

# Beyond the Status Quo: A Critical Assessment of Lifecycle Investment Advice

Aizhan Anarkulova\*

Scott Cederburg<sup>†</sup>

Michael S. O’Doherty<sup>‡</sup>

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## Abstract

We challenge two tenets of lifecycle investing: (i) diversify over stocks and bonds and (ii) reduce equity allocations with age. An optimal lifetime allocation of 33% domestic stocks, 67% international stocks, 0% bonds, and 0% bills vastly outperforms age-based, stock-bond strategies in building wealth, supporting retirement consumption, preserving capital, and generating bequests. Our lifecycle model preserves crucial time-series and cross-sectional dependencies in asset returns and addresses small sample issues in US data. Our investors prefer diversifying with international stocks, not bonds. Target-date fund investors need 61% more pre-retirement savings to match the all-equity strategy’s expected utility over retirement consumption and bequest.

**JEL classifications:** C15, D14, G11, G17, G51

**Key words:** Retirement, retirement savings, target-date funds, survivor bias, easy data bias

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\*Goizueta Business School, Emory University. Email: [aizhan.anarkulova@emory.edu](mailto:aizhan.anarkulova@emory.edu).

<sup>†</sup>Eller College of Management, University of Arizona. Email: [cederburg@arizona.edu](mailto:cederburg@arizona.edu).

<sup>‡</sup>Trulaske College of Business, University of Missouri. Email: [odohertym@missouri.edu](mailto:odohertym@missouri.edu).

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# 1 Introduction

Every year, Americans contribute about 5% of their total employee compensation to defined contribution (DC) pension plans, with contributions of \$621 billion in 2022 alone.<sup>1</sup> They then face a question that determines their financial fate: How should I invest my savings? Many consult financial advisors. These professionals impart two central tenets of lifecycle investing — people should diversify across stocks and bonds and the young should invest more heavily in stocks than the old — having learned them from investments textbooks [e.g., Bodie, Kane, and Marcus (2024)] or CFA study materials [e.g., Blanchett, Cordell, Finke, and Idzorek (2023)]. Self-directed savers seek answers, perhaps reading a popular book by Dave Ramsey, Suze Orman, or Tony Robbins. They receive similar advice [Choi (2022)]. Finance professors may closely study the literature on lifecycle investing and reach the same conclusions [e.g., Viceira (2001); Campbell and Viceira (2002); and Cocco, Gomes, and Maenhout (2005)]. A great many others are disinterested or overwhelmed, so they invest in the default option of their employer’s retirement plan. To safeguard these investors, the Pension Protection Act of 2006 (PPA) created safe harbors for employer DC plans. The most popular Qualified Default Investment Alternatives (QDIAs) are portfolios that provide “long-term appreciation and capital preservation through a mix of equity and fixed income exposures based on the participant’s age” [29 CFR § 2550.404c-5(e)(4)(i)].<sup>2</sup> As such, regulators rely on “generally accepted investment theories” [29 CFR § 2550.404c-5(e)(4)] that mirror the two principles to define QDIAs. In summary, these pieces of investment advice — split investments across stocks and bonds and invest more in stocks while young than while old — are close to being uniformly given and universally followed.

In this paper, we challenge these two tenets of lifecycle investing. In our setting, investors optimize utility over real retirement consumption and bequest within a lifecycle model with labor income risk, Social Security income, and longevity risk. Departing from the literature, we model asset class returns such that we maintain the time-series and cross-sectional properties of stock and bond returns that are evident in the data. We achieve this goal by using a block bootstrap approach that draws long time series of consecutive asset class returns from the historical record. This approach is essential for properly modeling the long-horizon returns that retirement savers may earn. These returns reflect the effects of changing investment opportunities during the holding

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<sup>1</sup>The total DC plan contribution is from the 2024 Private Pension Plan Bulletin from the Department of Labor. The 5% figure divides \$621 billion in 2022 DC contributions by \$13,437 billion in 2022 compensation of employees from Table 2.1 of the national income and product accounts (NIPA) from the Bureau of Economic Analysis.

<sup>2</sup>See <https://www.law.cornell.edu/cfr/text/29/2550.404c-5>. Vanguard (2024) reports that target-date funds, which are QDIAs under 29 CFR § 2550.404c-5(e)(4)(i), make up 98% of the QDIAs in DC plans.

period without the need to pre-specify which particular aspects are important to investors. Further, the investment horizons of retirement savers are very long, extending 75 years or beyond for young savers. The relatively short history of US financial markets poses a small sample problem given this setting, so we model forward-looking returns by examining the history of asset class returns from a broad cross section of developed economies. Our comprehensive dataset has returns on domestic stocks, international stocks, government bonds, and government bills from 39 developed countries and spans more than 2,600 years of country-month return data.<sup>3</sup>

In the base case of our lifecycle model, we allow the couple to choose a fixed-weight investment strategy with allocations to domestic stocks, international stocks, bonds, and bills. We impose constraints — no leverage and non-negative weights — that reflect reality for most retirement savers. The couple’s optimal fixed-weight portfolio allocation is an all-equity strategy: 33% domestic stocks, 67% international stocks, 0% bonds, and 0% bills. Although including international stocks in the investment opportunity set is rare in the lifecycle literature, the large weight on this asset class underscores the importance of allowing for international diversification.<sup>4</sup>

We compare this optimal fixed-weight strategy to two popular QDIA benchmarks: (i) a balanced strategy with 60% domestic stocks and 40% bonds and (ii) a representative target-date fund (TDF) that employs an age-based, stock-bond strategy. To achieve the same expected utility from retirement consumption and bequest as a couple investing in the optimal strategy and saving 10.0% of labor income, a couple using the balanced strategy must save 19.3% of income (i.e., nearly twice as much). Ex ante, the TDF may be preferable to the fixed-weight strategy because the TDF’s weights vary with age, in line with conventional wisdom. Despite this seeming advantage, a couple investing in the TDF must save 16.1% (i.e., 61% more) to match the expected utility of the optimal fixed-weight strategy that invests exclusively in equity.

We examine the determinants of expected utility by studying four retiree outcomes: wealth

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<sup>3</sup>We consider multiple risky assets but no risk-free asset, similar to Campbell and Viceira (2002, 2005); Campbell, Chan, and Viceira (2003); Sangivanatsos and Wachter (2005); Hoevenaars, Molenaar, Schotman, and Steenkamp (2008); Koijen, Nijman, and Werker (2010); and Duarte, Fonseca, Goodman, and Parker (2024). A large majority of studies include a risk-free asset, either with a single risky asset [e.g., Merton (1969); Samuelson (1969); Viceira (2001); Cocco, Gomes, and Maenhout (2005); Pástor and Stambaugh (2012); Dahlquist, Setty, and Vestman (2018); Gomes, Michaelides, and Zhang (2022); Gomes and Smirnova (2023); and Choukhmane and de Silva (2024)] or multiple risky assets [e.g., Merton (1971); Brennan, Schwartz, and Lagnado (1997); Lynch (2001); Lynch and Tan (2010); and Catherine (2022)].

<sup>4</sup>Baxter and Jermann (1997) consider international diversification in the context of non-tradeable human capital, concluding that investors should short domestic markets to hedge labor income risk. Michaelides (2003); Hnatkovska (2010); Coeurdacier and Rey (2013); and Bretscher, Julliard, and Rosa (2016) study these hedging motives in lifecycle asset allocation models featuring foreign stocks, concentrating on the roles of frictions, incomplete markets, and the strength of the relation between human capital and domestic asset returns in rationalizing home bias. Several researchers examine international diversification in settings without lifecycle features [e.g., Solnik (1974), Jorion (1985), Eun and Resnick (1988, 1994), French and Poterba (1991), and Ang and Bekaert (2002, 2004)].

at retirement, retirement income, conservation of savings, and bequest at death. The all-equity strategy dominates the QDIAs in long-term appreciation, with 50% more retirement wealth on average than the balanced strategy and 39% more than the TDF. This additional wealth generates a larger stream of income for the retirees.

A surprising result is that the all-equity strategy also compares favorably with the QDIAs in capital preservation. Households allocating 33% to domestic stocks and 67% to international stocks are much less likely to exhaust their savings. Under the common 4% rule for retirement spending [Bengen (1994)], a couple using the balanced strategy has a 16.9% probability of running out of wealth. The TDF is even worse at 19.7%. In comparison, the probability for the optimal, all-equity strategy is low at 7.0%. Finally, the optimal strategy produces much larger bequests than the QDIAs. Overall, the all-equity strategy — which is not a QDIA — beats the QDIAs across the board in achieving the PPA goals of long-term appreciation and capital preservation.

The base case analysis requires the couple to maintain a constant portfolio allocation throughout their lives. We also examine time-varying weight strategies based on age. First, we allow households to invest entirely in equity during the working years, but add bonds during retirement. They optimally allocate just 3% to bonds during retirement, and the expected utility for this strategy is virtually identical to the optimal fixed-weight strategy. Second, we consider age-based strategies similar to the “100-minus-age rule” that dictates investing 100 minus your age in stocks with the remainder in bonds. We specifically study whether there is any age at which the couple would like to begin investing in bonds and subsequently increase the bond allocation by 1% per year. None of these age-based rules generates higher expected utility than the optimal fixed-weight strategy. Contrary to conventional wisdom, bonds add little to nothing for retirees.

We also allow the couple to condition on the market state. Specifically, the couple can condition their strategy on the domestic stock price-dividend ratio by choosing different weights for each valuation quintile. In the bottom four quintiles of the price-dividend ratio, the couple maintains an all-equity strategy. Only in the highest-price quintile does the household allocate to bonds with a modest 9% weight. Conditioning allows for a small utility gain, as the couple can achieve the same expected utility as the optimal fixed-weight strategy by saving 9.7% of their income (relative to the base of 10.0%). We further show that nearly all of this utility gain is attributable to varying the domestic-international stock allocation rather than to buying bonds. Overall, these analyses provide two takeaways: (i) optimal allocations to bonds are small or zero across specifications and (ii) the time-varying strategies generate small, if any, utility gains relative to the optimal fixed-weight strategy.

We emphasize that we do not make any theoretical arguments that age does not matter for asset allocation, that diversification is unimportant, or that bonds are inherently inappropriate for investors. Rather, the inclusion of international stocks, the properties of long-horizon returns, household concerns over real buying power, and a realistic no-leverage constraint lead to our empirical results on the success of an all-equity strategy. Given this design, the couples are simply choosing to diversify using international stocks rather than bonds.

Table I illustrates why our couples prefer international stocks to bonds. Panel A reports the annualized mean and standard deviation of returns for bonds and international stocks based on our comprehensive dataset. Panel B shows variance ratios at horizons of one, ten, 20, and 30 years calculated as in Poterba and Summers (1988), and Panel C presents correlations of log returns and log inflation. Bonds offer modest average real returns (0.95% annually) compared with international stocks (7.03%), necessitating strong diversification benefits to make bonds attractive. At short horizons, bonds appear less risky with lower standard deviation (9.51% versus 23.26%) and lower correlation with domestic stocks (0.21 versus 0.33). At long horizons, the picture changes. Bonds' per-period variance increases to 2.30 times the one-year variance, but international stocks' decreases to 0.75 times. Bonds' correlation with domestic stocks rises to 0.45 at 30 years, whereas international stocks maintain a steady correlation.<sup>5</sup> International stocks also help preserve real buying power with a low correlation with inflation ( $-0.01$ ), whereas bonds do not ( $-0.78$ ). Bonds ultimately seem unattractive for long-horizon investors. They have low returns, high long-term variance, high long-term correlation with domestic stocks, and high exposure to inflationary periods.

The long-term properties of international stocks versus bonds significantly impact portfolio choice, even in a simple setting. Figure 1 illustrates this fact with optimal mean-variance weights. The second moments of the assets are calculated from a one-month horizon (Panels A and B) or a 30-year horizon scaled to a monthly level (Panels C and D). We consider cases without international stocks (Panels A and C) or with international stocks (Panels B and D). Much of the lifecycle literature focuses on domestic markets calibrated to short-term return moments.<sup>6</sup> Mean-variance investors confronted with this design (Panel A) allocate substantially to bonds and bills. With risk aversion of four, the optimal portfolio comprises 41% domestic stocks, 20% bonds, and 39% bills.

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<sup>5</sup>Also see Siegel (2014) and Campbell and Viceira (2002, 2005) for evidence on the risk of bonds over extended holding periods.

<sup>6</sup>It is possible to interpret the "stocks" asset class in previous lifecycle studies as representing a mix of domestic and international stock markets. If so, these studies make implicit assumptions about the relative weights across countries as well as the interdependencies of domestic and foreign stock returns, inflation, and exchange rates. By separately modeling domestic and international stocks, we allow investors to choose optimal weights and capture the rich patterns in returns, inflation, and exchange rates across markets.

When granted access to international stocks (Panel B), the optimal allocation substantially shifts towards equity with 26% in domestic stocks, 50% in international stocks, and 24% in bonds.

Figure 1 further shows that investors who consider long-horizon return properties shun bonds. Domestic market investors with long horizons (Panel C) and risk aversion of four optimally allocate 70% to domestic stocks and 30% to bills. The same long-horizon investors with access to international stocks (Panel D) invest 26% in domestic stocks, 73% in international stocks, and 1% across bonds and bills. This case is closest to our lifecycle model’s specification, which includes international stocks and preserves time-series dependencies in returns with the block bootstrap. The simple mean-variance setting thus captures the intuition behind our all-equity findings. In the remainder of our study, we use a rich lifecycle model to formalize the importance of properly modeling asset class returns.

Our couples’ aversion to bonds persists across a wide range of model specifications and parameter choices. Our results are insensitive to the bootstrap block length, risk aversion, strength of the bequest motive, retirement withdrawal strategy, retirement age, contribution rate, and household type (e.g., single versus couple). We study alternative investor types with low or high initial income and low or high human capital in Guvenen, Karahan, Ozkan, and Song’s (2021) model of stochastic labor income, and all household types choose similar all-equity strategies. We introduce correlation between labor income and domestic stock returns using a Gaussian copula; investors adjust their allocation across domestic and international stocks, but they do not buy bonds.

We impose a no-leverage constraint in our base case. This constraint is realistic for most retirement savers, but Asness (1996) nevertheless argues that investors should lever up a 60%/40% stock/bond strategy rather than invest 100% in stocks. We consider hypothetical couples who can borrow up to 100% of wealth at a margin rate equal to the bill yield plus a spread. With a margin spread of 1.4% (the lowest available spread as of April 2024), the couple borrows 55% of their wealth and optimally chooses an all-equity strategy with 34% in domestic stocks and 66% in international stocks.

Finally, we address potential concerns about our treatment of the consumption-saving decision and the retirement age decision. We adopt a static optimization approach out of necessity due to our block bootstrap simulation design, and our base case assumes a constant contribution rate and exogenous retirement age. When we replace the constant contribution rate with the age- and income-based contribution rates estimated by Parker, Schoar, Cole, and Simester (2023) for US households, the couple optimally chooses the same all-equity portfolio with 33% in domestic stocks and 67% in international stocks. If we allow couples to choose an optimal retirement age considering

their income, wealth level, and anticipated Social Security benefits, they choose the same optimal portfolio. Although these aspects of dynamic lifecycle models are important to investors’ utility, they do not interact strongly with portfolio choice in our setting.

We contribute to the recent normative literature on the optimal design of lifecycle investment strategies [e.g., Michaelides and Zhang (2017, 2022); Dahlquist, Setty, and Vestman (2018); Kraft, Munk, and Weiss (2019); Gomes, Michaelides, and Zhang (2022); and Duarte, Fonseca, Goodman, and Parker (2024)]. Our primary contribution to the lifecycle literature is our modeling of the investment opportunity set. Several classic studies assume constant investment opportunities with returns that are normally or lognormally distributed [e.g., Merton (1969); Viceira (2001); Cocco, Gomes, and Maenhout (2005); and Gomes and Michaelides (2005)]. Many other studies consider particular aspects of time-varying investment opportunities or non-normalities in returns. Optimal lifecycle asset allocation is affected by time-varying expected returns, time-varying return variance, and skewness in returns.<sup>7</sup> The studies in this literature typically introduce parametric assumptions to model a particular aspect of the investment opportunity set. Our block bootstrap approach preserves the empirically relevant features of investment opportunities and non-normalities that affect the return distribution without requiring ex-ante specification of which aspects matter most.

Although studies of time-varying investment opportunities provide important insights into optimal intertemporal hedging demands, Cochrane’s (2014, 2022) alternative perspective for long-term investors focuses on asset payoffs. Cochrane (2014) states, “the hedging demands emphasized by the portfolio approach are really means to an end — an optimal consumption stream — rather than the end itself.” Our approach focuses on the long-horizon asset payoffs that support a retirement consumption stream, allowing the data to speak directly about the investment opportunities investors face over their holding periods. We find that this more complete characterization of long-horizon outcomes enriches our understanding of how households should optimally invest over the lifecycle.

## 2 The status quo in lifecycle investing

In seminal studies, Merton (1969) and Samuelson (1969) provide a baseline for lifecycle asset allocation. They demonstrate that investors have constant optimal allocations to a risky asset and

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<sup>7</sup>See, e.g., Campbell and Viceira (1999); Barberis (2000); Lynch (2001); Wachter (2002); Campbell, Chan, and Viceira (2003); Pástor and Stambaugh (2012); Michaelides and Zhang (2017, 2022); and Gomes, Michaelides, and Zhang (2022) for evidence on time-varying expected returns; Lynch and Balduzzi (2000) and Chacko and Viceira (2005) for time-varying variance; and Fagereng, Gottlieb, and Guiso (2017); Catherine (2022); Bonaparte, Korniotis, Kumar, Michaelides, and Zhang (2024); and Shen (2024) for skewness.

a risk-free asset under the conditions that human capital is tradeable and investment opportunities are constant. Subsequent studies relax these assumptions to study investors' lifecycle problem.<sup>8</sup>

Human capital, as a dominant asset for many working-age individuals, is the focus of much of the literature on lifecycle portfolio choice. With complete markets, investors simply capitalize labor income and optimal asset allocation is unaffected [Merton (1971)]. In contrast, Cocco, Gomes, and Maenhout (2005) demonstrate that, with incomplete markets (e.g., non-tradeable, non-insurable labor income and borrowing constraints), investors optimally choose age-based allocations.<sup>9</sup> If labor income risk is idiosyncratic, human capital substitutes for the risk-free asset in the optimal portfolio and the young hold more in stocks than the old.<sup>10</sup> Bodie, Merton, and Samuelson (1992) consider endogenous labor supply and retirement with utility over consumption and leisure, finding that labor supply flexibility increases optimal financial portfolio risk. Because the young have more labor flexibility than the old, the young should hold more in stocks.<sup>11</sup> Reinforcing these human capital effects, mean reversion in stock returns also makes stocks more attractive for young investors with long horizons [e.g., Barberis (2000), Wachter (2002), and Siegel (2014)].

