)
s.t. $c_{0}^{i} = w_{0}^{i} - \sum_{j \in \mathcal{J}} p_{j}(a_{j}^{i} - e_{j}^{i})$ $c_{1}^{i}(z) = \sum_{j \in \mathcal{J}} y_{j}(z) a_{j}^{i} + w_{1}^{i}(z),$

where the utility function is now defined over pure, rather than effective units of, consumption. We assume that $G^i(\cdot)$ is twice continuously differentiable in each element. We also assume that it is strictly monotone in each element, but allow the marginal contribution to the non-pecuniary value to be positive for some assets and negative for others.

The main difference from the consumption-augmenting approach is that the nonpecuniary benefits of holding certain assets do not directly depend on the properties of the utility function u^i . For example, tastes do not necessarily induce wealth or substitution effects in portfolio choice. While this may be an advantage for particular applications, it also has the drawback that it is generally difficult to discipline the particular functional form of G^i . Yet, the functional form *will* generally determine the trade-off between pecuniary and non-pecuniary aspects of portfolio choice. Hence, portfolio choices will generically not be invariant to monotone linear transformations of G^i .

The rest of this section discusses two main implications of asset pricing with tastes: (i) that they may invalidate standard notions of no arbitrage that form the backbone theoretical asset pricing, and (ii) that they require cardinal interpretations of tastes (despite various characteristics being difficult to measure).

2.1 No Arbitrage with Tastes

Perhaps the most important theoretical concept in classical asset pricing theory is no arbitrage, which is the requirement that an equilibrium price system should not admit trades in which an investor receives something for nothing. Operationalizing this concept requires a theory of value. In the classical approach, the appropriate notion of value is cash flows, and an arbitrage is "something for nothing," or, more formally, "an investment strategy that guarantees a positive payoff in some contingency with no possibility of a negative payoff and no initial net investment" (Ross, 2004).

In the classical approach, a payoff space *X* (the set of attainable payoffs) and a pricing function *p* are sufficient to define no arbitrage.

Definition 1 (No Arbitrage (Cochrane, 2005)) A payoff space X and pricing function p leave no arbitrage opportunities if every payoff $x \in X$ that is always weakly positive, $x \ge 0$ almost surely, and strictly positive, x > 0, with some positive probability, has positive price, p(x) > 0.

This definition is critical for deriving basic properties of price systems, including the existence of positive stochastic discount factors.

When investors differ in their tastes, the aforementioned definition is not sufficient because investors do not evaluate investment opportunities on the basis of cash flows alone. In particular, investors may have subjective views on what constitutes "something for nothing." Because investors also evaluate the value of cash flows in part according to their *provenance* (i.e., which assets generated said cash flows), a payoff space alone is not sufficient for defining valuations. We therefore define no arbitrage under payoffaugmenting tastes as follows (Analogous results obtain with additive-separable tastes).

There are *J* assets indexed by $j \in \mathcal{J}$. The $J \times Z$ matrix of cash flows is *Y*. A vector of holdings in \mathbf{R}^J defines a portfolio denoted by $a \in \mathcal{A}$, where \mathcal{A} is the set of feasible portfolios. The price of asset *j* is p_j , and pricing function $P : \mathcal{A} \to \mathbf{R}$ maps portfolios into portfolio prices according to $P(a) = \sum_j p_j a_j$.

Investor *i* has a taste function $v^i : \mathcal{A} \to \mathbf{R}^Z$ that maps a portfolio *a* into a $1 \times Z$ vector of state-contingent taste-augmented payoffs. In the absence of tastes, all investors care only about cash flows as in standard asset pricing, $v^i(\omega) = a \cdot Y$.

Definition 2 (No Arbitrage with Payoff-Augmenting Tastes) Given a set of assets A and taste functions v^i for all investors *i*, pricing function P leaves no arbitrage opportunities if, for any

investor *i* and any portfolio such that the effective payoff is weakly positive, $v^i(a) \ge 0$, and strictly positive, $v^i(a) > 0$ with strictly positive probability, the associated price is positive, P(a) > 0.

We then have the following theorem regarding no arbitrage with tastes.

Theorem 1 (Generic Arbitrage Opportunities with Payoff-Augmenting Tastes) Fix a set of assets and taste functions v^i for all investors. There does not exist pricing function P such that there are no arbitrage opportunities for any investor if and only if:

there exist
$$a, v^i, v^{i'}$$
 such that $v^i(a) > 0$ and $v^{i'}(a) \le 0$. (C)

A sufficient but not necessary condition for (C) is that there exists assets j and j' such that

(i) both assets have identical cash flows

$$y_j(z) = y_{j'}(z)$$
 for all $z \in \mathcal{Z}$

(ii) there exist investors i and i' with sufficiently heterogeneous tastes with respect to these assets,

$$v_j^i \ge v_{j'}^i$$
 and $v_j^{i'} \le v_{j'}^{i'}$ with at least one inequality strict.

The central content of this result is that no arbitrage fails if tastes are sufficiently heterogeneous and the asset menu is sufficiently rich. This can be most transparently seen in a simple two asset economy.

Example 1 (Green and Red Assets) There is a green asset and a red asset with prices denoted by p_g and p_r , respectively. Both assets deliver a unit payoff with certainty. There are two investor types that differ in their relative taste for the two assets. Type 1's tastes-augmented payoffs for green and red assets satisfy $\theta_g^1 > \theta_r^1$, respectively; Type 2 has $\theta_g^2 < \theta_r^2$. We consider a long-short portfolio consisting of selling one unit of the green asset and buying one unit of the red asset. The price of this portfolio is $p(\{-1,1\}) = p_r - p_g$. Hence, investor i's taste-augmented payoff is $\theta_r^i - \theta_g^i$. The absence of arbitrage opportunities for Type 2 requires that $p^* < 0$. Since Type 1 can conduct the trade in reverse, no arbitrage for that type requires $p^* > 0$. Hence, there exists no prices $\{p_g, p_r\}$ such that no type has an arbitrage opportunity.

The result is a direct implication of the fact that investors care about non-pecuniary factors, that taste differences are invariant to quantities (i.e., marginal valuation differ-

ences are invariant to portfolio holdings), and that investors are free to short either asset. Price changes are then not sufficient to equilibriate asset markets.

Remark 1 (Implications for Law of One Price) Violations of no arbitrage may exist even if the law of one price (LOOP) holds conditional on the asset "color." In particular, in the presence of tastes LOOP can be defined as requiring that two assets which deliver identical taste-augmented payoffs must have the same price. However, the example shows that, even if LOOP holds for individual assets, one can still construct portfolios over which investors have strict disagreements (in the sense that any price system must offer a strict arbitrage opportunity to at least one investor.)

As we explain in more detail in later sections, this result has a number of theoretical and practical implications. On the theory side, one must worry about equilibrium existence and/or the precise nature of short-selling costs. In terms of practical implementation, it appears particularly difficult to identify taste parameters without observing short sales (as is often the case for common data sources). On the conceptual front, one must worry about whether redundant assets (in terms of their cash flows) inherit the non-pecuniary benefits of underlying assets. This raises a number of questions about the precise foundations of non-pecuniary tastes. Do they relate to the ultimate projects they fund? Are they fungible across financial instruments, or are they tied to governance aspects of e.g. stocks? If so, do they matter only to the extent that ownership allows the investor to affect equilibrium governance? Answers to these questions appear critical to be able to develop a full-fledged theory of taste-based asset pricing.

2.1.1 Restoring No Arbitrage under Tastes

There are two ways to potentially restoring no arbitrage under tastes. The first is to restrict the set of feasible portfolios, for example by not allowing short-sales on certain assets. A downside of doing so is that equilibrium prices will then closely related to the precise formulation of these constraints, and that equilibrium will be sensitive to potentially unobservable trading opportunities.⁶

A second approach is to introduce a notion of decreasing marginal tastes, whereby the marginal augmentation of payoffs by tastes converges to zero for sufficiently large

⁶There is a similarity between the failure of no arbitrage under tastes and equilibrium "mispricing" due to dogmatic differences in *beliefs* about cash flows. The key difference is that tastes are defined over assets characteristics rather than cash flows. Hence violations of no arbitrage may persist even when two assets are known to deliver identical cash flows.

asset holdings. This allows investors to potentially agree on marginal valuations even if they differ in their infra-marginal tastes. The downside of this approach is that it presents additional identification issues when trying to measure the taste function.

2.2 Sensitivity to Rank-Preserving Transformations of Tastes

Expected utility preferences imply a cardinal interpretation of utility functions, which means that the underlying preferences are preserved only under positive linear transformations of the utility function. We now show that this leads to a strong sensitivity of optimal portfolio choices to the intensity and functional form of tastes, even holding ordinal rankings of taste parameters fixed.

Define investor i's marginal rate of substitution between effective units of consumption in state z and state 1 as:

$$\tilde{\Lambda}^{i}(z) \equiv \frac{\pi_{z}\beta u^{i'}\left(\sum_{j}\theta^{i}_{j}y_{j}(z)a^{i}_{j}+w^{i}_{1}(z)\right)}{(1-\beta)u^{i'}\left(w^{i}_{0}-\sum_{j}p_{j}(a^{i}_{j}-e^{i}_{j})\right)},$$

where $u^{i'}$ is the derivative of the utility function. The analogue marginal rate of substitution with respect to pure consumption is:

$$\Lambda^{i}(z) \equiv \frac{\pi_{z}\beta u^{i'}\left(\sum_{j}y_{j}(z)a_{j}^{i}+w_{1}^{i}(z)\right)}{(1-\beta)u^{i'}\left(w_{0}^{i}-\sum_{j}p_{j}(a_{j}^{i}-e_{j}^{i})\right)}.$$

The optimality condition for a_i^i with consumption-augmenting tastes is:

$$\theta_j^i \sum_z y_j(z) \tilde{\Lambda}^i(z) = p_j.$$

The optimality condition for a_i^i under separable tastes is:

$$\sum_{z} y_j(z) \Lambda^i(z) + g_j^i\left((a_j^i)_{j \in \mathcal{J}}\right) = p_j,$$

where g_i^i is the partial derivative of G^i with respect to a_i^i .