Regulation stemming from the PPA favors an “investment fund product or model portfolio that applies generally accepted investment theories, is diversified so as to minimize the risk of large losses and that is designed to provide varying degrees of long-term appreciation and capital preservation through a mix of equity and fixed income exposures based on the participant’s age, target retirement date (such as normal retirement age under the plan) or life expectancy” [29 CFR § 2550.404c-5(e)(4)(i)]. TDFs meet these design criteria by following age-based, stock-bond asset allocation policies. They adopt aggressive allocations with higher exposures to equities for younger investors and become more conservative with increased exposures to fixed income assets as the investors age. Figure 2 illustrates this design with the advertised unconditional glidepath weights in domestic stocks, international stocks, bonds, and bills from a TDF offered by a major investment firm.

TDFs have exploded in popularity since the passage of the PPA [Parker, Schoar, and Sun

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<sup>8</sup>We refer readers to excellent reviews by Campbell (2006); Gomes (2020); and Gomes, Haliassos, and Ramadorai (2021).

<sup>9</sup>See also Heaton and Lucas (1997), Koo (1998), and Viceira (2001) for treatments of non-insurable labor income risk in infinite-horizon models.

<sup>10</sup>Human capital can act more like the risky asset with a systematic component in labor income [e.g., Viceira (2001); Campbell and Viceira (2002); Benzoni, Collin-Dufresne, and Goldstein (2007); and Lynch and Tan (2011)], which would decrease the optimal risky allocation if the correlation between labor shocks and stock returns were sufficiently high.

<sup>11</sup>Farhi and Panageas (2007); Chai, Horneff, Maurer, and Mitchell (2011); and Hubener, Maurer, and Mitchell (2016), among others, also study the implications for portfolio choice of flexible labor supply and endogenous retirement. Other studies consider the effects of nonemployment on portfolio decisions [e.g., Bremus and Kuzin (2014); Fagereng, Guiso, and Pistaferri (2018); and Bagliano, Fugazza, and Nicodano (2019)].



(2023)], with total assets under management (AUM) reaching \$3.5 trillion at year-end 2023 [Pacholok (2024)]. This growth reflects the aggressive trend of plan sponsors' adoption of automatic enrollment features, the overwhelming tendency to select TDFs as default funds, and the propensity for participants to retain the default elections [e.g., Madrian and Shea (2001) and Mitchell and Utkus (2022)].<sup>12</sup> Across all participants in Vanguard plans, for example, 83% use TDFs and 58% hold their entire account balance in a single TDF [Vanguard (2024)]. In short, TDFs now represent the status quo for retirement saving.

### 3 Data

In this section, we describe the underlying data on asset class returns used in our analyses. We take the perspective of a US couple saving for retirement. Given the paucity of statistical evidence on long-horizon asset class returns in the US data, we model forward-looking returns by examining the history of asset class returns from a broad cross section of developed economies. We follow Anarkulova, Cederburg, and O'Doherty (2022) in classifying countries as developed. The dataset includes monthly real returns for domestic stocks, international stocks, government bonds, and government bills for 39 developed countries. The data cover the period from 1890 to 2023, but the sample periods for individual countries vary based on data availability and the timing of economic development (i.e., a given country is included in the sample only for the period after it achieves developed status).

The starting point for constructing the dataset is the GFDDatabase from Global Financial Data. For each sample country, the GFDDatabase contains times series of total return indexes, price indexes, dividend-price ratios, and total market capitalization for stocks; yields for ten-year government bonds and short-term bills; consumer price indexes; and exchange rates. The internet appendix provides detailed descriptions of the appropriate GFD data series for each country, alternative sources used to fill gaps in the GFDDatabase, calculations of asset class returns, adjustments to these calculations for several periods surrounding major market disruptions (e.g., the closure of the New York Stock Exchange in 1914 at the onset of World War I and the Greek government bond default in 2012), and dataset validation.

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<sup>12</sup>As a default option, TDFs benefit investors who are inattentive, have behavioral biases, or who lack financial literacy by offering diversification benefits and automatic reallocations. A large literature shows adverse effects on decisions related to international diversification [Bekaert, Hoyem, Hu, and Ravina (2017)], asset allocation [Benartzi and Thaler (2001)], contribution levels [Lusardi and Mitchell (2007, 2011) and Goda, Levy, Manchester, Sojourner, and Tasoff (2020)], stock market participation [van Rooij, Lusardi, and Alessie (2011)], and account concentration in employer stock [Poterba (2003)]. Campbell (2016); Beshears, Choi, Laibson, and Madrian (2018); and Gomes, Haliassos, and Ramadorai (2021) provide comprehensive reviews of this evidence.

The dataset is a balanced panel in the sense that each country-month of data has non-missing returns for domestic stocks, international stocks, bonds, and bills. The nominal returns for domestic stocks, bonds, and bills for a given country are measured in the local currency; these nominal returns are then converted to real returns using the local inflation rate. The nominal international stock returns for a given country are market-capitalization-weighted averages of the nominal returns for all non-domestic stock markets, with appropriate adjustments for changes in exchange rates. Analogous to the calculations for the other asset classes, the nominal international stock returns are converted to real returns based on local inflation. As such, all asset class returns for a given country-month reflect the real investment outcomes of local investors in that month.

The broad sample of asset class returns allows for a more comprehensive characterization of potential investment outcomes relative to samples based on individual countries (e.g., the US or the UK). Although single-country samples are commonly used to calibrate inputs for investment simulations, such samples contain very few independent observations of long-horizon investment outcomes. Moreover, these samples are likely to suffer from both survivor bias [e.g., Brown, Goetzmann, and Ross (1995)] and easy data bias [e.g., Dimson, Marsh, and Staunton (2002)]. Fama and French (2002) and Avdis and Wachter (2017), for example, present direct evidence that the historical performance of US stocks over the postwar period likely exceeded *ex ante* expectations.<sup>13</sup>

In Table II, we list each individual sample country and the corresponding data coverage. Five countries — Denmark, France, Germany, the UK, and the US — are included in the sample over the full 1890 to 2023 period. The sample periods for the other countries are shorter owing to data availability and development classification status. Our data cover 91% of the potential country-months in the developed country sample. Table II also presents the geometric average real return and the standard deviation of real return for each combination of country and asset class. For the pooled sample of all 31,801 country-month observations, the geometric average returns for domestic stocks, international stocks, bonds, and bills are 0.37%, 0.44%, 0.04%, and  $-0.03\%$ , respectively (untabulated). Based on comparisons with the pooled sample, the average real returns in the US sample are higher for domestic stocks, bonds, and bills and lower for international stocks. But the US is not an extreme outlier relative to other countries for any of the four asset classes.

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<sup>13</sup>The dataset construction methods are intentionally designed to mitigate survivor bias and easy data bias. The sample inclusion dates for individual countries are based on *ex ante* measures of economic activity (e.g., the proportion of a country’s labor force employed in the manufacturing and services sectors and the country’s membership in global policy organizations like the Organisation for Economic Co-operation and Development), and we take significant steps to construct continuous monthly data series for each country.

## 4 Methods

In this section, we detail our lifecycle assumptions, present the household’s formal portfolio choice problem, and discuss our approaches for modeling uncertainty over labor income, investment returns, and longevity. Section 4.1 describes the lifecycle design. Section 4.2 defines and parameterizes household utility over retirement consumption and bequest and introduces our base case static optimization problem. Section 4.3 presents the stochastic process for labor income during the working period as well as Social Security income during the retirement period. Section 4.4 describes the Monte Carlo simulation procedure.

### 4.1 Lifecycle design

Households in our base specification are composed of a female and a male of equal age. The model periods are in months indexed by  $t = 1, 2, \dots, T_{max}$ , where  $T_{max}$  is the month of death of the last remaining survivor from the couple. Each member of the household is eligible to work and save starting from the first month of age 25 ( $t = 1$ ). The retirement date is denoted  $T_{ret}$ , and our base case specifies an exogenous retirement age of 65, such that  $T_{ret} = 480$ . An individual may, however, experience nonemployment during their potential working years, such that not all investors work the full 40 years. In the base case, we assume that individuals save  $r_c = 10\%$  of their income for retirement, and no contributions occur during nonemployment periods. The assumed 10% contribution rate is close to the mean and median contribution rates for participants in Vanguard defined-contribution plans (including both employee and employer contributions) in 2023 of 11.7% and 11.0%, respectively [Vanguard (2024)].<sup>14</sup> We also assume that individuals making less than  $Y_{min} = \$15,000$  (in 2022 dollars) in a given year do not contribute to their retirement plan, consistent with evidence of low retirement saving rates among this group [e.g., Vanguard (2024)].

At time  $T_{ret} + 1$ , each individual leaves the workforce (either ending employment or nonemployment) and begins to draw from retirement savings and Social Security. We assume that investors withdraw  $r_w = 4\%$  of their account balance at retirement in the first year and inflation-adjusted amounts calculated from this base withdrawal in subsequent years [i.e., the “4% rule” of Bengen (1994)]. In reality, retirees use a variety of withdrawal strategies. The 4% rule is ubiquitous in popular press and common retirement advice, so we use it as a simple heuristic for retirement withdrawals.<sup>15</sup> In robustness tests, we demonstrate that our main conclusions hold for alternative

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<sup>14</sup>Our assumed 10% contribution rate is also similar to Poterba, Rauh, Venti, and Wise’s (2005, 2009) assumed 9% contribution rate to household retirement accounts.

<sup>15</sup>In Choi’s (2022) review of the most popular personal finance books, he finds that seven of the 12 books offering

retirement withdrawal rules. We also note that the outcomes of households who choose to annuitize fully at retirement will be reflected by our wealth at retirement results.

The Social Security Administration (SSA) reports conditional death probabilities at each age for females and males.<sup>16</sup> Our simulations incorporate gender-specific longevity risk, and the lifespan of each individual is randomly determined. There is considerable uncertainty over longevity outcomes. The 5th percentile of age at death for the couple is 70.8 years, and the 95th percentile is 100.0 years. This uncertainty is an important feature to consider in assessing the ability of investment strategies to fund consumption through retirement (see the internet appendix for further details on the distribution of age at death). Both the female and the male in each couple are alive at age 25, but one or both may die before retirement at age 65. We retain couples in which both members die before retirement because these couples have bequests that depend on their investment strategies.

Given the simulation design, the (unmodeled) consumption and potential survivor benefits from Social Security during the pre-retirement period are independent of the retirement investment strategy. As such, we do not study consumption in the pre-retirement period and do not include it in the utility calculations.

## 4.2 Household utility and portfolio choice problem

Household utility is determined by monthly retirement consumption and a bequest. Following Duarte, Fonseca, Goodman, and Parker (2024), we scale household consumption by the square root of household size in the utility calculations to reflect differences in consumption needs for couples versus singles. The total utility from monthly consumption during retirement and the couple’s bequest is

$$U(C, B) = \sum_{t=T_{ret}+1}^{T_{max}} \delta^t \frac{(C_t/\sqrt{H_t})^{1-\gamma}}{1-\gamma} + \delta^{T_{max}} \theta \frac{(B+k)^{1-\gamma}}{1-\gamma}, \quad (1)$$

where  $C_t$  is the household’s real consumption in month  $t$ ,  $B$  is the household’s real bequest,  $\delta$  is the subjective discount factor,  $\theta$  and  $k$  are bequest utility parameters, and  $\gamma$  is the coefficient of relative risk aversion.

We follow Duarte, Fonseca, Goodman, and Parker (2024) by setting  $\delta$  to one, which equally weights utility in each month of retirement to reflect the flow of utility during the retirement period. Our bequest utility specification follows De Nardi, French, and Jones (2010). We use their estimate for risk aversion of  $\gamma = 3.84$ , and we assume that this risk aversion coefficient applies to

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explicit retirement spending advice recommend the 4% rule.

<sup>16</sup>See <https://www.ssa.gov/oact/HistEst/PerLifeTables/2022/PerLifeTables2022.html>.

both consumption and bequest. De Nardi, French, and Jones (2010) estimate a bequest intensity of  $\theta = 2,360$  when studying bequest utility alongside utility from annual consumption, and we multiply this parameter estimate by  $12^\gamma$  to reflect the mechanical difference in scaling between monthly and annual consumption levels.<sup>17</sup> Finally, we inflation-adjust their bequest curvature parameter  $k$ , which determines the extent to which bequests are viewed as luxury goods, and use  $k = \$490,000$  in 2022 USD.<sup>18</sup>

In our base specification, the household chooses a static asset allocation across domestic stocks, international stocks, bonds, and bills to maximize expected utility:

$$\max_{\{w\}} \mathbb{E}_0[U(C, B)] = \mathbb{E}_0 \left[ \sum_{t=T_{ret}+1}^{T_{max}} \frac{(C_t/\sqrt{H_t})^{1-\gamma}}{1-\gamma} + \theta \frac{(B+k)^{1-\gamma}}{1-\gamma} \right], \quad (2)$$

$$\text{s.t. } R_t^p = w' R_t, \quad (3)$$

$$\mathbb{1}' w = 1, \quad (4)$$

$$w \geq 0, \quad (5)$$

$$W_0 = 0, \quad (6)$$

$$W_{t+1} = \begin{cases} W_t(1 + R_{t+1}^p) + S_{t+1} & \text{for } t \leq T_{ret}, \\ (W_t - D_{t+1})(1 + R_{t+1}^p) & \text{for } t > T_{ret}, \end{cases} \quad (7)$$

$$D_{t+1} = \begin{cases} 0 & \text{for } t \leq T_{ret}, \\ \min\left(\frac{1}{12}(r_w W_{T_{ret}}), W_t\right) & \text{for } t > T_{ret}, \end{cases} \quad (8)$$

$$C_{t+1} = \max(D_{t+1} + SS_{t+1}, SSI_{t+1}) \quad \text{for } t > T_{ret}, \quad (9)$$

$$B = W_{T_{max}}, \quad (10)$$

where  $w$  is a  $4 \times 1$  vector of fixed portfolio weights,  $R_t$  is a  $4 \times 1$  vector of gross real returns on the four assets in month  $t$ ,  $R_t^p$  is the gross portfolio return,  $W_t$  is the household's end-of-month retirement wealth,  $S_t$  is the flow of savings for the couple,  $D_t$  is the monthly retirement account withdrawal,  $SS_t$  is the couple's monthly combined Social Security benefit, and  $SSI_t$  is their monthly Supplemental Security Income. Equations (4) and (5) capture the constraints faced by the couple on taking levered positions and short positions, respectively, in the risky assets. In subsequent analyses, we relax the restriction on portfolio leverage.

<sup>17</sup>Taking  $C_a$  as an arbitrary annual consumption level, the sum of utility from 12 months of consuming  $C_m = C_a/12$  is  $\sum_{t=1}^{12} \frac{C_m^{1-\gamma}}{1-\gamma} = 12 \frac{(C_a/12)^{1-\gamma}}{1-\gamma} = 12^\gamma \frac{(C_a)^{1-\gamma}}{1-\gamma}$ .

<sup>18</sup>We do not consider utility from housing, which is an important asset for many households. Venti and Wise (1991) and Poterba, Venti, and Wise (2011) show, however, that few households use reverse mortgages or otherwise decrease their home equity late in life.

The following subsections provide additional details on the optimization problem and our bootstrap simulation procedure used to compute expected utility.

### 4.3 Lifecycle income

We model labor income using the model of Guvenen, Karahan, Ozkan, and Song (2021). Their flexible framework allows for investor heterogeneity, permanent and transitory income shocks, and employment and nonemployment states. They estimate the model to fit a large number of cross-sectional moments and time-series properties of lifecycle earnings data on millions of US workers from the SSA.

The annual income level for investor  $i$  ( $i \in \{f, m\}$ ) at age  $\tau + 24$  is given by

$$Y_\tau^i = (1 - \nu_\tau^i) e^{(g(\tau) + \alpha^i + \beta^i f(\tau) + z_\tau^i + \varepsilon_\tau^i)}, \quad (11)$$

where  $g(\tau)$  is a quadratic polynomial that fits the well-known hump shape of lifecycle earnings,  $f(\tau)$  is a linear function increasing in  $\tau$ ,  $\alpha^i$  and  $\beta^i$  are investor-specific parameters that affect the expected level and slope of earnings, respectively,  $z_\tau^i$  is a persistent earnings component following

$$z_\tau^i = \rho z_{\tau-1}^i + \eta_\tau^i, \quad (12)$$

and  $\varepsilon_\tau^i$  is a transitory earnings shock. The persistent earnings component coefficient  $\rho$  is estimated to be 0.96 for annual earnings, which implies a half-life of about 17 years. The permanent and transitory shocks ( $\eta_\tau^i$  and  $\varepsilon_\tau^i$ , respectively) each follow a normal mixture distribution. Finally,  $\nu_\tau^i = 0$  represents full-year employment, whereas  $\nu_\tau^i = 1$  is full-year nonemployment. This nonemployment variable takes values as follows,

$$\nu_\tau^i = \begin{cases} 0 & \text{with prob. } 1 - p_\nu(\tau, z_\tau^i), \\ \min\{1, \phi_\tau\} & \text{with prob. } p_\nu(\tau, z_\tau^i), \end{cases} \quad (13)$$

where  $\phi_\tau$  follows an exponential distribution with mean  $1/\lambda$ ,  $p_\nu^i(\tau, z_\tau^i) = \frac{e^{\xi_\tau^i}}{1 + e^{\xi_\tau^i}}$  is the nonemployment probability, and  $\xi_\tau^i = a + bf(\tau) + cz_\tau^i + dz_\tau^i f(\tau)$  with  $b < 0$ ,  $c < 0$ , and  $d < 0$ . As such, the probability of nonemployment is negatively influenced by the level of the persistent earnings component, which produces persistence in the nonemployment state.

We assume annual income is divided evenly over months in the year. For months  $12(\tau - 1) <$

$t \leq 12\tau$ , savings is given by

$$S_t^i = \begin{cases} \frac{1}{12}(r_c Y_\tau^i) & \text{for } Y_\tau^i \geq Y_{min}, \\ 0 & \text{for } Y_\tau^i < Y_{min}, \end{cases} \quad (14)$$

and

$$S_t = S_t^f + S_t^m. \quad (15)$$

The heterogeneity in earnings processes across investors is captured by two income parameters,  $\alpha^i$  and  $\beta^i$ , and the initial state of the permanent income component,  $z_0^i$ . High (low) values for  $\alpha^i$  and  $\beta^i$  designate investor types with high (low) levels and growth rates, respectively, for expected lifetime earnings, whereas high (low)  $z_0^i$  captures a tendency for high (low) early-career earnings. In our base case analyses, we set all three parameters for both members of the couple equal to their median values in the Guvenen, Karahan, Ozkan, and Song (2021) calibration, i.e.,  $(\alpha^i, \beta^i, z_0^i) = (0, 0, 0)$ .

We simulate from the labor income model using the parameter estimates from the replication code of Guvenen, Karahan, Ozkan, and Song (2021) with the additional assumption that the income model applies equally to females and males. We scale the simulation output (which does not initially have a standard unit of measurement) to match the level of average log earnings in 2010 dollars [Figure C.36 in Guvenen, Karahan, Ozkan, and Song (2021)] and then convert to 2022 dollars by adjusting for the change in the consumer price index (CPI). Figure 3 plots the distribution of household income from our base case simulation as a function of household age. The mean reflects the well-known hump shape in earnings [e.g., Cocco, Gomes, and Maenhout (2005)], and the 10th and 90th percentiles imply considerable uncertainty in earnings.

Our simulations incorporate retirement income from Social Security benefits and the additional social safety net from Supplemental Security Income (SSI). Social Security benefits are calculated based on taxes paid on earnings during working years. We use the formulas effective in 2022 to calculate Social Security benefits based on each worker's earnings. We incorporate spousal and survivor benefits in the retirement period. In the internet appendix, we provide full details of the Social Security benefit calculations, including the calculation of average indexed monthly earnings (AIME), the bend points in the benefit formula, the scenarios for spousal and survivor benefits, and the effects of retirement age on benefits. Finally, SSI is available to retirees with little other income. The maximum monthly benefit in 2022 is \$1,261 for couples and \$841 for singles.

## 4.4 Simulation procedure

We simulate lifecycle outcomes for couples using a Monte Carlo simulation approach. We solve for the optimal static investment policy by maximizing expected household utility following equation (2). We initially find the base case optimal strategy by considering all possible strategies that allocate across domestic stocks, international stocks, bonds, and bills with fixed weights throughout the lifecycle, subject to the no-leverage and non-negative weight constraints. We consider a grid with 1% increments for each weight to perform a grid search for the optimal strategy. We then compare simulation results for the optimal fixed-weight strategy and several benchmark strategies.

Each investment strategy is adopted by otherwise identical couples in each draw (i.e., the couples in each draw have the same longevity, income, and savings and realize the same asset class returns). As such, our simulation design draws inferences about the differences in expected utility and retirement outcomes that derive from the chosen investing strategy.

Our simulation design includes the following steps in each draw.

1. We determine the lifespan of the household. We generate random longevity using conditional mortality probabilities, and we assume that the probability of death is equal across the 12 months at a given age. In the base case, we denote the realized lifespans of the female and the male as  $T_f$  and  $T_m$ , respectively, and the couple's lifespan in months (starting from age 25) as  $T_{max} = \max(T_f, T_m)$ .
2. We adopt a stationary block bootstrap approach in the spirit of Politis and Romano (1994) to draw a full time series of monthly real returns for the four asset classes. We draw blocks of consecutive monthly returns from the same country to capture time-series dependencies in asset returns. Block lengths are drawn from a geometric distribution. The average block length is 120 months in the base case, so the blocks reflect long-term time-series properties of returns. A set of all four asset class returns are drawn from each selected country-month to preserve cross-sectional dependencies across assets, and we denote a monthly real return vector as

$$R_t = [R_t^{Domestic\ stocks} \quad R_t^{International\ stocks} \quad R_t^{Bonds} \quad R_t^{Bills}]'. \quad (16)$$

We repeatedly draw blocks of returns from random countries and periods until we produce a time series of asset class returns that spans the viable investment period for the couple. This period extends from month  $t = 2$  (i.e., the second month of age 25, which is the first month in which the couple may have a positive beginning-of-month account balance) to month



$t = T_{max}$ . In the event that the asset return data from a country-period is insufficient to fill a return block (i.e., the end of the sample for that country occurs before the number of monthly return vectors in the block equals the drawn block length), we draw a random country and continue to fill the return block with return data from the beginning of that country’s sample [i.e., we use a stationary block bootstrap approach to avoid undersampling]. The final bootstrap draw of asset class returns in the iteration is  $R = \{R_2, R_3, \dots, R_{T_{max}}\}$ .

3. Given the couple’s chosen investment strategy, we compute a monthly time series of portfolio returns over the couple’s lifetime. Denote the investment weights for the chosen strategy in month  $t$  as  $w_t$ , where  $w_t = w$  for a fixed-weight strategy. The portfolio return in month  $t$  is  $R_t^p = w_t' R_t$ .
4. The couple begins with no wealth in savings,  $W_0 = 0$ . We calculate the evolution of wealth during the working years, considering savings from labor income and returns on invested wealth as previously described.
5. At retirement, the couple stops working and saving. We calculate the evolution of wealth during the retirement years, considering withdrawals and return on invested wealth as previously described. If the household’s wealth is depleted at any time during retirement, it remains at zero until death. The household also receives monthly Social Security benefits. The couple is supported by SSI if their income falls below the threshold (\$1,261 for couples and \$841 for singles). Finally, the couple’s bequest is all remaining wealth at the death of the last surviving spouse.

We repeat this process 1,000,000 times for each investment strategy. We use the time series of monthly consumption during retirement and the household’s bequest to compute utility for each draw following equation (1). Given our assumptions, consumption prior to retirement is unaffected by the couple’s choice for their investment strategy, so pre-retirement consumption is irrelevant for inferences about the strategies. The mean utility across draws is our Monte Carlo estimate of expected utility for a given investment strategy.

We use the simulation approach outlined above to compare the optimal fixed-weight strategy and benchmark strategies. For each strategy, we compile distributional statistics for wealth at retirement, the portfolio drawdown during the household’s working period, the income replacement rate during retirement, the portfolio drawdown during the household’s retirement period, and

wealth at death (i.e., the bequest).<sup>19</sup>

We make utility comparisons across strategies by running simulations as described above but with a range of potential savings rates. Given the retirement utility from a current investment strategy (e.g., the optimal fixed-weight allocation) with the 10% base savings rate during working years, we find the savings rate associated with an alternative investment strategy that provides the same expected retirement utility. An equivalent savings rate of 15%, for example, would indicate that a couple would need to increase their savings rate from 10% to 15% to maintain their expected utility in retirement.

## 5 Results

In this section, we examine optimal portfolio choice in our lifecycle model. Section 5.1 presents the optimal fixed-weight strategy and the utility gains relative to four benchmark strategies. Section 5.2 details strategy performance in the pre-retirement and post-retirement periods. Section 5.3 discusses results for a wide variety of alterations to our base fixed-weight specification. Section 5.4 considers the effects of consumption-saving decisions, optimal retirement ages, and time-varying asset allocation strategies.

### 5.1 Optimal investment strategy

Panel A of Table III shows the optimal fixed-weight strategy that maximizes expected retirement utility. The couple optimally invests 33% in domestic stocks, 67% in international stocks, 0% in bonds, and 0% in bills throughout their lifetimes. At this optimum, the couple eschews fixed income investments and chooses an all-equity strategy. This result may seem surprising given the vaunted diversification potential and safety offered by bonds. As discussed in the context of Table I, however, bonds become riskier and more correlated with domestic stocks as the horizon grows, whereas the optimal allocation reflects the superior diversification benefits and growth potential of international stocks.

Panel B of Table III shows asset weights for four benchmark strategies and provides an analysis of the economic differences compared with the optimal strategy. The benchmark strategies are a 100% allocation to bills (a common default in the pre-PPA era), a 100% allocation to domestic

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<sup>19</sup>Maximum portfolio drawdowns are calculated as the largest real negative cumulative return relative to the previous peak. The working-period drawdown occurs entirely within the working years. The retirement-period drawdown begins with a peak that could occur either during the working years or the retirement years. That is, falling asset prices in the late working years can contribute to our measured retirement drawdown.

stocks, a balanced strategy with a 60% allocation to domestic stocks and a 40% allocation to bonds, and the TDF shown in Figure 2. The table reports the savings rates that would provide households with the same expected utility over retirement consumption and bequest as a 10.0% savings rate in the optimal, all-equity strategy. The bills, domestic stocks, and balanced portfolios each follow fixed-weight strategies, such that the optimal fixed-weight strategy provides higher expected utility by construction. These equivalent savings rates reveal the economic magnitudes of utility differences. To achieve the same degree of wellbeing, the couple would be required to save 56.2% (Bills), 16.3% (Domestic Stocks), or 19.3% (Balanced) of its annual income during the working years.

Based on the conventional wisdom, we may expect the TDF to improve expected utility relative to the optimal fixed-weight strategy. The TDF specifies age-based weights, and relaxing our focus on fixed-weight strategies introduces the potential for utility gains. Further, TDF glidepaths are typically designed within lifecycle models that consider investors' long-term goals and risk preferences.<sup>20</sup> We find, however, that a couple must save 16.1% (i.e., 61% more) in the TDF to achieve the same expected utility as with the optimal fixed-weight strategy. Overall, Table III implies that a household currently heeding conventional advice can achieve an economically large increase in utility from investing in an internationally diversified, all-equity strategy.<sup>21</sup>

Figure 4 explores deviations from the optimal strategy and the associated equivalent savings rates. Panel A varies the allocation to domestic stocks between 0% and 100% within an all-equity design (i.e., the weights in bonds and bills are 0% and the remainder of the portfolio not invested in domestic stocks is allocated to international stocks). Expected utility as a function of the allocation to domestic stocks is relatively flat, and all allocations ranging from 11% domestic and 89% international to 55% domestic and 45% international have equivalent savings rates below 10.5%. This finding gives real-world investors the latitude to choose an all-equity strategy that reflects the size of their local market relative to the global market. That is, an American investor may feel comfortable investing over half of their wealth in the domestic market given the US's large global weight, whereas a Canadian investor may wish to invest less than the optimal 33% in domestic stocks. Our results indicate that, as long as investors avoid overly large domestic equity

<sup>20</sup>See, e.g., [https://corporate.vanguard.com/content/dam/corp/research/pdf/Vanguard-Life-Cycle-Investing-Model-VLCM-A-general-portfolio-framework-US-ISGVLCM\\_032021\\_online.pdf](https://corporate.vanguard.com/content/dam/corp/research/pdf/Vanguard-Life-Cycle-Investing-Model-VLCM-A-general-portfolio-framework-US-ISGVLCM_032021_online.pdf).

<sup>21</sup>The magnitude of the estimated utility gains depends on household risk aversion, but our conclusion that the all-equity strategy provides economically large gains holds for all  $\gamma$  values between zero and ten. As is tradition in the literature [e.g., Campbell, Cocco, Gomes, and Maenhout (2001)], we take a partial equilibrium view of lifecycle investing and do not consider equilibrium effects of investor shifts across investment strategies. As Heaton and Lucas (2000) state, "Although one could look at portfolio choice in a general equilibrium framework, unless the model were capable of generating a realistic returns process the results on portfolio choice would also be suspect."

allocations, the utility costs are relatively small if they deviate from the optimum but maintain an all-equity approach.

Figure 4 also presents analogous results for adding bonds (Panel B) or bills (Panel C). In these analyses, the weight in bonds or bills ranges from 0% to 40% and the equity portion of the portfolio is split with relative 33% and 67% weights in domestic and international stocks, respectively. Our investors dislike even relatively small allocations to bonds. An allocation of 12% to bonds produces an 11.0% equivalent savings rate, which implies that the couples feel they need to increase their savings rate by 10% if they allocate 12% of their wealth to bonds. To achieve the same expected utility as saving 10.0% with the all-equity strategy, the couples must save 20% more to invest 20% in bonds, 35% more to invest 30%, and 54% more to invest 40%. The equivalent savings rates are even higher for allocations to bills in Panel C.

## 5.2 Lifecycle investment strategy performance

The previous section shows that the optimal fixed-weight strategy is an all-equity allocation with international diversification. In this section, we examine simulation results for couples who adopt the optimal investment strategy or one of the alternatives. For each strategy, we study four retirement saving outcomes: (i) the distribution of wealth at retirement, (ii) the distribution of the income replacement rate to describe the consumption stream in retirement, (iii) the probability of exhausting financial wealth prior to death, and (iv) the distribution of wealth at death. We also consider two intermediate outcomes: (i) the distribution of the maximum drawdown during working years and (ii) the distribution of the maximum drawdown during retirement years. Recall that our simulation design allows for a direct comparison across five investment strategies used by otherwise identical couples, in that the couples in each draw have the same longevity, income and saving, and asset class return realizations. As such, any differences in retirement outcomes are directly attributable to the investment strategies.

### 5.2.1 Pre-retirement period

Panel A of Table IV summarizes the distribution of wealth built up through the savings period for each investment strategy. For each strategy, the table shows the mean, standard deviation, and percentiles of the wealth distribution at the beginning of the retirement period across 1,000,000 bootstrap simulations. Panel A of Figure 5 provides a visual summary for each of the five strategies. The figure shows box-and-whiskers plots of the distribution of retirement wealth. The middle line

is the median, the box designates the interquartile range, and the whiskers extend to the 10th and 90th percentiles. The exhibits report wealth at retirement in millions of 2022 dollars.

For context in interpreting the wealth levels, couples save \$0.24 million on average. As reported in Table C.I, about 2.3% of households have both members die prior to retirement age. For these couples, bequests occur prior to retirement and wealth at retirement is \$0.

The optimal strategy outperforms the alternatives in generating wealth at retirement. Bills perform poorly in wealth accumulation, with an average retirement wealth balance of only \$0.27 million. This poor performance demonstrates that money market and stable value funds are ineffective retirement saving tools when used in isolation and provide support for the changes to default DC plan options brought about by the PPA. Investing in domestic stocks produces \$1.02 million in wealth on average (annual withdrawal of \$40,800 given our 4% retirement withdrawal rule), which is higher than the averages for the two QDIAs of \$0.71 million for the balanced portfolio (withdrawal of \$28,400) and \$0.77 million for the TDF (withdrawal of \$30,700). The optimal, all-equity strategy generates the most wealth on average at \$1.07 million, which supports an annual real withdrawal of \$42,600.

More important, the percentiles in Panel A of Table IV show that the distribution of wealth for the optimal strategy is preferable to the distributions of the other strategies. Concentrating on the poor outcomes, the optimal strategy has a 5th percentile of \$0.13 million versus \$0.03 million for bills, \$0.07 million for domestic stocks, \$0.06 million for the balanced strategy, and \$0.09 million for the TDF. The optimal strategy provides households with impressive upside, and its international diversification limits downside risk during the saving years.

### 5.2.2 Retirement period

Panel B of Table IV and Panel B of Figure 5 summarize the distribution of income replacement rates during retirement for each strategy. The reported replacement rates are calculated as the mean of monthly household retirement consumption divided by the mean of monthly household income during the working ages of 25 to 65.<sup>22</sup> The table reports that the two QDIA strategies (along with Social Security benefits) allow couples to achieve full income replacement, on average. The mean replacement rate is 1.03 for the TDF, for example, although 62% of couples have a replacement rate below one. The optimal strategy performs best by generating a mean replacement rate of 1.24, and more than half (55%) of couples achieve full replacement or better. The left-tail outcomes for the optimal portfolio also exceed those for the QDIA strategies that diversify into fixed income in

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<sup>22</sup>The replacement rate is equal to 0.00 for couples who are deceased prior to retirement.

an attempt to preserve wealth and retirement consumption.

### 5.2.3 Wealth preservation and bequest

Preserving wealth in retirement is crucial, and retirees who exhaust their savings must rely solely on their Social Security and SSI benefits. Panel C of Figure 5 plots the probability of financial ruin, defined as reaching \$0 in wealth prior to the death of the last survivor in the household. Couples investing in “safe” bills and using the 4% rule have a 38.9% chance of running out of savings. Fully investing in domestic stocks is better with a 17.1% ruin probability, but this risk is likely higher than most households would like to face. The two QDIAs, which invest in fixed income to preserve wealth, also fail to generate reliable streams of retirement income. The balanced strategy has a ruin probability of 16.9%, and the status quo TDF has an even higher probability of 19.7%. Recall from Figure 2 that the TDF invests just 17% in equity throughout much of retirement. The large allocations to bonds and bills do little to prevent poor retirement outcomes. The ruin probability for the optimal strategy is low in comparison at 7.0%. Equity offers strong potential for additional investment gains during retirement, and international diversification is crucial for capital preservation.

Panel C of Table IV and Panel D of Figure 5 display distributional statistics for real wealth upon the death of the last survivor in the household. A couples’s bequest is \$0 if they experience financial ruin prior to death. The optimal strategy substantially outperforms the other strategies in providing bequests. The mean wealth at death is \$2.94 million for the optimal strategy versus \$0.08 million for bills, \$2.61 million for domestic stocks, \$1.10 million for the balanced strategy, and \$0.72 million for the TDF. The means are heavily affected by right-tail outcomes. The median couple generates a bequest of \$1.03 million with the optimal strategy, which is far larger than for bills (\$0.02 million), domestic stocks (\$0.59 million), the balanced strategy (\$0.35 million), or the TDF (\$0.27 million). The large bequests for the optimal strategy reflect both the tendency for stocks to help to build wealth and the relative safety of the optimal strategy in preserving wealth during retirement.<sup>23</sup>

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<sup>23</sup>In the appendix, we further study household retirement outcomes. We specifically show that the favorable performance of the all-equity strategy relative to the benchmarks in achieving retirement outcomes: (i) is partially attributable to continued wealth generation in retirement to combat longevity risk, (ii) persists even if the domestic stock market has a poor realization during retirement, (iii) is strongest if realized inflation in retirement is high, and (iv) occurs whether the realized correlation between domestic and international stock markets is low or high.

## 5.2.4 Portfolio drawdowns

The focus on the four retirement outcomes summarized in Panels A to D of Figure 5 glosses over important intermediate portfolio performance measures. Regulations focus on short-term losses as a metric for the suitability of retirement strategies. To study this issue, Table V and Panels E and F of Figure 5 report the largest portfolio drawdowns during the working years and retirement years for each strategy. The reported drawdowns are the largest peak-to-trough declines in real asset values for a given strategy, and they are expressed in decimal form in the table.

During the working years, each strategy produces large real drawdowns on average. Panel A of Table V shows average maximal drawdowns of 42% for bills, 67% for domestic stocks, 54% for the balanced strategy, 52% for the TDF, and 55% for the optimal strategy. The optimal strategy's 55% average drawdown would cause discomfort for even the most stalwart investors, but each strategy that attempts to provide long-term appreciation is subject to similarly large average losses. Investors, advisors, and regulators are likely most concerned about the largest potential drawdowns, and the optimal strategy outperforms the alternatives in the right tail of the drawdown distribution. The 95th percentile drawdown of 76% for the optimal strategy is favorable relative to drawdowns of 96% (bills), 96% (domestic stocks), 92% (balanced), and 81% (TDF).

Retirement-period drawdowns are likely an even larger concern for households. Panel B of Table V reports average maximal drawdowns during retirement of 46% for bills, 62% for domestic stocks, 50% for the balanced portfolio, 40% for the TDF, and 48% for the optimal strategy. According to this metric, the TDF achieves better capital preservation compared with the optimal portfolio. In terms of avoiding the largest drawdowns, however, the optimal strategy is the best. The 95th percentile for the optimal portfolio is a 74% drawdown versus an 89% drawdown for the TDF.