These conditions relate the standard risk-return tradeoff, measured by the distribution of marginal utility across states, to investor tastes. The solutions to these programs depend on the functional form of tastes and are not invariant to simple monotone transformations. This means that identifying demand systems requires measuring the intensity of tastes, not just their ordinal ranking.

Proposition 1 (Sensitivity to Rank-preserving Transformations) The solution to Program (P-CA) is sensitive to rank-preserving transformations of $\theta^i = (\theta_1^i, \theta_2^i, \dots, \theta_J^i)$, including positive linear transformations and mean-preserving spreads of taste parameters. The solution to Program (P-AS) is sensitive to monotone transformations of $G^i(\cdot)$, including linear positive transformations such as the re-scaling of the units in which tastes are measured.

The intuition for this result is straightforward. In the case of consumption-augmenting tastes, increasing the nominal value of θ_j increases the consumption-equivalent value of holding asset *j*, leading the investor to allocate more funds to this asset and distorting the overall portfolio. Increasing tastes for all assets simultaneously raises the value of holding all assets, which leaves portfolio weights unchanged but alters the consumption-saving decision between dates 0 and 1.

In the case of separable tastes, portfolio choices trade off the marginal increase in non-pecuniary values against the risk-return trade-off as measured by marginal utility over consumption. Since expected utility is cardinal, any rank-preserving transformation (such as a change in units) will alter the optimal portfolio. This is concerning because non-pecuniary tastes do not have natural units of measurement.

3 Framework

The previous section discussed some foundational conceptual issues related to incorporating non-pecuniary tastes in an asset pricing framework. In this section, we construct a tractable framework with consumption-augmenting tastes to study issues of identification and counterfactuals using state-of-the-art methods.

We use the consumption-augmenting approach because it allows us to use much of the theoretical scaffolding of expected utility theory and it aligns closely with existing approaches in the literature (Koijen, Richmond, and Yogo, 2022). However, the main results carry over to the case of additive-separable tastes.

3.1 A Taste-Augmented Lucas (1978) Economy

Environment. There is a single period of trading and all information is public. Asset payoffs depend on the realization of an aggregate state $z \in \{1, 2\}$ that is realized at date 1. The probability of state z is given by $\pi_z \in (0, 1)$. Associated with each state z is a Lucas tree that pays off y_z if the state is z. Trees are perfectly divisible, and the aggregate supply of each tree is equal to one. The tree associated with state 1 is composed of two equally sized branches: *red* and *green*. Conditional on aggregate state 1, the green branch pays $y_g(\iota)$ and the red branch pays $y_r(\iota)$, where $\iota \in \{r, g\}$ is a distributional shock that determines which of the two branches offers more cash flows. In particular, let

$$y_g(\iota) = \begin{cases} y_1 - \epsilon & \text{if } \iota = r \\ y_1 + \epsilon & \text{if } \iota = g \end{cases} \quad \text{and} \quad y_r(\iota) = \begin{cases} y_1 + \epsilon & \text{if } \iota = r \\ y_1 - \epsilon & \text{if } \iota = g \end{cases}$$

Given these payoffs, it is clear that the distributional shock is fully diversifiable because

$$\frac{1}{2}y_g(\iota) + \frac{1}{2}y_r(\iota) = y_1 \text{ for all } \iota.$$

Parameter $\epsilon \in [0, y_1)$ determines the substitutability of red and green trees. If $\epsilon = 0$, then red and green trees are perfect substitutes with respect to their cash flows. If $\epsilon > 0$, they are complements because holding both serves to diversify distributional risk. The probability of the red tree doing better is denoted by $Pr(\iota = r) = \rho$.

Given this structure, one can think of assets as having two "characteristics:" the aggregate state of the world in which their payoffs accrue (i.e., 1 or 2), and their color. These characteristics determine in which states cash flows accrue (and thus serve as useful statistical summaries of the overall cash flow distribution), and they can also be used to define non-pecuniary tastes. When Type 1 trees are perfect substitutes ($\epsilon = 0$), the color characteristic is irrelevant for cash flows and *only* matters through its link with tastes. When Type 1 trees are imperfect substitutes ($\epsilon > 0$), even investors without tastes ($\theta_j^i = 1$) care about the color characteristic because it summarizes cash flow risk.

To focus on variation in the price of red and green trees, we use the following assumption in all of our numerical examples.

Assumption 1 (Aggregate symmetry) Aggregate payoffs are $y_1 = y_2 = 1$, and the probability of each aggregate state is equal to one half, $\pi_1 = \frac{1}{2}$.

Investors. There are two types of investors indexed by *i*. (Our results readily generalize to many types, or to a continuum of types.) Types determine an investors' endowment, tastes, and mandates. Specifically, let investor *i* be endowed with e_g^i, e_r^i and e_2^i units of green, red, and state 2 trees, where aggregate feasibility dictates that

$$\sum_{i} e_j^i = \frac{1}{2} \text{ for } j \in \{g, r\} \text{ and } \sum_{i} e_2^i = 1.$$

Although this is not necessary, it is helpful to work with a relatively symmetric setting. Hence we will typically assume that Type 1 owns share $\omega \ge \frac{1}{2}$ of the aggregate endowment of each tree, $e_g^1 = e_r^1 = \frac{\omega}{2}$ and $e_2^1 = \omega$.

Investor *i* takes positions a_j^i in asset $j \in \mathcal{J} \equiv \{g, r, 2\}$, and may be subject to short sale constraints: $a_j^i \ge 0$. The investor evaluates the payoffs of his portfolio using *effective units of consumption*. In particular, this object is defined as

$$\tilde{c}^i(z) = \sum_{j \in \mathcal{J}} \theta^i_j y_j(z) a^i_j.$$

In this expression, θ_j^i represents agent *i*'s private *tastes* over assets. Taste parameters allow us to nest characteristics-based demand as distinct from cash-flowed based risk-return considerations. For simplicity, we assume that tastes are irrelevant for Tree 2: $\theta_2^i = 1$ for all *i*. Hence, tastes only affect relative preferences for red and green trees. Since Section 2 has shown that rank-preserving variation in the taste distribution can have independent effects on portfolio choice, for transparency we will mainly focus on the sparse specification $\theta_g^1 = 1 + t$ and $\theta_r^1 = 1 - t$, while $\theta_g^2 = 1 - t$ and $\theta_r^2 = 1 + t$. This means that Type 1 prefers green while Type 2 prefers red. However, this choice is not necessary.

Investors care only about consumption at date 1. Relative to Section 2, this simplifies matters in that variation in the level of non-pecuniary tastes cannot distort any consumption savings decision. Hence, we can focus on identifying cross-sectional asset pricing and portfolio choice effects. We work within the expected utility framework. In particular, we assume that investor preferences over state-contingent effective units of consumption are given by CRRA utility. Our numerical examples use log utility.

Investors (or delegated managers) may also face different *mandates*, which are exogenous restrictions on permissible portfolios. We incorporate mandates because they have been argued to be useful for identification of demand systems (Koijen and Yogo, 2019). Since tastes only affect red and green trees, we define mandates over these assets as well. Given market prices p_g and p_r , define the portfolio share of green trees among red and green trees as

$$w_g^i = \frac{p_g a_g^i}{p_g a_g^i + p_r a_r^i}$$

A mandate is a restriction that imposes, for parameters \overline{w}_{g}^{i} and \underline{w}_{g}^{i} , that

$$w^i \in [\underline{w}^i_g, \overline{w}^i_g].$$

Decision Problem. We normalize the price of Tree 2 to $p_2 = 1$. The budget constraint is

$$a_2^i + p_g a_g^i + p_r a_r^i = e_2^i + p_g e_g^i + p_r e_r^i.$$

Substituting consumption in state 2, the decision problem of investor *i* is:

$$\max_{\substack{a_g^i, a_r^i \ge 0}} \pi_1 \left[\rho u \left(\theta_g^i y_g(r) a_g^i + \theta_r^i y_r(r) a_r^i \right) + (1 - \rho) u \left(\theta_g^i y_g(g) a_g^i + \theta_r^i y_r(g) a_r^i \right) \right] \\ + \pi_2 u \left(e_2^i + p_g(e_g^i - a_g^i) + p_r(e_r^i - a_r^i) \right) \quad \text{s.t.} \quad w_g^i \in [\underline{w}_g^i, \overline{w}_g^i].$$
(1)

Our equilibrium concept is competitive equilibrium.

Definition 3 (Competitive Equilibrium) A competitive equilibrium consists of asset prices p_g and p_r and portfolios (a_g^i, a_r^i, a_2^i) for each *i* such that:

- 1. Given asset prices, portfolios solve decision problem (1) for each i.
- 2. Markets clear for every asset:

$$\sum_i a^i_j = rac{1}{2}$$
 for $j \in \{r,g\}$ and $\sum_i a^i_2 = 1.$

3.2 Optimal Portfolio Choice

We now solve for equilibrium demand systems and prices and discuss identification challenge. It is instructive to begin with the case where mandates do not bind for any investor. In this case, demand functions are determined by the following first-order conditions:

$$\begin{aligned} a_g^i : & \pi_1 \rho \theta_g^i (y_1 - \epsilon) u' \left(\frac{\tilde{c}_{1r}^i}{\tilde{c}_2^i} \right) + \pi_1 (1 - \rho) \theta_g^i (y_1 + \epsilon) u' \left(\frac{\tilde{c}_{1g}^i}{\tilde{c}_2^i} \right) &\leq (1 - \pi_1) p_g; \\ a_r^i : & \pi_1 \rho \theta_r^i (y_1 + \epsilon) u' \left(\frac{\tilde{c}_{1r}^i}{\tilde{c}_2^i} \right) + \pi_1 (1 - \rho) \theta_r^i (y_1 - \epsilon) u' \left(\frac{\tilde{c}_{1g}^i}{\tilde{c}_2^i} \right) &\leq (1 - \pi_1) p_r, \end{aligned}$$

where \tilde{c}_{1r}^i and \tilde{c}_{1g}^i represent effective units of consumption depending on whether the red or green tree offers relatively high cash flows, respectively. These conditions hold with equality whenever the investor chooses a positive quantity of the associated asset. Whether this is the case in equilibrium depends on the distribution of tastes.