We note that the superior performance of the optimal strategy in the four outcomes (generating wealth at retirement, producing retirement income, avoiding financial ruin, and providing a bequest) occurs despite the potential for relatively large drawdowns. Experiencing large drawdowns is painful for investors, and many investors will have a natural inclination to abandon their strategies at inopportune times when drawdowns occur. An important policy issue is how to manage this behavior, as the optimal strategy relies on maintaining the all-equity approach through good times and bad. An additional policy issue is the extent to which the average drawdown is emphasized relative to the worst potential drawdowns. The TDF provides lower average and median retirement drawdowns than the all-equity approach, but it has poor worst-case scenarios when high inflation

erodes the real value of bonds in retirement.

### 5.3 Lifecycle problem modifications

The all-equity strategy with 33% in domestic stocks and 67% in international stocks is optimal given our lifecycle design described in Section 4. We now explore the sensitivity to a large variety of modifications to this base case. Table VI presents the results, with the base case summarized in Panel A for convenience. The remaining panels correspond to alternative designs. For each specification, we report the optimal asset class weights. The last column of the table shows the amount of borrowing as a percentage of wealth for the specifications in which leverage is allowed.

#### 5.3.1 Lifecycle simulation design

Panels B through I of Table VI show that our results are insensitive to modifying several aspects of the simulation design:

- the average block length in the stationary block bootstrap from 12 months to 240 months,
- the risk aversion parameter from  $\gamma = 0.5$  to  $\gamma = 10.0$ ,<sup>24</sup>
- the weight on bequest from  $\theta = 0$  (i.e., no bequest motive) to  $\theta = \infty$  (i.e., utility only from bequest),
- the household utility specification to scale consumption by  $H_t$  (i.e., household size) instead of  $\sqrt{H_t}$ ,
- the household utility specification to include a subjective discount factor that we set to  $\delta = 0.98^{1/12}$  (i.e., an annualized discount factor of 0.98),
- the withdrawal strategy to be a constant, real withdrawal rate of  $r_w = 3\%$  or  $r_w = 5\%$ ,
- the withdrawal strategy to withdraw an annualized 4% of wealth at the beginning of each month,
- the retirement age to be 62, 67, or 70 (i.e.,  $T_{ret}$  to equal 444, 504, or 540, respectively),
- the contribution rate to be  $r_c = 5\%$  or  $r_c = 15\%$ ,

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<sup>24</sup>Calvet, Campbell, Gomes, and Sodini (2023) estimate risk aversion in the cross section of Swedish households. The risk aversion parameter of 7.5 is close to their mean of 7.57, and their standard deviation of 1.06 implies that the parameters of 5.0 and 10.0 are roughly two standard deviations from the mean. Therefore, the large majority of households lie in this range according to Calvet, Campbell, Gomes, and Sodini’s (2023) estimates.



- the lower income limit for retirement saving to be  $Y_{min} = \$0$  or  $Y_{min} = \$45,000$ , and
- the household type to be a single female, a single male, a female couple, or a male couple.

None of these alterations has a meaningful impact on the optimal strategy. The weights in bonds and bills are 0% in every case, and the weight in domestic stocks varies between 32% and 35%.

### 5.3.2 Investor types

Our base analysis studies households with median parameter values in the lifetime earnings model of Guvenen, Karahan, Ozkan, and Song (2021). Panel J of Table VI studies four household types differing along two dimensions: low or high human capital  $[(\alpha^i, \beta^i)]$  and low or high initial income  $[z_0^i]$ . These parameter combinations produce a variety of lifetime earnings profiles (i.e., saving more early versus later in the working years) and total lifetime earnings levels. Despite the differences in savings behavior across households and the relative importance of savings versus Social Security, the optimal strategies are virtually identical. The optimal strategy provides a one-size-fits-all approach to lifetime savings.

### 5.3.3 Correlated labor income and stock returns

Panel K of Table VI explores the effect of correlation between the persistent earnings shock in Guvenen, Karahan, Ozkan, and Song’s (2021) model and domestic stock returns. The earnings shocks are annual, so we relate them to annual stock returns. We first generate a distribution of compounded annual stock returns from our block bootstrap approach. We then use a Gaussian copula approach to introduce correlation between this bootstrap distribution of annual stock returns and the mixture of normals distribution of the earnings shock. The base case has 0.0 correlation, and Panel K considers correlations ranging from 0.1 to 0.5.<sup>25</sup>

Introducing positive correlation between income and domestic stock returns decreases the optimal weight in domestic stocks. At a modest correlation of 0.1, the optimal strategy is 30% in domestic stocks and 70% in international stocks. At high correlation of 0.5, the weight in domestic stocks is down to 18% with the remaining 82% allocated to international stocks. None of the

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<sup>25</sup>The literature diverges on the magnitude of the correlation between income and stock returns. Fama and Schwert (1977) and Cocco, Gomes, and Maenhout (2005), among others, find little relation. Davis and Willen (2000) and Campbell, Cocco, Gomes, and Maenhout (2001) estimate heterogeneous correlations with educational attainment, with highly educated individuals having correlations as high as 0.3 to 0.5. Benzoni, Collin-Dufresne, and Goldstein (2007) model cointegration between dividends and labor income, and their model implies a correlation around 0.5 between stock returns and returns to human capital throughout most of the working years. We consider a wide range of correlations that spans these estimates.

couples switches to bonds when shifting their portfolios away from domestic stocks.<sup>26</sup>

### 5.3.4 Leverage

Our base case considers couples that are constrained from taking on leverage in their retirement savings vehicles. This constraint is realistic for most investors and can affect the optimal portfolio allocation [e.g., Frazzini and Pedersen (2014)]. Asness (1996) argues, however, that investors should use leverage to invest in a balanced portfolio with 60% stocks and 40% bonds rather than investing 100% in equities. We relax the constraint on leverage to study hypothetical investors who are able to borrow in their retirement savings accounts.

We model leverage as follows. We assume that investors will pay the prevailing yield on local government bills plus a spread. We consider a high spread of 6.50% that is estimated by Davis, Kubler, and Willen (2006) from household borrowing costs and near the median broker margin rates as of April 2024, a medium spread of 1.40% that equals the lowest broker margin rate as of April 2024, and a low spread of 0.37% estimated by van Binsbergen, Diamond, and Grotteria (2022) as an average risk-free rate spread over government bills using derivative prices. Leverage makes it possible for a couple's wealth to fall below zero. If the couple is still working, we require the household to repay their debts before they can resume saving for retirement. If the couple is in their retirement years, we set their wealth to zero. Finally, we limit borrowing to 100% of wealth, in line with US regulations on margin for stocks.

The couples who have access to leverage optimize over both asset class weights and leverage levels. Panel L of Table VI reports that a couple subject to the high borrowing spread does not wish to use leverage. As such, the no-leverage constraint in our base case is not binding for this household. At the medium spread, the couple optimally borrows 55% of their wealth. Rather than levering up a balanced portfolio, as advocated by Asness (1996), the couple continues to choose an all-equity portfolio with 34% in domestic stocks and 66% in international stocks. In unreported results, we find that a couple who borrows 60% of their wealth would invest a small amount in bonds. Given the medium spread, the couple optimally uses leverage right up to the point that they still prefer an all-equity approach, and they do not cross the threshold that would cause them to buy bonds. The results for the high and medium spread show that households who pay margin rates optimally choose all-equity strategies.

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<sup>26</sup>We also study the extreme case of perfect correlation considered by Bodie, Merton, and Samuelson (1992) and find that the optimal allocation is 3% domestic stocks, 97% international stocks, 0% bonds, and 0% bills, which is consistent with the pattern in Panel K.

Panel L of Table VI shows that bonds enter the optimal allocation with the lowest spread of 0.37%. This borrowing spread is more reflective of the borrowing costs for large institutions rather than for households. As such, one could think of this scenario as involving an investment product with leverage that is created by an institution and sold to households. We assume the institution provides this service without charging a fee. With risk-free borrowing rates, optimal borrowing hits the cap at 100% of wealth. The optimal portfolio allocations are 28% in domestic stocks, 57% in international stocks, 15% in bonds, and 0% in bills. Thus, the couples remain heavily invested in stocks, but they do have a small allocation to bonds. Given this allocation and our bootstrap approach, the worst monthly return to the portfolio is  $-48\%$  such that the couple just remains solvent.

#### 5.4 Static versus dynamic optimization

Our lifecycle simulation design allows investors to perform static optimization. The base case has a constant savings percentage, a fixed retirement date, and fixed-weight investment strategies. Dynamic optimization is common in the lifecycle asset allocation literature, and past studies allow investors to choose optimal savings rates and retirement ages. Many studies allow for age-dependent asset allocations, and some incorporate asset market state variables to allow for state-dependent strategies.

We acknowledge that dynamic optimization can be important for portfolio choice. The commonly used numerical methods for dynamic optimization, however, require a relatively small number of state variables. Studies with dynamic optimization often make simplifying assumptions that asset returns are IID and normally distributed or that expected stock returns are linear functions of state variables. We strongly believe that it is crucial to preserve the time-series and cross-sectional dependencies of asset class returns when evaluating lifecycle strategies. Given the complexities of the patterns in the data, it is not possible to describe adequately the current market state using a small number of state variables. We therefore prioritize our bootstrap approach and adopt a static optimization problem as our base case.

In this section, we consider feasible alterations to our base case to gain a sense of the importance of dynamic optimization in our setting. We retain our careful treatment of the asset class return data and study investors who are constrained from using leverage or shorting asset classes. We then consider age- and income-based savings rates, optimal retirement dates, strategies that condition on the domestic stock market price-dividend ratio, and age-based rules for investing. Overall, we

find little evidence that full dynamic optimization, if it were possible to implement, would change our conclusion that an all-equity strategy is optimal (or close to it).

#### 5.4.1 Income- and age-based contribution rates

We first consider contribution rates that vary based on income and age. We use the realized contribution rates estimated by Parker, Schoar, Cole, and Simester (2023) using account-level data on US retirement savers. They divide savers into terciles based on income and examine investor contribution rates by age groups (i.e., ages 25-27, ages 28-30, and so on).<sup>27</sup> We place each retirement saver in our simulation into one of the income- and age-based groups based on the income cutoffs reported by Parker, Schoar, Cole, and Simester (2023), such that each individual in our simulation has a time-varying contribution rate that matches the behavior of US investors. Panel M of Table VI shows that this modification to our static design with a constant savings rate does not affect the optimal strategy.

#### 5.4.2 Optimal retirement timing

We next examine optimal retirement timing. We allow households to optimize across portfolio choice and retirement age, with potential ages ranging from 62 to 70 (the earliest and latest ages to claim Social Security). We assume that the couple claims Social Security upon retirement. The investors optimize utility over a bequest and consumption from age 62 until death, and their consumption prior to retirement is set to their earnings minus their retirement contributions.

To find the optimal retirement ages, we first classify couples into terciles across each of three dimensions as they are turning 62: (i) current earnings, (ii) current retirement wealth, and (iii) expected Social Security benefit level. This procedure classifies each couple into one of 27 groups. We find the optimal retirement age for each group. The optimal retirement ages range from 62 to 70. The optimal age is increasing in current income, decreasing in current wealth, and decreasing in Social Security benefits.<sup>28</sup> We then find the optimal fixed-weight asset allocation strategy, while allowing each couple to retire at their optimal age. Panel M of Table VI shows that incorporating optimal retirement ages does not influence the optimal asset allocation.

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<sup>27</sup>Fagereng, Holm, Moll, and Natvik (2021) show that the net contribution rate is relatively constant as a function of wealth, so we concentrate on the age- and income-based savings rates in Parker, Schoar, Cole, and Simester (2023).

<sup>28</sup>We report the optimal retirement ages in the internet appendix.

### 5.4.3 Time-varying investment allocations

A portfolio with 33% in domestic stocks and 67% in international stocks is the optimal fixed-weight strategy. We demonstrate in Sections 5.1 and 5.2 that this fixed-weight strategy is superior to the age-based, stock-bond strategy employed by a representative TDF. There may be, however, alternative strategies with weights that are based on age or market conditions that improve performance relative to our base case. We investigate these alternatives in this section, noting that specific strategy rules are required to make the optimization computationally feasible.

#### *Conditional asset allocations*

We study whether the couples in our simulations would prefer to adopt time-varying allocations that depend on the market state. The price-dividend ratio of the domestic stock market is a prominent state variable in the asset allocation literature, so we allow our investors to consider this variable. We first augment our dataset of asset class returns with a lagged price-dividend ratio for each country-month before running the bootstrap. As such, entering each month, the couple is aware of the current valuation level of domestic stocks. We divide the market states into quintiles, creating groups of country months that range from low price-dividend ratios (from 0.00 to 18.76) to high price-dividend ratios (from 43.67 to infinity).<sup>29</sup> We allow the couples to choose a different asset allocation in each quintile, and we find the jointly optimal set of state-dependent weights.

Table VII shows the optimal conditional allocations. Panel A repeats allocations for the base case, which does not condition on market state. Panel B shows the optimal weights for each price-dividend quintile. When the domestic price-dividend ratio is low, investors weight domestic stocks heavily at 65% and allocate the remaining 35% to international stocks. In the middle three quintiles, the allocations are similar to the unconditional optimal strategy. In the quintile with the highest price-dividend ratios, the couple wishes to reduce the domestic stock allocation to 16%, increase international stock exposure to 75%, and invest 9% in bonds. Thus, when domestic stock prices are very high, the couples optimally allocate a small portion of their wealth to bonds.

To measure the economic gains from considering the market state relative to a fixed-weight strategy, we calculate the equivalent savings rate. To achieve the same expected utility as the couple saving 10.0% with the optimal fixed-weight strategy, a couple using the conditional strategy saves 9.7%. In untabulated results, we find that nearly 80% of the economic gain from conditioning is attributable to varying the domestic-international stock allocation rather than to including bonds.<sup>30</sup>

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<sup>29</sup>Relative to imposing a linear relation between weights and the state variable, the quintile approach provides more flexibility in the relation but ignores variation in the state variable within the quintiles.

<sup>30</sup>The equivalent savings rate for the optimal all-equity conditional strategy, which invests 64%, 27%, 30%, 30%, and 22% in domestic stocks across the five states with the remainder in international stocks, is 9.79% compared with

### *Age-based, stock-bond rules*

A natural starting point for age-based, stock-bond rules is to allow households to invest fully in equity during their working years and then add an allocation to bonds in retirement. Panel A of Figure 6 explores the addition of bonds in retirement. The figure plots the equivalent savings rates for strategies that allocate 33% to domestic stocks and 67% to international stocks in the equity portion of the portfolio and allocate from 0% to 100% in bonds beginning at age 65. The 0% bonds case corresponds to the optimal fixed-weight strategy.

Figure 6 indicates that a small, positive weight in bonds during retirement is optimal, but that economic differences are minimal. The optimal retirement-period weight in bonds is 3%, with the remaining 97% in equity. To achieve the same expected utility as the optimal fixed-weight strategy with a 10.0% savings rate, the couple with bonds in retirement needs to save 10.0% (rounded from 9.995%). The gain from adding bonds during retirement is virtually zero, and the optimal fixed-weight strategy remains very near optimal in this setting.

The equivalent savings rates in Figure 6 also suggest that retirees will have relatively small utility losses from moderate allocations to bonds. The equivalent savings rate for a 20% bond allocation in retirement is 10.1%. It is possible that investors who are, for example, averse to short-term volatility may prefer this option. The savings rates increase to 10.3% at a 30% allocation, 10.6% at 40%, and 10.9% at 50%. Larger allocations to bonds in retirement seem undesirable because couples would need to increase their savings rate substantially to achieve the same utility as the all-equity strategy.

The prior analysis allows for a single change in allocations at retirement. As an alternative, age-based rules of thumb for investing across stocks and bonds, such as the “100-minus-age rule” that advocates investing 100 minus your age in stocks with the remainder in bonds, are prominent in popular financial advice [Choi (2022)]. The 100-minus-age rule implies large bond allocations that our investors dislike, so we consider rules with which the couple begins investing in bonds later in life.

Panel B of Figure 6 shows equivalent savings rates for strategies that begin to invest in bonds at any age ranging from 25 to 75. The equity portion of the portfolio is 33% domestic and 67% international, and the bond allocation increases by 1% each year once the couple begins to invest in bonds. An age-based, stock-bond strategy beginning at age 25 (i.e., the “125-minus-age” rule) has an equivalent savings rate of 13.7%. The couples who delay bond investments fare better, but there is no age at which the age rule is preferred to the optimal fixed-weight strategy.

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9.73% for the conditional strategy in Table VII.

The analyses in Figure 6 may be surprising given conventional wisdom. Our lifecycle simulation methods detailed in Section 4 have two important differences compared with common approaches: (i) we adopt a block bootstrap with long blocks (120 months on average) to preserve time-series dependencies in returns and (ii) we include international stocks in the investment opportunity set. In Figure 7, we study the impact of these two design choices. In each specification, we use either a block bootstrap or an IID bootstrap, and we include either domestic assets or all assets (i.e., including international stocks). The figure considers each of the four combinations of these two choices, with the all-block approach being our base case.

Panel A of Figure 7, analogous to Panel B of Figure 4, considers the addition of bonds to the full-life, fixed-weight strategies. Across all four designs, investors prefer a fixed-weight allocation of 0% to bonds. As such, the conclusion that the optimal fixed-weight strategy is all-equity is not sensitive to the bootstrap or investment opportunity set choices. The disutility from adding bonds differs across specifications, with a 15.4% equivalent savings rate at 40% bonds for our base case versus a 12.0% rate in the domestic-IID case.

Panels B and C of Figure 7 repeat the analyses in Figure 6 with the additional designs. If we maintain the block bootstrap but only include domestic assets, our couples perceive utility gains from investing in bonds later in life. The optimal allocation to bonds in retirement is 30% (9.7% equivalent savings rate), and the couples have a preference to begin an age-based strategy at age 56 (9.8% rate) versus maintaining a fixed weight. An IID assumption also makes bonds appear more favorable. In the domestic-IID case, for example, the retired couples wish to invest 42% in bonds and have a 9.5% equivalent savings rate. When considering an age-based rule, the couple begins to invest in bonds at age 53 with the domestic-IID design (9.7% equivalent savings rate).

Figure 7 helps to reconcile our findings of strong performance for an all-equity strategy with the conventional wisdom. The domestic-IID approach is similar in nature to the analyses in many academic and practitioner outlets (e.g., those that use monthly return moments from US stocks and bonds to calibrate models), which may explain the broad-based support for age-based, stock-bond investing. It is important to note, however, that this method misses two important aspects of modeling the investment opportunity set. First, returns are not IID. Given that the time-series dependencies in returns have an important impact on optimal asset allocation, it seems difficult to justify an IID assumption. Second, international stocks are an attractive asset class. They provide diversification to domestic stock investors and also offer high expected returns. The all-equity, fixed-weight strategy is supported by the data when we model these features of the investment opportunity set.