This demand system is non-linear and exhibits complementarities: a change in the price of one asset alters the demand for all other assets. This is because, when $\epsilon > 0$, there is a diversification benefit to holding both red and green trees. As Berry and Haile (2021) point out, in settings with demand complementarities it is generally not enough to have a valid instrument for a particular price.

Asset demand is also sensitive to the *intensity* of tastes. By this we mean that variation in θ_j^i will drive changes in portfolios even when the ordinal ranking of preferences is preserved. This is not necessarily true in some discrete choice models of durable good purchases, where the outcome of interest is a binary choice. An implication is that the estimation of counterfactual asset demands generally requires identifying *cardinal* values of tastes. In the language of asset pricing, defining a stochastic discount factor requires incorporating the exact value of the marginal investor's tastes. The difficulty in identifying such a SDF is that tastes may be latent given equilibrium play.

3.3 Representative Agent Benchmark: No Tastes

To build intuition, we first solve the model without tastes: $\theta_j^i = 1$. As in Lucas (1978), this leads to the representative agent framework in which the representative agent holds the aggregate endowment in equilibrium, and is thus well-diversified in aggregate state 1. Given that total output in state 1 is a constant, we can define by

$$p_1 \equiv \frac{p_g + p_r}{2}$$

the price of a sure claim on one unit of consumption in state 1. Then prices are

$$p_1 = \frac{\pi_1}{1 - \pi_1} y_2$$
 and $p_r - p_g = 2\epsilon \frac{\pi_1}{1 - \pi_1} (2\rho - 1) \frac{y_2}{y_1}.$

The prices of claims on aggregate states reflect the relative scarcity of aggregate consumption across the two states, and price differences between red and green assets are driven by the distribution over the distributional shock ρ . With symmetric aggregate states (i.e., $y_1 = y_2 = 1$ and $\pi_1 = \frac{1}{2}$), this yields

$$p_1 = 1$$
 and $p_r = 1 + (2\rho - 1)\epsilon$.

3.4 Equilibrium with Tastes: Endogenous Sorting

Next, we consider the case with tastes. Because different investors may disagree on the marginal value of investing in a particular asset, there may be endogenous sorting in equilibrium. By this, we mean that investors with a taste for green assets will hold only green assets, while those with a taste for red assets will buy only red assets. (Of course, both will hold Tree 2 as well.)

We guess and verify that Type 1 specializes in green assets, and vice versa. Then Type 1's consumption in state 1 is $\theta_g^1 y_g(\iota) a_g^1$, and vice versa for Type 2. By the first-order conditions for optimal portfolios, demand functions are

Type 1:
$$a_g^1 = \frac{1}{p_g} \frac{\pi_1}{1 - \pi_1} a_2^1;$$

Type 2: $a_r^2 = \frac{1}{p_r} \frac{\pi_1}{1 - \pi_1} a_2^2;$

and consider only the trade-off between Tree 2 and one specific color. Observe that these demand functions are independent of the particular intensity of tastes: since the equilibrium features sorting, only the ordinal ranking of tastes matters.

While this is reminiscent of consumer good settings (for example, discrete choice over automobiles), sparse portfolio choices may be particularly problematic in financial markets. In particular, under the cardinal interpretation of preferences required for expected utility framework, the intensity of tastes will affect portfolio choice when investors hold both assets in equilibrium. Since taste intensities are latent on the equilibrium path when sorting occurs (in particular, they can only be identified up to their ordinal properties), counterfactuals are vulnerable to incorrect inference about taste intensities.

To illustrate this issue, we verify whether sorting can be sustained in equilibrium. Evaluating the first-order condition for a_r^1 at $a_r^1 = 0$, it is indeed optimal for Type 1 to refrain from purchasing red trees if and only if

$$\frac{p_r}{p_g} \geq \frac{\theta_r^1}{\theta_g^1} \left[\rho \frac{y_1 + \epsilon}{y_1 - \epsilon} + (1 - \rho) \frac{y_1 - \epsilon}{y_1 + \epsilon} \right].$$

This inequality states that the relative price of red trees must be high enough relative to the relative taste for red trees, adjusted by the benefits of diversification within state 1. In the equilibrium with sorting, moreover, relative prices are driven by wealth shares:

$$\frac{p_r}{p_g} = \frac{1-\omega}{\omega}$$

Hence, shocks to the wealth distribution or the degree of complementarity of red and green trees (as determined by ϵ and ρ) can lead sorting to break down. Shocks to wealth are precisely the type of counterfactual entertained by Koijen, Richmond, and Yogo (2022).

We now demonstrate that equilibrium demand functions do indeed depend on the intensity of taste parameters once there is partial sorting. We obtain simple closed-form solutions for the case where trees are perfect substitutes, i.e., when $\epsilon = 0$.

Proposition 2 (Equilibrium with and without sorting) Let $\theta_r^1/\theta_g^1 < 1$ and assume $\epsilon = 0$. There exists a threshold $\overline{\omega}$ for Type 1's wealth share ω such that Type 1 buys only green trees if $\omega \leq \overline{\omega}$ and buys both red and green trees if $\omega > \overline{\omega}$. When this is the case, prices are determined by Type 1's taste parameters, $p_g = \theta_g^1$ and $p_r = \theta_r^1$, and the quantity of red trees held by Type 2 is

$$a_r^2 = \frac{1-\omega}{2} \frac{\frac{1}{2}E_2 + \theta_g^1 E_g + \theta_r^1 E_r}{\theta_r^1}$$

where $E_2 = 1$ and $E_g = E_r = \frac{1}{2}$ denote the aggregate endowments of each tree.

The result highlights that, conditional on a shock to the wealth distribution, prices and quantities are now determined by the *intensity* of Type 1's tastes, not just their ordinal ranking. Since these are latent conditional on equilibrium play, it is difficult to conduct counterfactuals based on observational data that feature sparse portfolios. While we illustrate this in a setting with only two types, the insight naturally generalizes to many types. In this case, equilibrium portfolios do not reveal preferences of infra-marginal investors. Yet, identifying preference parameters of inframarginal investor is critical for any counterfactual in which investors may rebalance their portfolios on the extensive margin.

Figure 1 shows equilibrium prices for the entire range of wealth ω and substitutability ϵ . The left panel shows the green price p_g , and the right panel shows the price of a sure claim on state 1, $p_1 = \frac{p_g + p_r}{2}$. In the left panel, sorting occurs in the linear region near the origin, but breaks down as either Type 1 becomes too wealthy or the diversification benefits become too large. In response to shocks, the relative price of green trees (and thus the underlying demand system) is highly non-linear in fundamentals. In contrast, the aggregate price of state 1 is flat in the entire region, as in the representative agent benchmark. This is because, *conditional* on a well-specified stochastic discount factor that takes into account tastes, every investor remains well-diversified within state 1.



Figure 1: Green Price (Left) and State 1 Price $\frac{p_g + p_r}{2}$ (Right).

4 Identification and Counterfactuals

4.1 On Identification and Counterfactuals with Mandates

In addition to tastes, investors may be distinguished by their mandates. In practice, this means that certain investors only invest in the S&P500, or only in companies that have high Environmental, Social, and Governance (ESG) scores. While mandates may be a contributing factor to observed portfolio choices, they have also been put forth as helpful for *identifying* demand systems (Koijen and Yogo, 2019). As the argument goes, an asset

that is inside the "investment universe" of many investors will be in higher demand, and thus see higher prices, than an otherwise similar asset that is not widely held in many investment universes. Putting aside for a minute the concern that mandates may be chosen in response to investment opportunities, we can consider the implications of this identification strategy in our model.

In particular, say there is a share *m* of Type 1 investors that are not permitted to invest in red trees. While this is a very stark mandate, it is the type of mandate that would be ideal for the identification strategy in Koijen, Richmond, and Yogo (2022) because it is entirely inflexible with respect to changes in investment opportunities. Mandates are observed by investors, but not by the econometrician.

The demand function of *mandate investors* (superscript *M*) trades of green trees with Tree 2. Under log utility, they spend share π_1 of their wealth on green trees:

$$a_g^M = \pi_1 \cdot \frac{e_2^M + p_g e_g^M + p_r e_r^M}{p_g}.$$

Mandates and tastes for green assets are observationally equivalent to the extent that the equilibrium features sorting; that is, tastes and mandates are both valid microfoundations for sparse portfolios. However, they differ when non-mandate investors choose to hold both red and green assets. This threatens the validity of counterfactuals. In particular, shocks to the wealth of Type 1 will result in different counterfactual prices when many investors are subject to mandates versus when they are not.

Proposition 3 The equilibrium response to shocks to wealth ω or complementarity ϵ may be qualitatively different depending on the share of mandate investors *m*.

Figure 2 illustrates this result. When there are almost no investors with mandates, a shock to ϵ creates more demand for diversification. Hence, the price of green trees is decreasing in ϵ if Type 1 investors choose to hold both types of trees. Mandate investors do not buy red trees at any price. Hence, shocks to ϵ do not reduce their demand for green trees even as Type 2's demand increases. Hence, the price of green trees may be increasing in ϵ when there are sufficiently many mandate investors. Hence, the observational equivalence of tastes and mandates creates the risk that counterfactuals are misspecified.

More broadly, identification based on mandates is threatened by the lack of a theory of delegation. In practice, mandates are typically imposed on funds (such as mutual



Figure 2: Green Price. Left: Low Mandate Share ($m \approx 0$). Right: High Mandate Share ($m \approx 0.85$).

funds), not end investors (such as households). This means that even very tight mandates are irrelevant as long as end investors can flexibly reallocate investments across funds.