## 6 Conclusion

We challenge two central tenets of lifecycle investing — savers should diversify across stocks and bonds and the young should invest more heavily in stocks than the old. These principles underly mainstream investment advice and permeate regulations for DC pension plans. We find that investors with access to international stocks and subject to a realistic no-leverage constraint on retirement savings optimally choose an all-equity portfolio. There is no economically meaningful gain from holding bonds at any point during their lifetimes. The long-horizon return data suggest that diversifying with international stocks, rather than with bonds, improves investor outcomes for long-term appreciation and capital preservation. Our block bootstrap approach is key for modeling these long-horizon outcomes when investors are faced with changing investment opportunities and non-normalities.

Despite the dominance of the internationally diversified, all-equity strategy in achieving retirement outcomes, investors and regulators may be uncomfortable with the risk of large intermediate drawdowns in stocks. Drawdowns can inflict intense psychological pain, and one worry is that some investors will abandon their investments rather than stay the course. Contrary to common intuition, however, the QDIAs, with their large bond allocations, carry the potential for even larger drawdowns in real terms. Investors are more likely to exhaust their savings if they use these strategies. Our results, as a whole, do not suggest that the all-equity strategy is safe; they merely suggest that it is safer than common alternatives. Given the relative safety and strong growth potential of equities, retirement savers and retirees would likely benefit from adopting a “set it and forget it” strategy that fully invests in domestic and international stocks.



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TABLE I: EMPIRICAL PROPERTIES OF REAL RETURNS FOR BONDS AND INTERNATIONAL STOCKS

The table summarizes the empirical properties of real returns for bonds and international stocks. The underlying data are a monthly panel of real asset class returns for 39 developed countries covering the period from 1890 to 2023. The dataset formation details are provided in Section 3. Panel A reports the annualized mean and standard deviation of real returns for each asset class. Panel B reports variance ratios for each asset class at horizons of one, ten, 20, and 30 years. Panel C reports the correlation of the log return for each asset class with the log return for domestic stocks (based on monthly and 30-year returns) and with log inflation (based on 30-year returns). The statistics reported in Panels B and C are based on a bootstrap simulation.

Measure	Asset class	
	Bonds	International stocks
Panel A: Moments of annualized real returns		
Mean (%)	0.95	7.03
Standard deviation (%)	9.51	23.26
Panel B: Variance ratios		
VR(1)	1.00	1.00
VR(10)	2.09	0.88
VR(20)	2.26	0.80
VR(30)	2.30	0.75
Panel C: Log real return correlations		
Correlation with domestic stocks (monthly returns)	0.21	0.33
Correlation with domestic stocks (30-year returns)	0.45	0.34
Correlation with inflation (30-year returns)	-0.78	-0.01



TABLE II: SAMPLE COVERAGE AND SUMMARY STATISTICS

For each developed country, the table reports the sample period start date, the sample period end date, and summary statistics (i.e., geometric mean return and standard deviation of return) for monthly real returns for domestic stocks, international stocks, bonds, and bills. The development classification and sample formation criteria are described in the internet appendix.

Country	Sample start	Sample end	Asset class returns													
			Domestic stocks				International stocks				Bonds				Bills	
			Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev		
Denmark	1890:01	2023:12	0.46	3.58	0.39	3.93	0.20	1.89	0.17	0.73	0.20	1.89	0.17	0.73		
France	1890:01	2023:12	0.26	5.40	0.45	6.58	-0.08	2.28	-0.16	1.74	-0.08	2.28	-0.16	1.74		
Germany	1890:01	2023:12	0.25	8.29	0.56	10.20	-0.14	45.61	0.15	0.86	-0.14	45.61	0.15	0.86		
United Kingdom	1890:01	2023:12	0.41	4.25	0.43	4.09	0.13	1.98	0.06	0.87	0.13	1.98	0.06	0.87		
United States	1890:01	2023:12	0.52	5.01	0.34	3.85	0.12	1.76	0.06	0.61	0.12	1.76	0.06	0.61		
Canada	1891:01	2023:12	0.48	4.26	0.44	3.48	0.18	1.66	0.11	0.57	0.18	1.66	0.11	0.57		
New Zealand	1896:01	2023:12	0.47	3.66	0.44	4.06	0.13	1.84	0.14	0.58	0.13	1.84	0.14	0.58		
Belgium	1897:01	2023:12	0.21	5.03	0.39	4.57	0.02	1.79	-0.04	1.13	0.02	1.79	-0.04	1.13		
Australia	1901:01	2023:12	0.57	3.96	0.43	3.75	0.14	1.73	0.06	0.53	0.14	1.73	0.06	0.53		
Sweden	1910:01	2023:12	0.47	4.85	0.45	4.15	0.14	1.86	0.08	0.96	0.14	1.86	0.08	0.96		
Netherlands	1914:01	2023:12	0.41	5.19	0.41	4.39	0.13	1.73	0.01	0.80	0.13	1.73	0.01	0.80		
Norway	1914:01	2023:12	0.37	4.62	0.44	4.19	0.13	1.73	0.02	0.85	0.13	1.73	0.02	0.85		
Switzerland	1914:01	2023:12	0.38	4.30	0.37	4.47	0.14	1.41	0.02	0.61	0.14	1.41	0.02	0.61		
Austria	1920:01	2023:12	0.22	7.27	0.57	12.79	-0.38	4.51	-0.46	3.68	-0.38	4.51	-0.46	3.68		
Czechoslovakia	1922:05	1945:05	-0.14	6.56	0.41	6.24	0.69	3.28	0.34	3.06	0.69	3.28	0.34	3.06		
Chile period I	1927:01	1970:12	0.13	6.15	0.60	8.55	-0.92	3.38	-0.86	2.34	-0.92	3.38	-0.86	2.34		
Japan	1930:01	2023:12	0.31	6.59	0.51	15.92	-0.18	3.40	-0.32	2.60	-0.18	3.40	-0.32	2.60		
Italy	1931:01	2023:12	0.19	7.38	0.43	12.98	-0.14	2.56	-0.25	1.68	-0.14	2.56	-0.25	1.68		
Portugal	1934:01	2023:12	0.14	7.82	0.44	4.10	0.02	2.81	-0.06	1.34	0.02	2.81	-0.06	1.34		
Ireland	1936:01	2023:12	0.45	4.77	0.46	4.08	0.16	2.40	0.01	0.59	0.16	2.40	0.01	0.59		
Argentina	1947:02	1966:12	-0.18	8.53	0.64	15.33	-1.66	2.84	-1.56	2.73	-1.66	2.84	-1.56	2.73		
Spain	1959:01	2023:12	0.28	5.58	0.39	4.25	0.14	2.22	0.00	0.69	0.14	2.22	0.00	0.69		
Finland	1969:01	2023:12	0.73	6.24	0.41	4.35	0.24	2.26	0.03	0.47	0.24	2.26	0.03	0.47		
Greece	1981:02	2023:12	0.48	10.23	0.55	4.77	0.30	5.41	0.14	1.29	0.30	5.41	0.14	1.29		
Luxembourg	1982:01	2023:12	0.55	5.75	0.57	4.51	0.28	1.88	0.10	0.57	0.28	1.88	0.10	0.57		
Singapore	1998:07	2023:12	0.44	5.74	0.29	4.09	0.14	2.04	-0.04	0.48	0.14	2.04	-0.04	0.48		

(Continued on next page)

TABLE II (Continued)

Country	Sample start	Sample end	Asset class returns													
			Domestic stocks			International stocks			Bonds			Bills				
			Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev				
Hungary	1999:02	2023:12	0.35	6.52	0.26	4.21	0.17	3.49	0.09	0.53	0.26	4.21	0.17	3.49	0.09	0.53
Poland	1999:06	2023:12	0.26	6.20	0.21	3.78	0.22	2.92	0.12	0.53	0.21	3.78	0.22	2.92	0.12	0.53
Slovakia	2000:01	2023:12	0.17	5.10	-0.02	4.26	0.24	2.96	-0.12	0.62	-0.02	4.26	0.24	2.96	-0.12	0.62
Czech Republic	2000:05	2023:12	0.77	6.92	0.01	4.23	0.08	2.45	-0.10	0.63	0.01	4.23	0.08	2.45	-0.10	0.63
South Korea	2000:11	2023:12	0.63	6.18	0.34	3.93	0.25	2.02	0.06	0.36	0.34	3.93	0.25	2.02	0.06	0.36
Mexico	2001:08	2023:12	0.62	4.94	0.44	3.72	0.29	2.67	0.14	0.40	0.44	3.72	0.29	2.67	0.14	0.40
Iceland	2002:01	2023:12	0.02	7.27	0.33	4.80	0.16	3.16	0.14	0.54	0.33	4.80	0.16	3.16	0.14	0.54
Chile period II	2010:01	2023:12	-0.01	4.89	0.70	3.99	-0.05	1.93	-0.02	0.43	0.70	3.99	-0.05	1.93	-0.02	0.43
Israel	2010:01	2023:12	0.01	5.07	0.68	3.63	0.28	2.14	0.09	0.87	0.68	3.63	0.28	2.14	0.09	0.87
Slovenia	2010:01	2023:12	0.41	4.75	0.72	3.79	0.09	2.69	-0.10	0.80	0.72	3.79	0.09	2.69	-0.10	0.80
Türkiye	2010:02	2023:12	0.36	7.42	1.11	6.49	-0.79	6.92	-0.47	1.77	1.11	6.49	-0.79	6.92	-0.47	1.77
Latvia	2016:01	2023:12	0.47	4.42	0.40	4.09	-0.51	2.23	-0.36	0.82	0.40	4.09	-0.51	2.23	-0.36	0.82
Lithuania	2018:01	2023:12	0.02	3.85	0.26	4.57	-0.82	2.76	-0.46	0.80	0.26	4.57	-0.82	2.76	-0.46	0.80
Colombia	2020:01	2023:12	-0.79	9.02	0.46	4.75	-0.44	4.27	-0.08	0.51	0.46	4.75	-0.44	4.27	-0.08	0.51

TABLE III: ECONOMIC VALUE OF ALTERNATIVE INVESTMENT PLANS

Panel A reports the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal static asset allocation policy, and Panel B reports the asset allocation weights for the other four investment strategies. The weights for the TDF strategy change over the lifecycle following the glide path shown in Figure 2. The column titled “QDIA” indicates whether the strategy is classified as a Qualified Default Investment Alternative under the Pension Protection Act of 2006. The final column of the table reports equivalent savings rates to quantify relative economic value in pairwise comparisons of the optimal static asset allocation strategy with each alternative asset allocation strategy. Each comparison is based on expected household utility over retirement consumption and bequest across 1,000,000 bootstrap simulations. Household income in the pre-retirement period is stochastic and follows the process estimated by Guvenen, Karahan, Ozkan, and Song (2021). The household’s savings rate for the base strategy (i.e., the optimal static asset allocation strategy) in the pre-retirement period is 10%. The equivalent savings rate is the household’s savings rate for the alternative strategy that equates the expected utility from retirement consumption and bequest for the two strategies. For each strategy, household income in the post-retirement period is the sum of Social Security income and a constant real withdrawal of 4% of the household’s investment account value at retirement (as long as the account has not been depleted). The household faces uncertainty over labor income, investment returns, and longevity.

Strategy	QDIA	Asset class weights				Equivalent savings rate
		Domestic stocks	International stocks	Bonds	Bills	
Panel A: Base strategy						
Optimal	No	33%	67%	0%	0%	—
Panel B: Alternative strategies						
Bills	No	0%	0%	0%	100%	56.2%
Domestic Stocks	No	100%	0%	0%	0%	16.3%
Balanced	Yes	60%	0%	40%	0%	19.3%
TDF	Yes	[10%, 54%]	[7%, 36%]	[10%, 73%]	[0%, 10%]	16.1%

TABLE IV: RETIREMENT SAVING OUTCOMES

The table summarizes the distributions of real wealth at retirement (Panel A), the real income replacement rate (Panel B), and real wealth at death (Panel C) across 1,000,000 bootstrap simulations for households adopting various asset allocation strategies. Household income in the pre-retirement period is stochastic and follows the process estimated by Guvenen, Karahan, Ozkan, and Song (2021). The household’s savings rate in the pre-retirement period is 10%. Household income in the post-retirement period is the sum of Social Security income and a constant real withdrawal of 4% of the household’s investment account value at retirement (as long as the account has not been depleted). The household faces uncertainty over labor income, investment returns, and longevity. For each asset allocation strategy, the table reports the mean, standard deviation, and distribution percentiles of each measure of investment performance. Real wealth at retirement and real wealth at death are reported in millions of 2022 USD.

Strategy	Moments		Percentiles								
	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Panel A: Wealth at retirement (\$MM)											
Bills	0.27	0.21	0.00	0.03	0.06	0.13	0.23	0.36	0.53	0.65	0.98
Domestic Stocks	1.02	1.75	0.00	0.07	0.14	0.30	0.61	1.18	2.14	3.13	6.79
Balanced	0.71	1.94	0.00	0.06	0.12	0.26	0.48	0.85	1.40	1.91	3.73
TDF	0.77	1.30	0.00	0.09	0.16	0.31	0.55	0.93	1.49	2.00	3.78
Optimal	1.07	1.43	0.00	0.13	0.22	0.41	0.74	1.27	2.11	2.91	5.94
Panel B: Income replacement rate											
Bills	0.70	1.27	0.00	0.43	0.49	0.57	0.66	0.78	0.93	1.06	1.73
Domestic Stocks	1.18	1.63	0.00	0.50	0.59	0.73	0.95	1.31	1.90	2.52	4.82
Balanced	0.99	1.71	0.00	0.50	0.57	0.70	0.86	1.10	1.45	1.79	3.07
TDF	1.03	1.55	0.00	0.52	0.60	0.73	0.90	1.15	1.51	1.86	3.16
Optimal	1.24	1.56	0.00	0.59	0.68	0.83	1.05	1.39	1.91	2.43	4.56
Panel C: Wealth at death (\$MM)											
Bills	0.08	0.14	0.00	0.00	0.00	0.00	0.02	0.11	0.23	0.34	0.64
Domestic Stocks	2.61	14.71	0.00	0.00	0.00	0.10	0.59	1.93	5.29	9.76	31.65
Balanced	1.10	8.35	0.00	0.00	0.00	0.07	0.35	0.98	2.29	3.83	10.65
TDF	0.72	5.21	0.00	0.00	0.00	0.04	0.27	0.73	1.57	2.47	6.00
Optimal	2.94	11.25	0.00	0.00	0.07	0.37	1.03	2.59	5.99	10.18	30.54

TABLE V: PORTFOLIO DRAWDOWNS

The table summarizes the distributions of the maximum portfolio drawdown during the pre-retirement period (Panel A) and the maximum portfolio drawdown during the post-retirement period (Panel B) across 1,000,000 bootstrap simulations for households adopting various asset allocation strategies. Household income in the pre-retirement period is stochastic and follows the process estimated by Guvenen, Karahan, Ozkan, and Song (2021). The household’s savings rate in the pre-retirement period is 10%. Household income in the post-retirement period is the sum of Social Security income and a constant real withdrawal of 4% of the household’s investment account value at retirement (as long as the account has not been depleted). The household faces uncertainty over labor income, investment returns, and longevity. For each asset allocation strategy and drawdown period, the table reports the mean, standard deviation, and distribution percentiles of the maximum portfolio drawdown.

Strategy	Moments		Percentiles								
	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%
Panel A: Working-period drawdown											
Bills	0.42	0.26	0.04	0.09	0.13	0.22	0.37	0.57	0.84	0.96	1.00
Domestic Stocks	0.67	0.16	0.32	0.41	0.46	0.55	0.66	0.78	0.91	0.96	0.99
Balanced	0.54	0.19	0.23	0.29	0.32	0.41	0.50	0.66	0.85	0.92	0.99
TDF	0.52	0.16	0.22	0.28	0.32	0.41	0.51	0.61	0.74	0.81	0.94
Optimal	0.55	0.12	0.24	0.31	0.38	0.48	0.56	0.61	0.68	0.76	0.91
Panel B: Retirement-period drawdown											
Bills	0.46	0.30	0.00	0.03	0.09	0.22	0.42	0.69	0.94	0.98	1.00
Domestic Stocks	0.62	0.21	0.00	0.27	0.35	0.49	0.63	0.77	0.91	0.96	0.99
Balanced	0.50	0.23	0.00	0.18	0.24	0.34	0.47	0.66	0.85	0.92	0.98
TDF	0.40	0.23	0.00	0.11	0.15	0.23	0.35	0.54	0.76	0.89	0.99
Optimal	0.48	0.17	0.00	0.19	0.25	0.38	0.52	0.59	0.66	0.74	0.88

TABLE VI: OPTIMAL STATIC ASSET ALLOCATION POLICIES UNDER ALTERNATIVE DESIGN PARAMETERS

Panel A reports the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal static asset allocation policy under the base case design. The lifecycle design and bootstrap assumptions are detailed in Section 4. The subsequent panels report the optimal asset allocation policies under alternative design assumptions, as described in the table.

Description	Asset class weights				Borrowing (% of wealth)
	Domestic stocks	International stocks	Bonds	Bills	
Panel A: Base case					
Base case	33%	67%	0%	0%	N/A
Panel B: Average block length					
Block length parameter $b = 12$	32%	68%	0%	0%	N/A
Block length parameter $b = 60$	33%	67%	0%	0%	N/A
Block length parameter $b = 240$	33%	67%	0%	0%	N/A
Panel C: Risk aversion					
Risk aversion parameter $\gamma = 0.5$	32%	68%	0%	0%	N/A
Risk aversion parameter $\gamma = 1.0$	35%	65%	0%	0%	N/A
Risk aversion parameter $\gamma = 2.0$	35%	65%	0%	0%	N/A
Risk aversion parameter $\gamma = 5.0$	33%	67%	0%	0%	N/A
Risk aversion parameter $\gamma = 7.5$	33%	67%	0%	0%	N/A
Risk aversion parameter $\gamma = 10.0$	33%	67%	0%	0%	N/A
Panel D: Bequest					
No bequest motive ( $\theta = 0$ )	33%	67%	0%	0%	N/A
Utility only from bequest ( $\theta = \infty$ )	34%	66%	0%	0%	N/A
Panel E: Household utility specification					
Consumption scaled by household size	33%	67%	0%	0%	N/A
Subjective discount factor $\delta = 0.98^{1/12}$	33%	67%	0%	0%	N/A
Panel F: Withdrawal strategy					
3% rule ( $r_w = 3\%$ )	34%	66%	0%	0%	N/A
5% rule ( $r_w = 5\%$ )	34%	66%	0%	0%	N/A
4% of current account value	34%	66%	0%	0%	N/A
Panel G: Retirement age					
Retirement at age 62 ( $T_{ret} = 444$ )	33%	67%	0%	0%	N/A
Retirement at age 67 ( $T_{ret} = 504$ )	33%	67%	0%	0%	N/A
Retirement at age 70 ( $T_{ret} = 540$ )	33%	67%	0%	0%	N/A
Panel H: Contribution rules					
Contribution rate of 5% ( $r_c = 0.05$ )	33%	67%	0%	0%	N/A
Contribution rate of 15% ( $r_c = 0.15$ )	33%	67%	0%	0%	N/A
Lower income limit $Y_{min} = \$0$	33%	67%	0%	0%	N/A
Lower income limit $Y_{min} = \$45,000$	33%	67%	0%	0%	N/A

(Continued on next page)

TABLE VI (Continued)

Description	Asset class weights				Borrowing (% of wealth)
	Domestic stocks	International stocks	Bonds	Bills	
Panel I: Household type					
Single female	33%	67%	0%	0%	N/A
Single male	33%	67%	0%	0%	N/A
Both female	33%	67%	0%	0%	N/A
Both male	33%	67%	0%	0%	N/A
Panel J: Investor type					
Low initial income and low human capital	33%	67%	0%	0%	N/A
Low initial income and high human capital	33%	67%	0%	0%	N/A
High initial income and low human capital	33%	67%	0%	0%	N/A
High initial income and high human capital	32%	68%	0%	0%	N/A
Panel K: Correlation between persistent earnings shocks and domestic stock returns					
Income-domestic stock correlation of 0.1	30%	70%	0%	0%	N/A
Income-domestic stock correlation of 0.2	27%	73%	0%	0%	N/A
Income-domestic stock correlation of 0.3	24%	76%	0%	0%	N/A
Income-domestic stock correlation of 0.4	21%	79%	0%	0%	N/A
Income-domestic stock correlation of 0.5	18%	82%	0%	0%	N/A
Panel L: Leverage					
Borrowing spread of 6.50% (High)	33%	67%	0%	0%	0%
Borrowing spread of 1.40% (Medium)	34%	66%	0%	0%	55%
Borrowing spread of 0.37% (Low)	28%	57%	15%	0%	100%
Panel M: Consumption-savings decisions					
Contribution rate based on income and age	33%	67%	0%	0%	N/A
Endogenous retirement timing	33%	67%	0%	0%	N/A

TABLE VII: OPTIMAL DYNAMIC ASSET ALLOCATION POLICY

The table reports the investment positions in domestic stocks, international stocks, bonds, and bills for the optimal dynamic asset allocation policy that conditions on the aggregate price-dividend ratio.

Market state	Aggregate $P_t/D_t$	Asset class weights			
		Domestic stocks	International stocks	Bonds	Bills
Panel A: Static asset allocation					
All	$[0, \infty)$	33%	67%	0%	0%
Panel B: Dynamic asset allocation					
Low $P_t/D_t$	$[0, 18.76]$	65%	35%	0%	0%
2	$(18.76, 23.47]$	28%	72%	0%	0%
3	$(23.47, 29.94]$	30%	70%	0%	0%
4	$(29.94, 43.67]$	31%	69%	0%	0%
High $P_t/D_t$	$(43.67, \infty)$	16%	75%	9%	0%

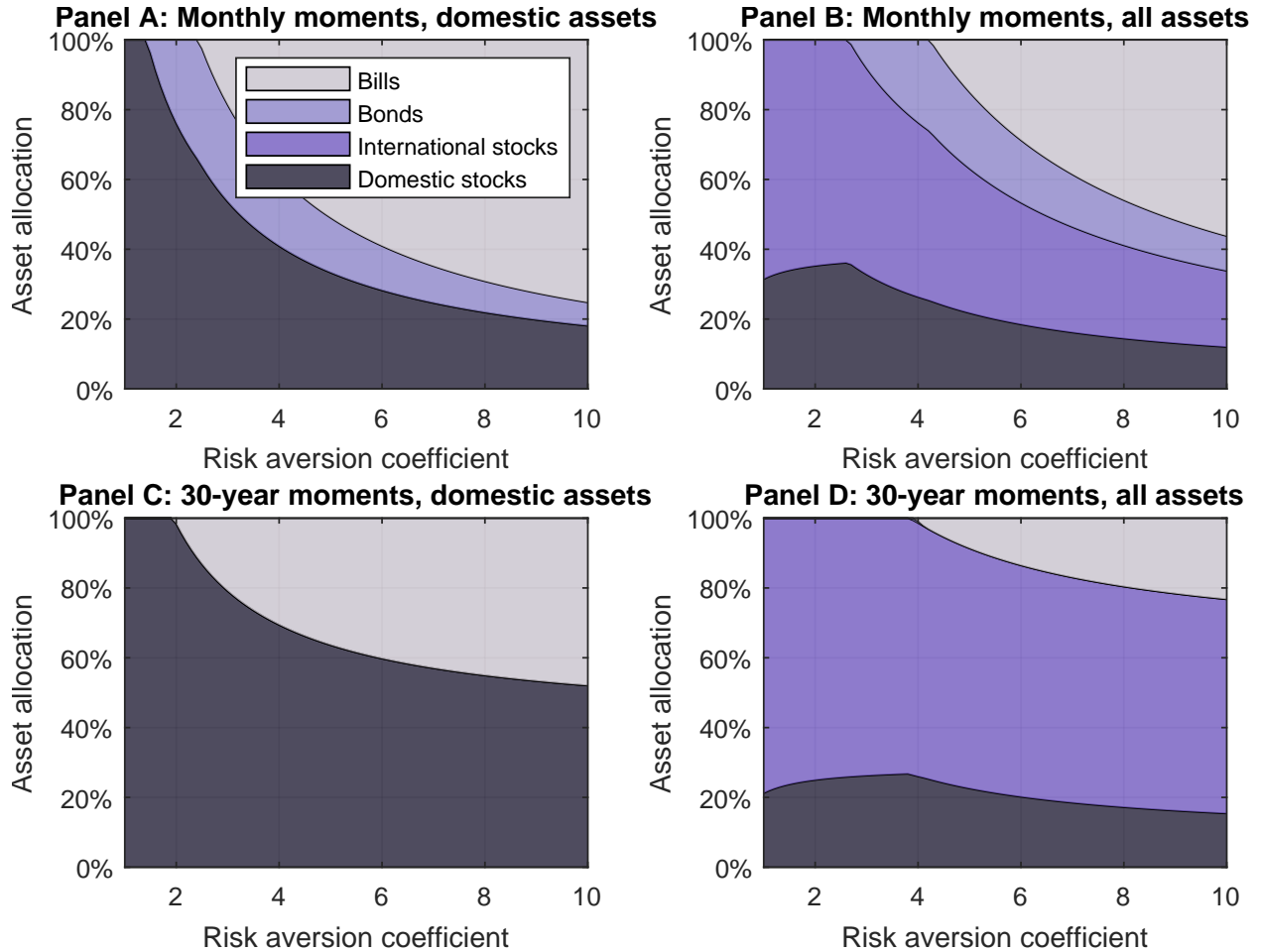


FIGURE 1. MEAN-VARIANCE ASSET ALLOCATION. The figure shows the optimal asset allocation for a mean-variance investor as a function of investor risk aversion. The strategies in Panels A and C invest in domestic stocks, bonds, and bills. The strategies in Panels B and D add international stocks. For the results in Panels A and B, the investor estimates portfolio risk using a covariance matrix based on the realized monthly log real returns from a panel of 39 developed countries covering the period from 1890 to 2023. For the results in Panels C and D, the investor estimates portfolio risk using simulated 30-year log real returns from the same developed-country sample by dividing the covariance matrix by 360 to scale to a monthly level.



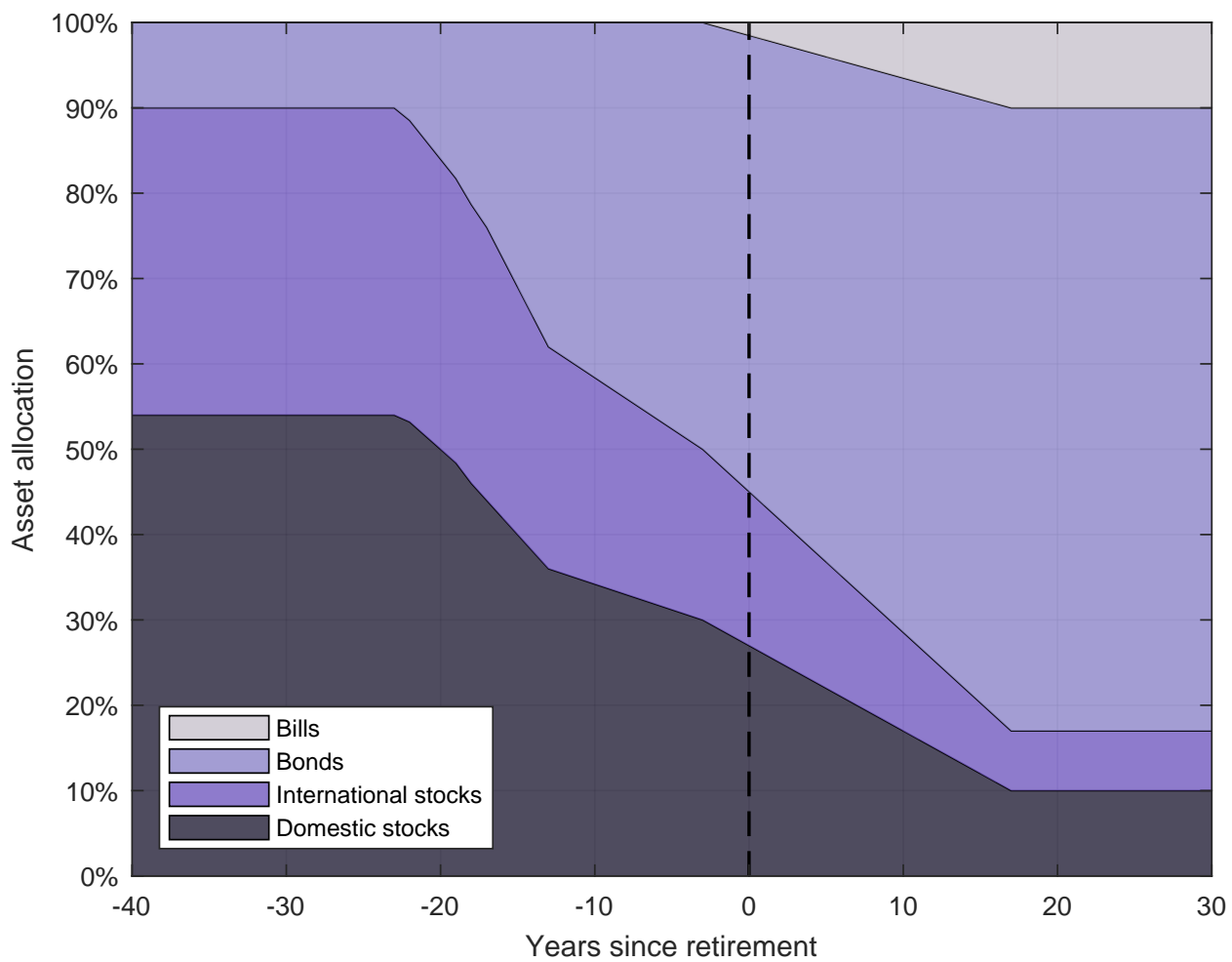


FIGURE 2. GLIDEPATH WEIGHTS FOR THE TARGET-DATE FUND. The figure shows the asset allocation of the target-date strategy as a function of time since retirement. The strategy invests in domestic stocks, international stocks, bonds, and bills.

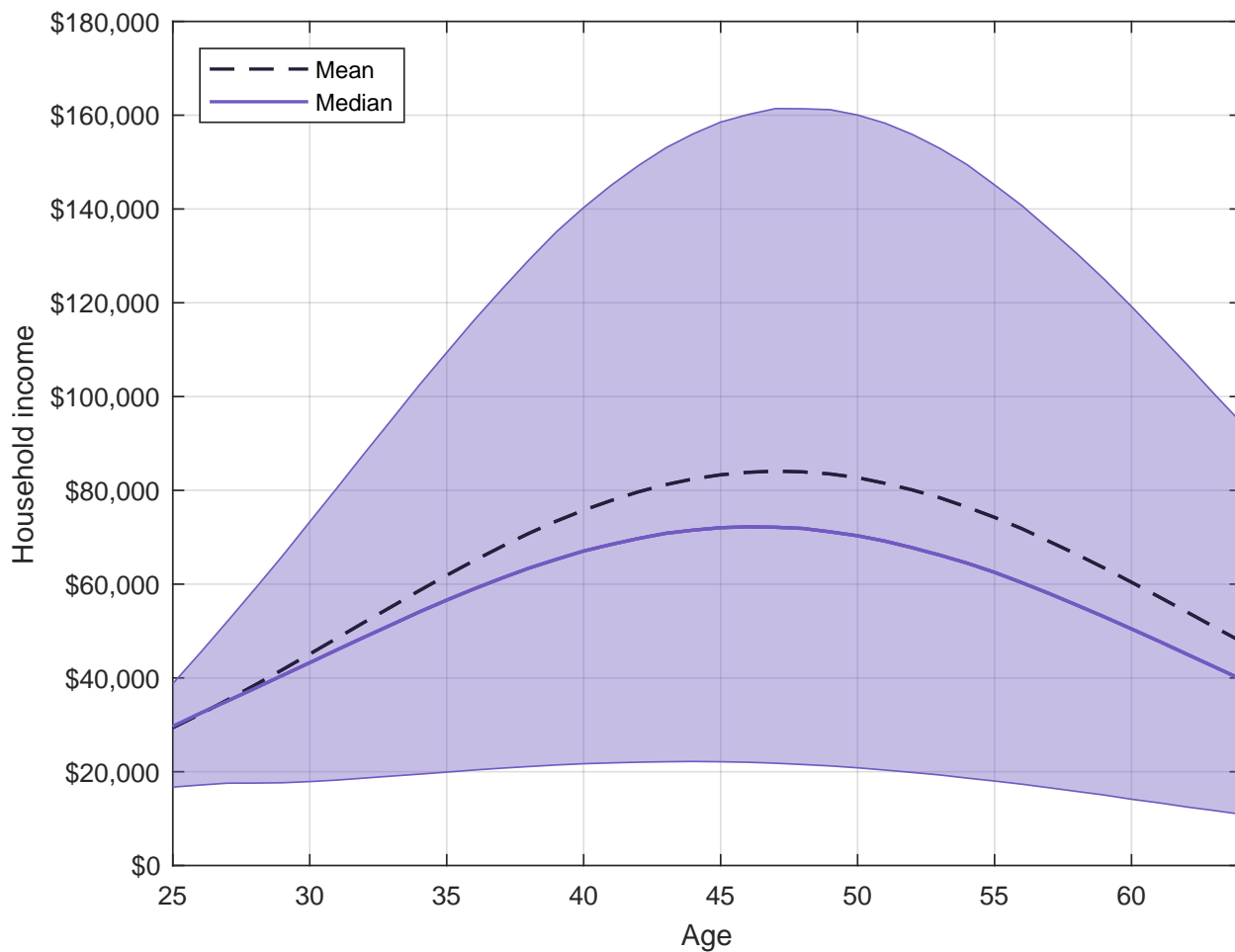


FIGURE 3. DISTRIBUTION OF HOUSEHOLD INCOME. The figure shows the distribution of real household income across 1,000,000 bootstrap simulations in 2022 USD as a function of age. Household income is stochastic and follows the process estimated by Guvenen, Karahan, Ozkan, and Song (2021) with an initial income parameter of  $z_0^i = 0$  and human capital parameters of  $(\alpha^i, \beta^i) = (0, 0)$ . The solid (dashed) line corresponds to the median (mean) household income as a function of age. The shaded region covers the 10th through 90th percentiles of the distribution.

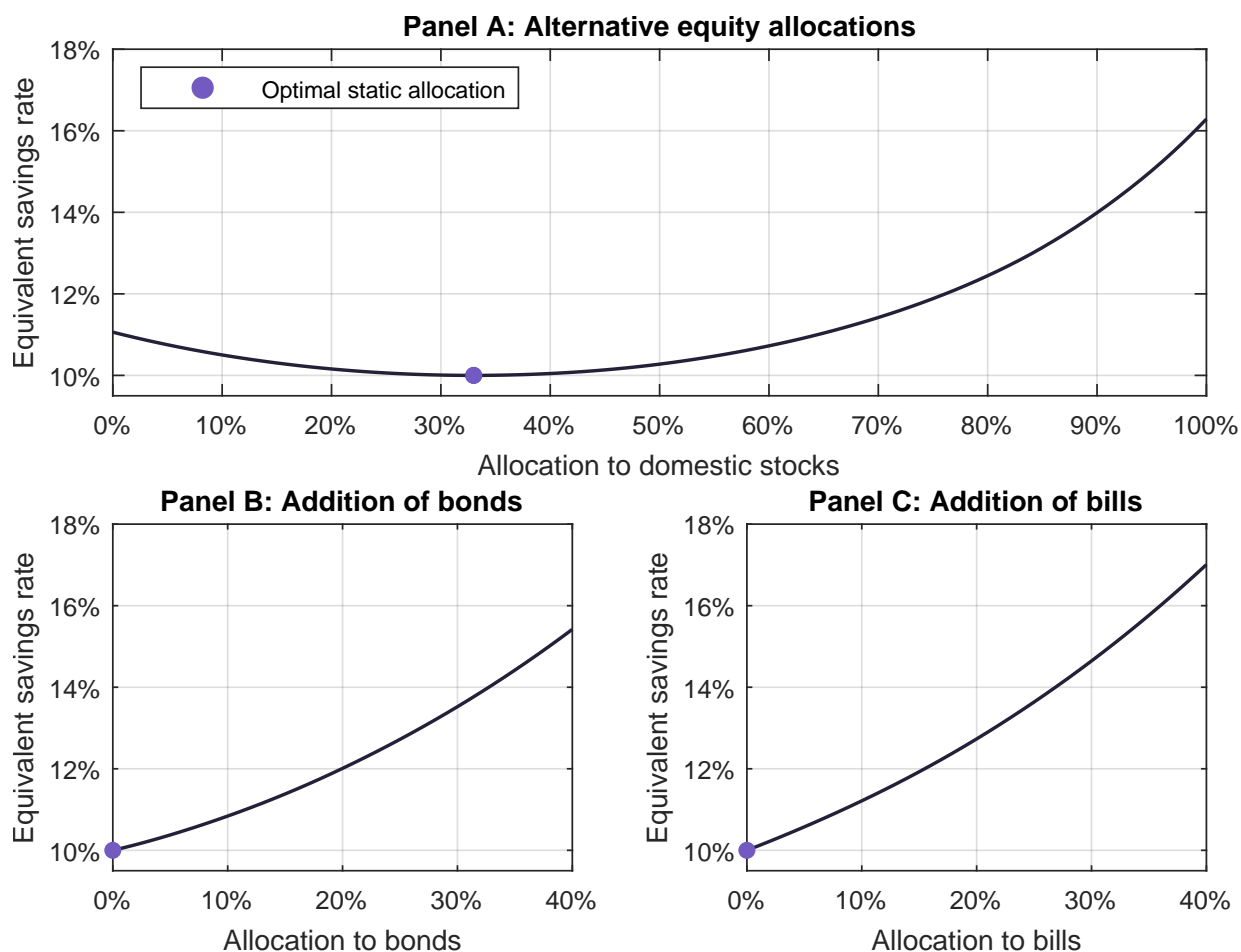


FIGURE 4. EQUIVALENT SAVINGS RATES FOR DEVIATIONS FROM THE OPTIMAL STATIC PORTFOLIO. The figure shows equivalent savings rates to quantify relative economic value in pairwise comparisons of the optimal static asset allocation strategy with alternative asset allocation strategies. Each comparison is based on expected household utility over retirement consumption and bequest across 1,000,000 bootstrap simulations. In each plot, the base strategy corresponds to the optimal static allocation of 33% in domestic stocks and 67% in international stocks with a pre-retirement period savings rate of 10%. The alternative strategies in Panel A adopt static allocations to domestic stocks and international stocks, but deviate from the base strategy in weighting the two asset classes. The alternative strategies in Panel B (Panel C) adopt the same relative allocation to domestic stocks and international stocks as does the base strategy, but these strategies add a fixed allocation to bonds (bills). Each panel shows the household's equivalent savings rate for the indicated alternative strategy (i.e., the value that equates the expected utility from retirement consumption and bequest for the alternative and base strategies).