Proposition 4 Consider a two-layer structure where households invest through funds, and funds are subject to mandates. Suppose further that there exist at least one red and one green fund. Absent other frictions, equilibrium is invariant in mandates.

In practice, researchers have pointed out that households may be slow to rebalance, or do so in predictable manners at regular intervals (i.e., quarter end). While it is possible that this may help with identification, the argument is incomplete: if some investors are known to rebalance intermittently, other investors may trade preemptively only to later sell. In this sense, intermittent rebalancers' tastes may be reflected in market demand even when they are not actively trading.

4.2 Equilibrium Consequences of Short Sales

As discussed in Section 2, the classical law of one price may fail to hold when investors differ in terms of their tastes. We now illustrate consequences of this fact by introducing a set of investors who can freely sell short any asset. Such investors are likely to have outsize implications for equilibrium prices. We show this mechanism under the assumption that red and green trees are perfect substitutes in terms of their cash flows, $\epsilon = 0$. In this case, optimal portfolio choices are bang-bang, and the investor takes an infinite short position whenever the relative price of red and green trees is misaligned with her tastes.

Proposition 5 Let $\epsilon = 0$. We enrich the model with a single investor type, indexed by *S*, who can freely short. Then the relative price of red and green trees is given by

$$\frac{p_g}{p_r} = \frac{\theta_g^S}{\theta_r^S}$$

and is independent of any other parameters in the model.

The proposition states that an agent who can freely short trades until relative prices are aligned with her tastes *irrespective* of any other parameters. In practice, there may be short-sale constraints or other limits to arbitrage that prevent large short positions. However, this merely means that researchers have to measure *when* they might bind.

Two considerations make this difficult in practice: researchers may not observe short positions, nor do they have universal coverage of all investors in a given market. These data limitations make it difficult to infer taste parameters from equilibrium play.

4.3 Endogenous Wealth Effects

We now discuss another important feature of financial markets—portfolios are regularly marked to market. This means that, even holding preferences fixed, an individual who already owns a particular stock will exhibit different demand elasticities in response to a price change than an investor who does not. Hence standard instruments that may work well in consumer good settings (where purchases are one shot) will not be sufficient to identify asset demand systems.

This simple logic also has implications for the case of "index deletions." In particular, assume that an investor is mandated to hold only assets that are in a particular index. Assume that green and red were initially the index, before red surprisingly drops out. Hence, the investor must divest upon deletion. We call the short-run demand curve the one that determines demand right upon deletion, and the long-run demand curve the one that obtains once short-run adjustments have occurred. Assume for illustrative purposes that the investor is not forward-looking, and that $\epsilon = 0$. Then demand functions satisfy

Short-run demand:
$$a_g^M = \pi_1 \cdot \frac{e_2^M + p_g e_g^M + p_r e_r^M}{p_g};$$

Long-run demand: $a_g^M = \pi_1 \cdot \frac{e_2^M + p_g e_g^M}{p_g}.$

The difference is due to the valuation of endowments. In the short run, changes in the red price affect demand because wealth is marked to market. In the long run, demand is independent of the red price because the investor was forced to divest. In general, there are thus important dynamic considerations that differ from consumer markets, where most purchases are generally not resold or marked to market.

5 On The Structural Interpretation of Demand Elasticities

Demand elasticities are one of the main objects of interest in industrial organization. The reason is that a well-identified demand elasticity which can be related to, e.g., preferences for automobiles may inform a policymaker of the quantity response to a tax policy that raises automobile prices. In line with this view, researchers in demand-system asset pricing often argue that demand elasticities are a useful diagnostic that might distinguish their method from more classical approaches. Against this background, we now discuss the structural interpretability of demand elasticities in financial markets.

An important difference between consumer goods and financial markets is that portfolio choice is generally modeled, at least in part, using preferences over state-contingent payoffs rather than asset characteristics alone. We will therefore argue that *asset-level* demand elasticities may not be informative about preference parameters whenever multiple (portfolios) of assets can deliver the same state-contingent payoff stream. In particular, the structural interpretation of demand elasticities depends on the security menu, as well as on whether there are "outside options" for an investor to pursue in response to price changes. This in turn is linked to the general equilibrium consequences of price changes.

5.1 Elasticities and the Security Menu

To establish a clean benchmark, we first show an example where asset-level demand elasticities *are* informative about preferences. In particular, we assume that the security menu consists *only* of the full set of Arrow securities, and that investors do not exhibit tastes over assets. Since each asset is uniquely tied to a particular state, asset-demand elasticities are then informative about state-contingent valuations. In particular, consider a generic investor choosing Arrow security positions $\{a(z)\}_z$ to solve:

$$\max_{a(z)} \qquad u(c_0) + \beta \sum_{z} \pi(z) u(c_1(z))$$

s.t.
$$c_0 = w - \sum_{z} p(z) a(z)$$
$$c_1(z) = y(z) + a(z).$$

where p(z) is the price of security z and y(z) an exogenous state-contingent endowment. If we define the marginal rate of substitution (or the state price) associated with state z to be $\Lambda(z) = \beta \pi(z) u'(c_1(z)) / u'(c_0)$, the first-order condition is $\Lambda(z) = q(z)$. The implicit function theorem yields an equation linking demand elasticities to preference parameters,