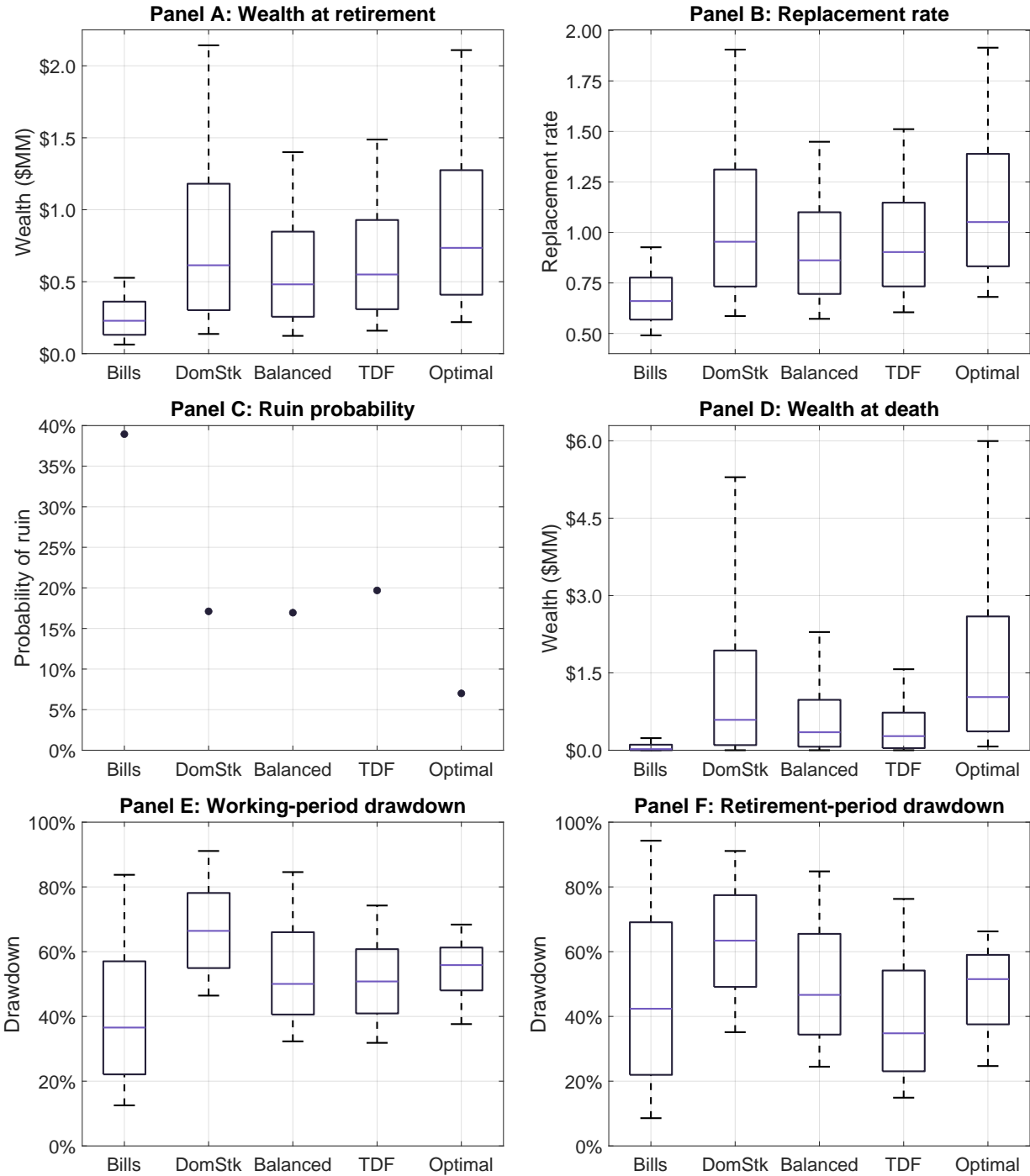


FIGURE 5. MEASURES OF INVESTMENT PERFORMANCE. The figure summarizes the distribution of real wealth at retirement (Panel A), the distribution of the real income replacement rate (Panel B), the probability of financial ruin (Panel C), the distribution of real wealth at death (Panel D), the distribution of the working-period drawdown (Panel E), and the distribution of the retirement-period drawdown (Panel F) across 1,000,000 bootstrap simulations for households adopting various asset allocation strategies. In each box-and-whiskers plot, the middle line corresponds to the median, the box covers the interquartile range, and the whiskers cover the 10th through 90th percentiles.

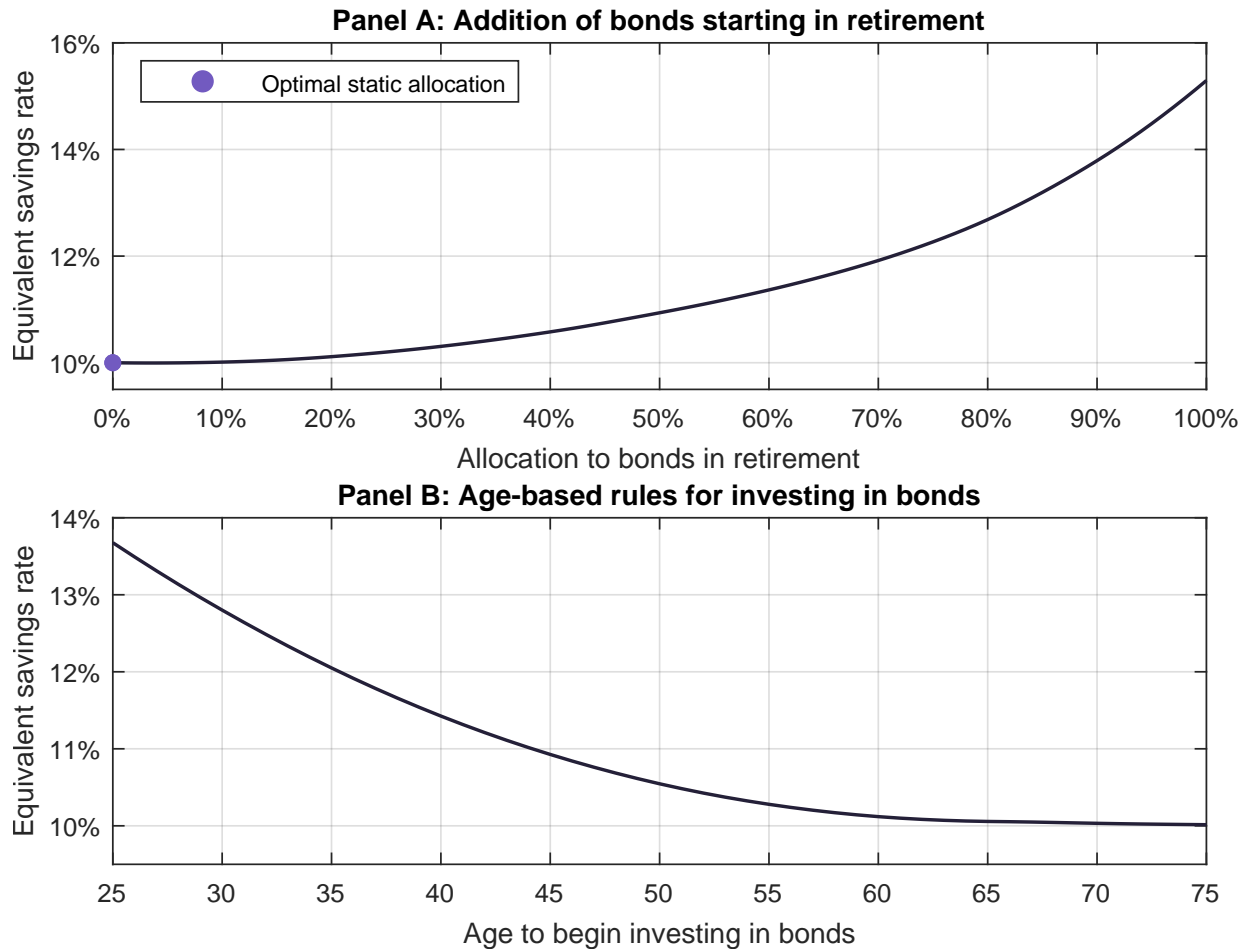


FIGURE 6. EQUIVALENT SAVINGS RATES FOR STRATEGIES THAT SHIFT INTO BONDS. The figure shows equivalent savings rates to quantify relative economic value in pairwise comparisons of the optimal static asset allocation strategy with alternative asset allocation strategies. Each comparison is based on expected household utility over retirement consumption and bequest across 1,000,000 bootstrap simulations. In each plot, the base strategy corresponds to the optimal static allocation of 33% in domestic stocks and 67% in international stocks with a pre-retirement period savings rate of 10%. The alternative strategies in Panel A follow an allocation of 33% in domestic stocks and 67% in international stocks in the pre-retirement period, but shift into bonds at retirement (while maintaining the same relative weights in domestic stocks and international stocks). The alternative strategies in Panel B begin investing in bonds at the indicated age and increase the allocation to bonds at a rate of 1% per year. Each panel shows the household's savings rate for the indicated alternative strategy (i.e., the value that equates the expected utility from retirement consumption and bequest for the alternative and base strategies).

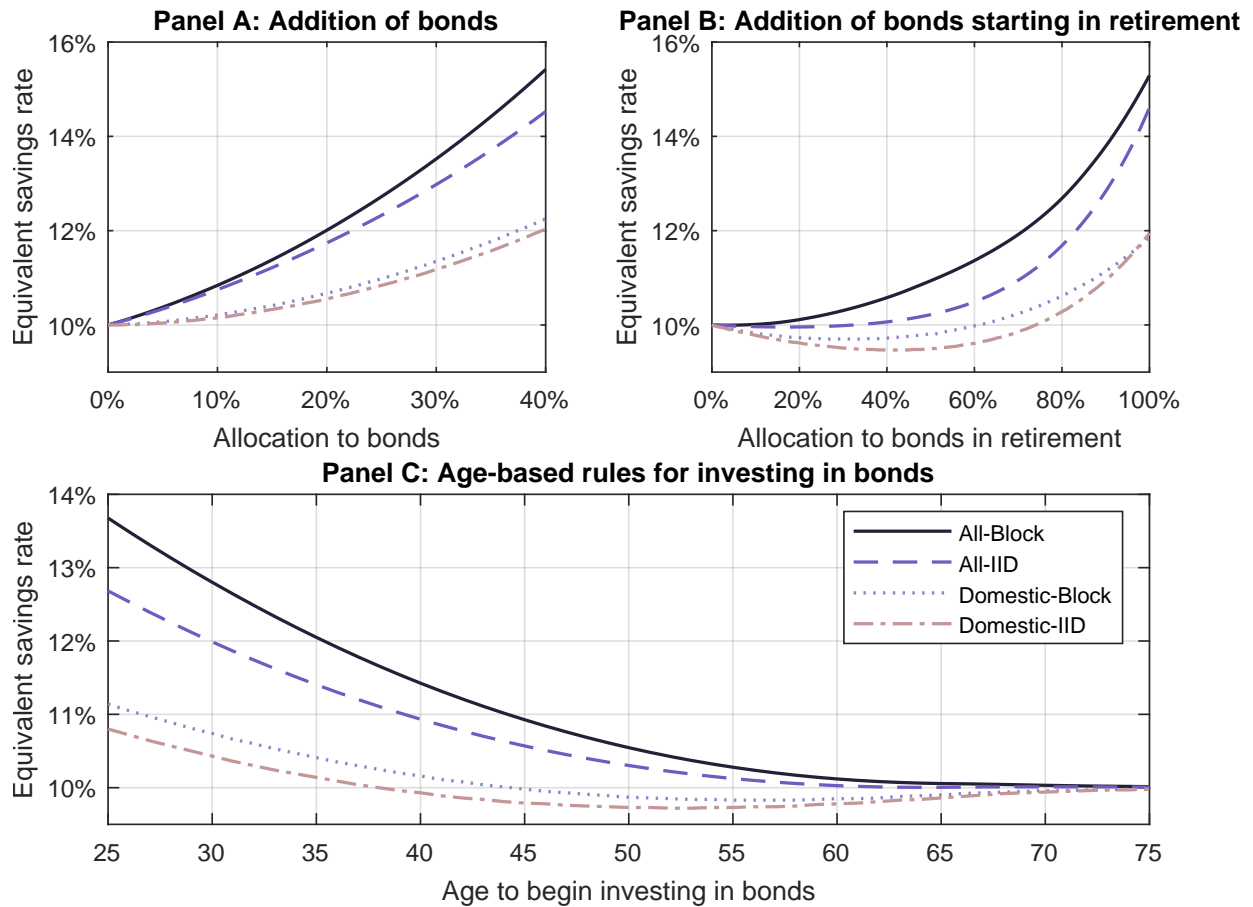


FIGURE 7. EQUIVALENT SAVINGS RATES FOR STRATEGIES THAT SHIFT INTO BONDS: ALTERNATIVE INVESTMENT OPPORTUNITY SETS AND BOOTSTRAP SPECIFICATIONS. The figure shows equivalent savings rates to quantify relative economic value in pairwise comparisons of the optimal static asset allocation strategy with alternative asset allocation strategies for various underlying investment opportunity sets and bootstrap specifications. Each comparison is based on expected household utility over retirement consumption and bequest across 1,000,000 bootstrap simulations, and each simulation corresponds to a specific investment opportunity set and bootstrap sampling approach (i.e., a block bootstrap with an average block length of 120 months or an IID bootstrap). The investment opportunity set in the “All” (“Domestic”) cases include domestic stocks, international stocks, bonds, and bills (domestic stocks, bonds, and bills). In each plot, the base strategy corresponds to the optimal static asset allocation strategy and excludes bonds and bills (the optimal asset allocation differs across investment opportunity set and bootstrap method). The base strategy also corresponds to a pre-retirement period savings rate of 10%. The alternative strategies in Panel A add a fixed allocation to bonds to the corresponding optimal strategies. The alternative strategies in Panel B follow the optimal allocation in the pre-retirement period, but shift into bonds at retirement. The alternative strategies in Panel C begin investing in bonds at the indicated age and increase the allocation to bonds at a rate of 1% per year. Each plot shows the household’s savings rate for the indicated alternative strategy (i.e., the value that equates the expected utility from retirement consumption and bequest for the alternative and base strategies).

# Internet Appendix

## “Beyond the Status Quo: A Critical Assessment of Lifecycle Investment Advice”

### A Data appendix

This appendix describes our development classification approach, data sources, calculations of asset class returns, special data issues, and dataset validation. Section A.1 outlines our development classification and data sources used to compute asset class returns. Sections A.2 to A.5 provide details on the calculations of domestic stocks, international stocks, government bonds, and government bills, respectively, and special data issues related to each asset class. Section A.6 presents data details of other variables. Section A.7 compares our data on stock and bond returns with data from alternative sources.

#### A.1 Development classification and data sources

We follow Anarkulova, Cederburg, and O’Doherty (2022) to classify countries as developed. We classify a given country as developed early in the sample period if its agricultural labor share is less than 50% based on evidence about labor patterns from the economics literature [e.g., Kuznets (1973)]. Beginning with the formation of the Organisation for European Economic Co-operation (OEEC) in 1948, we use membership in the OEEC and the Organisation for Economic Co-operation and Development (OECD) to identify development dates.

In order to form a balanced panel, a developed country can not enter into our sample until its government issues ten-year bonds. Sample eligibility postdates development for several countries on this basis. The sample eligibility date is the latest of 1890 (i.e., the sample period start date for our study), the country development year, and the year in which the country first issued long-term bonds.

Table A.I displays the development date, reason for classification, sample eligibility date, and data coverage for each country. In three instances, a previously developed country is reclassified as developing. These instances occur in Argentina, Chile, and Czechoslovakia, and each reclassification results from substantial changes in governments and markets in these countries. Chile, the Czech Republic, and Slovakia reenter the sample with membership in the OECD. We include the early periods in these countries to avoid survivor bias.

For some countries, we have missing data at the beginning of the eligible period. Returns on a diversified domestic stock index are the binding data constraint in each of these instances. No country has data gaps in the middle or at the end of its series.

The primary source of data for our study is the GFDdatabase from Global Financial Data (GFD). This database contains long time series of times series of total return indexes, price indexes, dividend-price ratios, and total market capitalization for stocks; yields for ten-year government bonds and short-term bills; consumer price indexes; and exchange rates for a broad set of countries. Table A.II reports the data series we use to compute monthly stock, bond, and bill returns for each country. As noted in the footnotes to Table A.II, we supplement the data from GFD with data from other sources.

## A.2 Domestic stocks

The GFDatabase contains data for total return indexes, price indexes, and dividend-price ratios. It includes stock market indexes that are created and calculated by stock exchanges (e.g., the Tokyo Stock Price Index from the Tokyo Stock Exchange), by well-known index providers (e.g., the S&P 500 Index), or by GFD directly from original source documents. Multiple stock indexes are available in the database for some countries and periods. We select a single index in these cases by considering the breadth of market coverage and the length of historical coverage. We use a total return index whenever one is available, and we otherwise use a price index and a dividend-price ratio to calculate returns.