$$\epsilon(z)\Lambda(z)\left[-rac{lpha(c_1(z))}{q(z)}-lpha(c_0)
ight]=rac{1}{a(z)}+\Lambda(z)lpha(c_0),$$

where $\alpha(c)$ is the coefficient of absolute risk aversion and $\epsilon(z) = \frac{\partial a(z)}{q(z)} \frac{q(z)}{a(z)}$ is the price elasticity of demand for Arrow security *z*. Thus, when the security menu is the case of Arrow securities, suitable price instruments allow researchers to estimate preference parameters $\Lambda(z)\alpha(c_0)$ from portfolio data. With multiple price instruments, it may also be possible to disentangle both components under mild parametric assumptions.

Next, consider the effects of changes in the security menu. To remove direct effects on prices, we hold the set of marketable payoffs fixed. In particular, we begin with the Arrow security menu and, for some generic state of the world z^* , introduce a new Arrow security that also pays off only in state z^* . Denote the demand for the original security by $a_0(z^*)$, and the demand for the new security by $a_1(z^*)$. If the Law of One Price holds, we must have that both assets have the same initial price, $p_0(z^*) = p_1(z^*)$, and so the investor will be indifferent between holding both assets.

Proposition 6 (Elasticities with Redundant Assets) Suppose that the investor holds a positive position in both securities referencing state z^* . Now consider an exogenous increase in price of the new security, holding all other prices fixed. Then the demand elasticity for the new security is $-\infty$, while the demand for consumption in state z^* is unchanged. Hence, estimated demand elasticities for the new security are uninformative about preference parameters.

While this example is deliberately stark, it is sufficient to highlight the critical role of the security menu, and redundant assets, for the structural interpretability of demand elasticities. Outside the case of Arrow securities, moreover, investors may need to combine multiple assets in certain proportions to achieve a certain consumption stream, and asset-level demand elasticities may only have structural interpretations when multiple assets are considered jointly. Finally, when there are redundant assets, investor quantities may not be uniquely pinned down by optimality conditions, yet they will still affect measured elasticities. It is precisely because of this indeterminacy that classical asset pricing has relied so extensively on no arbitrage relationships between prices.

5.2 Outside Goods

Next, we consider the role of outside options, and how they are shaped by general equilibrium forces. In industrial organization, it has long been recognized that the interpretation of demand elasticities depends critically on the assumed notion of an "outside good," which is the alternative use of money available to a consumer that, e.g., chooses not to buy a car (Berry and Haile, 2021). In consumer good settings, a common strategy is to model preferences as quasi-linear, with the implicit understanding that the outside good is a consumption bundle whose utility scales approximately linearly with wealth.

Such an approach may be less viable in financial markets, at least as long as one is interested in *general equilibrium* economies. To see this, return to our baseline Lucas Tree economy from Section 3 and assume that the equilibrium features perfect sorting, with only Type 1 investors buying green trees and only Type 2 investors buying red trees. In this case, Type 1's demand function for green trees satisfies:

$$a_g^1 = \frac{1}{p_g} \frac{\pi_1}{1 - \pi_1} a_2^1.$$

Differentiating this expression with respect to the price p_g yields

$$\begin{aligned} \frac{\partial a_g^1}{\partial p_g} &= -\frac{1}{(p_g)^2} \frac{\pi_1}{1 - \pi_1} a_2^1 + \frac{1}{p_g} \frac{\pi_1}{1 - \pi_1} \frac{\partial a_2^1}{\partial p_g} \\ &= -\frac{a_g^1}{p_g} + \frac{a_g^1}{a_2^1} \frac{\partial a_2^1}{\partial p_g}. \end{aligned}$$

The first term is the own price effect that is negative; the second is the cross-demand elasticity of good 2. As is well known, under natural assumptions, the overall response is negative, with investors substituting away from green trees as their price increases.

However, this cannot be true in equilibrium for any *fundamental* shock that might increase the price of green trees. Consider for example an increase in Type 1's wealth share ω . Our preceding analysis shows that this must lead to an increase in p_g . However, since Type 1 agents are symmetric within type and only Type 1 investors buy green trees, market clearing ensures that $a_g^1 = 1/2$ in equilibrium. Hence the *equilibrium* elasticity is zero, not negative, because there is no outside good for investors to move to. In applied contexts, it is therefore critical to assess the degree of substitutability between "inside" and "outside" assets. For example, corporate bonds may be a better subsitute for Treasury bonds than equities, and the degree of substitutability may differ depending on the level of aggregation.

6 Conclusion

We present a synthesis between classical asset pricing and recent demand-system approaches to asset pricing, using multiple methods of incorporating non-pecuniary tastes into equilibrium models of portfolio choice. Our analysis highlights important conceptual concerns, including the definition of no arbitrage and the law of one price, the pricing of redundant assets, and the cardinal interpretation of taste parameters. Based on these concerns, we highlight several barriers to identification of demand systems that may threaten the validity of counterfactuals based on estimated demand systems.

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