For sample months in which a total return index is available, we calculate the monthly nominal return,

$$R_{i,t}^{Nominal\ stocks} = \frac{I_{i,t}^{Total}}{I_{i,t-1}^{Total}}, \quad (A1)$$

where  $I_{i,t}^{Total}$  is the total return index for country  $i$  at the end of month  $t$  and  $R_{i,t}^{Nominal\ stocks}$  is the gross nominal return for country  $i$  in month  $t$ . If no total return index is available, we use price index and dividend-price ratio data to calculate returns. We assume that the annual dividend reflected by the reported dividend-price ratio is paid equally across months in the year. If  $I_{i,t}^{Price}$  is the price index and  $\hat{D}_{i,t}$  is the estimated dividend (appropriately scaled to the level of the price index) for country  $i$  in month  $t$ , then we calculate the monthly nominal return,

$$R_{i,t}^{Nominal\ stocks} = \frac{I_{i,t}^{Price} + \hat{D}_{i,t}}{I_{i,t-1}^{Price}}. \quad (A2)$$

Nominal returns reflect diversified investments in a broad country-level index. To calculate real returns, we first calculate gross inflation,

$$\Pi_{i,t} = \frac{I_{i,t}^{CPI}}{I_{i,t-1}^{CPI}}, \quad (A3)$$

where  $I_{i,t}^{CPI}$  is the consumer price index (CPI) for country  $i$  at the end of month  $t$ . We then calculate the gross real return on domestic stocks given the gross nominal return,  $R_{i,t}^{Nominal\ stocks}$ , and gross inflation,

$$R_{i,t}^{Stocks} = \frac{R_{i,t}^{Nominal\ stocks}}{\Pi_{i,t}}. \quad (A4)$$

This return calculation produces real returns that are denominated in the local currency of country  $i$ .

### A.2.1 Data issues related to domestic stocks

Our treatment of special data issues mirror those in Anarkulova, Cederburg, and O’Doherty (2022) with minor exceptions. Details on the data adjustments required to compute nominal and real stock returns for our developed country sample are available in Anarkulova, Cederburg, and O’Doherty (2022) and the corresponding internet appendix, in addition to the ones described below.

We measure returns that are denominated in the primary home currency with one exception. Our real returns for Germany are denominated in gold marks (rather than paper marks) for the 1917 to 1923 period. Extraordinary hyperinflation during this period complicates the calculation of real returns based on nominal returns in paper marks, and the GFDatabase reports a series of stock market returns denominated in gold marks.



The internet appendix for Anarkulova, Cederburg, and O’Doherty (2022) outlines the smoothing procedures used to fill gaps in return series for short periods in a few sample countries. In addition to those cases, we apply a smoothing procedure to convert five- and seven-month return data for Austria over the period from January 1920 to February 1922 and quarterly return data for Belgium over the period from May 1919 to January 1926 into a time series of monthly returns. In particular, we make the assumption of constant monthly returns within each period. We apply a similar procedure to quarterly return data for Switzerland from February 1914 to July 1914 and from August 1916 to January 1921. Czechoslovakia is missing return data for July 1921. We estimate returns for July 1921 and August 1921 using price index data for June 1921 and August 1921 under the assumption of a constant return for July and August of 1921.

One difference between our sample construction approach and the one in Anarkulova, Cederburg, and O’Doherty (2022) relates to the handling of multi-month return observations associated with stock market disruptions and closures. There are 35 instances in which stock exchanges closed for extended periods, typically as the result of a major war, political revolution, or banking crisis. Investors tend to earn negative real returns in these periods, such that omitting countries or periods because of these stock return data gaps induces an easy data bias. Table A.III reports cases of exchange closures or heavily restricted trading during our sample period along with the corresponding nominal and real returns. The bootstrap procedure in Anarkulova, Cederburg, and O’Doherty (2022) treats each of these events as a single return observation covering a multi-month period. This treatment reflects that most investors would have been unable to trade during these periods, such that they could only wait for the eventual realizations of the longer-period returns. This treatment is not ideal for our multi-asset analysis, however, as we would like to maintain a balanced panel of monthly asset returns for each country. At the same time, we need the data to reflect the economic outcomes of stock market investors.

In our current approach to handling multi-month returns, we take the perspective on an investor in a hypothetical fund attempting to track the market index for a given country. Although this investor could not directly liquidate her stock holdings via exchange trades during times of market closure, she could sell her shares in the hypothetical fund. The fund’s managers, in turn, could either rely on black market data for valuation purposes or produce an estimate of the historical event’s impact on asset prices at the beginning of the closure period. Based on this perspective, we apply one of two approaches to handling multi-month returns:

1. For events during which GFD provides black market prices, we use these values to estimate stock market index returns.
2. For events without corresponding data in GFD, we assign the total multi-month real return to the first monthly observation and zero real return to the remaining monthly observations.

The three exceptions to this general approach correspond to Austria’s 113-month return from July 1939 to November 1948, Switzerland’s 24-month return from August 1914 to July 1916, and Czechoslovakia’s 26-month return from April 1943 to May 1945. For Austria, GFD reports limited black market data in January 1943, April 1946, and from November 1946 to November 1948. We use these intermittent values and assign the remaining part of the total real return to July 1939. Similarly for Switzerland, we use GFD’s black market data in January 1916 and July 1916 and assign the remaining part of the total real return to August 1914. For Czechoslovakia, the April 1943 to May 1945 period corresponds to an episode that starts with severe trading restrictions and price controls and ends with the permanent stock exchange closure in Prague on May 5, 1945. For this period, we assign a terminal nominal return of  $-90.00\%$  to May 1945 and zero nominal returns to the other months. This treatment is consistent with the economic experience of investors over this period, as detailed in Anarkulova, Cederburg, and O’Doherty (2022).

### A.3 International stocks

We calculate real returns on a portfolio of international stocks from the perspective of an investor in a developed country. For each country, the international stock portfolio is a weighted investment across all developed stock markets excluding the local stock market. The international stock portfolio is value weighted by total market capitalization, and the returns are expressed in the domestic currency such that they reflect the exchange rate risk incurred by investing in assets denominated in foreign currencies.

The return calculation for international stocks uses the gross nominal stock market returns described in Section A.2. We convert the nominal return for each country  $j \neq i$  into a real return that is denominated in the domestic currency of country  $i$  and calculate the weighted average across countries  $j \neq i$ ,

$$R_{i,t}^{International\ stocks} = \sum_{j \neq i} w_{j,t-1} \frac{R_{j,t}^{Nominal\ stocks}}{\Pi_{i,t}} \left( \frac{E_t^{i,j}}{E_{t-1}^{i,j}} \right), \quad (A5)$$

where  $E_t^{i,j}$  is the exchange rate at the end of month  $t$  expressed in country  $i$  currency per country  $j$  currency,  $w_{j,t-1}$  is country  $j$ 's weight in the international stock portfolio in month  $t$ ,

$$w_{j,t-1} = \frac{M_{j,t-1}}{\sum_{j \neq i} M_{j,t-1}}, \quad (A6)$$

and  $M_{j,t-1}$  is the total market capitalization for the stock market in country  $j$  at the end of month  $t - 1$  expressed in US dollars.

### A.4 Bonds

We calculate bond returns using monthly data on bond yields. For comparability across countries and periods, we focus on ten-year government bonds. The GFDdatabase has variables for ten-year bond yields for most countries and periods in our sample, and we supplement these data to achieve full data coverage.

We first estimate ten-year bond prices given bond yields. We assume that each bond has exactly ten years to maturity, semiannual coupons, and a coupon rate equal to the greater of the bond yield and zero at the end of month  $t - 1$ . We then reprice the bond at the end of month  $t$  given the month- $t$  yield and the one month shorter maturity. We calculate the gross nominal return,

$$R_{i,t}^{Nominal\ bonds} = \frac{P_{i,t}}{P_{i,t-1}}, \quad (A7)$$

where  $P_{i,t}$  is the calculated dirty bond price (i.e., inclusive of accrued interest) for country  $i$  at the end of month  $t$ . Finally, we calculate the gross real bond return,<sup>31</sup>

$$R_{i,t}^{Bonds} = \frac{R_{i,t}^{Nominal\ bonds}}{\Pi_{i,t}}. \quad (A8)$$

Sections A.4.1 to A.4.7 describe several issues related to the underlying bond yield data.

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<sup>31</sup>This return calculation requires assumptions about the maturity and the coupon rate of the underlying bond. We validate this calculation in Section A.7 by comparing our calculated returns with returns from Datastream over the period of overlap between the two data samples. Our return calculations are very highly correlated with and have similar moments to those from Datastream.

#### A.4.1 Bond data availability

For several countries in our sample, there are no ten-year government bonds in circulation at the time the country is initially classified as developed. For example, ten-year government bonds are first issued in Iceland in 1992, Singapore in 1998, Hungary in 1999, Poland in 1999, the Czech Republic in 2000, South Korea in 2000 [Kang, Kim, and Rhee (2005)], Mexico in 2001 [Jeanneau and Verdia (2005)], and Türkiye in 2010.<sup>32</sup> These circumstances create gaps between the development dates and the sample eligibility dates for these countries.

Estonia issued its only domestic bond in 1993, and all tranches were redeemed by 2004.<sup>33</sup> As a result, Estonia is excluded from our sample because the country has no domestic bond data for the developed period.

#### A.4.2 Data gaps and errors

Table A.IV shows periods over which we are missing monthly bond yields. In these cases, we use a smoothing procedure to fill gaps in the monthly bond return series. This procedure uses the country-level yield data from before and after the missing observations to produce a series of constant monthly returns across a given period.

Since there are no data from GFD or alternative sources, we use the last non-missing yield of 4.33% in June 1944 to fill the data gap in bond yield data for the Czechoslovakia from July 1944 to May 1945.

We adjust an apparent error in the GFD bond yield data for Switzerland. The stated source for the GFD data is the Swiss National Bank. In comparing the GFD data to the Swiss National Bank data, however, the yields match only through December 1941. The Swiss National Bank reports yields of 3.11% in January 1942, 3.14% in February 1942, 3.12% in March 1942, and 3.08% in April 1942. GFD reports yields of 3.14%, 3.12%, and 3.07% for January through March 1942. From April 1942 to December 1990, the GFD data lead the Swiss National Bank data by one month. We adjust the GFD data by entering a 3.11% yield for January 1942 and shifting the original GFD data from January 1942 to November 1990 so that it covers February 1942 to December 1990.

#### A.4.3 Merging multiple sources

As shown in Table A.II, constructing a series of bond returns for a given country often requires us to combine yield data from multiple sources. We make additional adjustments in linking the data series for two sample countries. The GFD data for Chile end in March 2015, and we use data from Federal Reserve Economic Data (FRED) from April 2015 to December 2023. GFD reports a yield of 2.23% for March 2015, whereas the yields from FRED are 4.34% for March 2015 and 4.49% for April 2015. Merging these data series without adjustment would result in a return calculation of  $-17.76\%$  for April 2015. This return likely provides a poor characterization of investment outcomes, given the relative stability in yields in the FRED data. To address this issue, we use March 2015 and April 2015 yields from FRED to compute the April 2015 bond return. We make an analogous adjustment for Iceland in March 2004, as well as for Lithuania, Luxembourg, Poland, Portugal,

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<sup>32</sup>See [http://www.lanamal.is/asset/12732/special-report-markadsvidskipti\\_agust-2019.pdf](http://www.lanamal.is/asset/12732/special-report-markadsvidskipti_agust-2019.pdf) for Iceland, <https://eservices.mas.gov.sg/statistics/fdanet/BenchmarkPricesAndYields.aspx> for Singapore, [https://stats.oecd.org/OECDStat\\_Metadata/ShowMetadata.ashx?Dataset=GOV\\_DEBT&Coords=%5BCOU%5D.%5BBHUN%5D&ShowOnWeb=true&Lang=en](https://stats.oecd.org/OECDStat_Metadata/ShowMetadata.ashx?Dataset=GOV_DEBT&Coords=%5BCOU%5D.%5BBHUN%5D&ShowOnWeb=true&Lang=en) for Hungary, <https://www.gov.pl/web/finance/transaction-database> for Poland, <https://www.cnb.cz/en/financial-markets/treasury-securities-market/government-bonds/> for the Czech Republic, and <https://www.tcmb.gov.tr/wps/wcm/connect/EN/TCMB+EN/Main+Menu/Statistics/Markets+Data/Treasury+Auction/> for Türkiye.

<sup>33</sup>See <https://www.rahandusministeerium.ee/en/objectivesactivities/state-treasury/financial-reserves-and-liabilities/debt-management>.

Slovakia, and Türkiye in the corresponding months of 2023 when we merge data from GFD and FRED.

#### **A.4.4 Alternative bond return calculations**

Our primary bond return calculations use yield data with an assumption that the coupon rate is equal to the bond yield for a hypothetical new ten-year bond. In the cases described below, we use an alternative approach of separately measuring the capital gain and the coupon income due to data availability. We use data on current yields and coupon rates from the Central Bank of Argentina to infer bond prices for each month end from January 1947 to December 1966. We compute the capital gain based on the change in bond price and add one month of coupon income based on the 3% coupon rate from February 1947 to July 1960 and the 8% coupon rate from August 1960 to December 1966. We use London quotes from the International Center for Finance at Yale for Chile (December 1926 to September 1929) and Czechoslovakia (April 1922 to January 1927). We compute monthly bond returns based on price changes and monthly coupon income at the coupon rate of 4.5% for Chile and 8.0% for Czechoslovakia. Similarly, we compute monthly bond returns based on price changes and monthly coupon income at the coupon rate of 4% for Chile from April 1965 to December 1970. We use bond price data from the Central Bank of Chile over this period.

#### **A.4.5 Bond conversion in Argentina**

Argentina issued a 3% bond in 1955. In August 1960, the government allowed for a voluntary conversion of these old bonds to new 8% bonds. The conversion was favorable for bondholders, as they could receive bonds with higher interest payments. According to Duggan (1963), the 3% bonds were exchanged at 79 pesos for the nominal value of 100 pesos. Because the terms of the conversion were favorable, the majority of existing bondholders took the offer. In constructing our bond series for Argentina, we assume conversion at the 79:100 rate. We compute the price change and multiply by 0.79 to reflect the conversion when computing the capital gain for August 1960, and we add one month of coupon income at the 8% coupon rate to calculate the return.

#### **A.4.6 Bond default in Greece**

The bond return calculation must be adjusted in the event of a default or bond exchange that produces a change in par value. Defaults on domestic sovereign bonds are rare relative to external defaults, particularly for developed countries [Reinhart and Rogoff (2011)]. Rather, inflation is a more commonly used tool for eroding the real value of debt.

A notable event that produced a change in par value is the Greek default in 2012. Greece undertook a debt exchange in March 2012 in which creditors exchanged their existing bonds for a package of new government securities with a lower face value. Zettelmeyer, Trebesch, and Gulati (2013) provide an issue-by-issue estimate of the haircut for existing bondholders. We use the 53.8% haircut estimate for the bond with maturity closest to ten years. The ten-year bond yield declined substantially from 36.6% to 21.0% in March 2012, such that our calculation based on bond yields produces a nominal net return of 67.1%. Our calculation of the nominal gross return that incorporates the haircut is  $1.671 \times (1 - 0.538) = 0.772$  to produce a nominal net return of  $-22.8\%$  for ten-year bonds in March 2012.

#### **A.4.7 Germany in 1919 to 1924 and 1948**

To maintain consistency with our treatment of stock returns in Germany in the inflationary period from 1917 to 1923, we also compute bond returns in gold marks. We use bond prices in

paper marks from Fischer (1923, 1924, 1925) and convert paper mark prices to gold marks by using the USD exchange rate because the United States was on the gold standard during that period. The change in gold mark bond prices provides an estimate of the capital appreciation of the bonds. We compute the total bond return by including interest payments based on the 3% coupon rate of the bonds. We use this approach from February 1919 to January 1924.

Germany exchanged Reichsmarks for Deutschemarks in June 1948. For government bonds, the exchange was 10:1 [Schnabl (2019)]. To reflect the economic value of the currency exchange, we adjust the bond price at the end of June 1948 by dividing the price of the bond by ten. The resulting nominal bond return in June 1948 is  $-90.0\%$ .

## A.5 Bills

We estimate returns on bills using short-term yields and rates. For most countries and periods, the GFDdatabase has coverage with yield data on short-term (typically three-month) government bills. When these data are missing, we next use central bank rates when available and then interbank rates from the GFDdatabase. We supplement these data with hand-collected, short-term rates from original source documents to achieve full coverage. We convert annual nominal rates on bills into monthly nominal returns denoted by  $R_{i,t}^{Nominal\ bills}$  and then calculate real returns,

$$R_{i,t}^{Bills} = \frac{R_{i,t}^{Nominal\ bills}}{\Pi_{i,t}}. \quad (A9)$$

We compute monthly nominal bill returns from annual yields or rates as

$$R_{i,t}^{Nominal\ bills} = (1 + R_{i,t-1}^{Annual\ rate})^{1/12}, \quad (A10)$$

where  $R_{i,t-1}^{Annual\ rate}$  is the annualized short-term government bill yield, central bank rate, or interbank rate reported at the end of month  $t - 1$ .

### A.5.1 Data issues related to bills

We have a few periods over which there are no bill data from GFD or alternative sources, and we are required to make assumptions to fill these gaps in the data. For Canada, we use a yield of 5.75% for the seven-month period from January 1914 to July 1914. This value is an average of the 6.50% interbank rate for December 1913 from GFD and the 5.00% advance rate for August 1914 from Shearer and Clark (1984). The Netherlands is missing data for February 2014, so we average the short-term government bill yields from GFD of 0.09% for January 2014 and 0.13% for March 2014.

For New Zealand from January 1896 to December 1914, we use short-term yields on bills held by the Post Office Savings Bank Fund. The Post Office Savings Bank Fund did not hold Treasury bills in 1913, so we are missing data for that year. The yields are 3.00% in December 1912 and 4.00% in January 1914, and we use the average of 3.50% to fill in the data gap. We are also missing yield data for New Zealand from January 1915 to December 1919. The yields for December 1914 and January 1920 are both 4.00%, however, so we assume a 4.00% yield over the adjoining period with missing data.

## A.6 Other variables

We follow the data adjustments noted in Anarkulova, Cederburg, and O'Doherty (2022) and the corresponding internet appendix to estimate country-level inflation and exchange rate changes.

The data for market capitalization are from GFD. These series are typically reported at an annual frequency. There are missing data for some country-year observations of these series. For market capitalization, which is reported in USD, we fill data gaps by interpolating changes in proportion to USD nominal stock index returns. We use market capitalization series for Germany from 1917 to 1923 that are denominated in gold marks rather than paper marks. This approach is consistent with the calculation of the total return index for domestic stocks for Germany over this period. We fill a data gap in the 2023 GFD market capitalization data for the UK using an alternative source.<sup>34</sup>

Table A.V shows dividend-price ratio data for each country in the sample. We use annual dividend-price ratio data. We rely on external sources to calculate dividend-price ratios for Slovakia and Latvia. In both cases, GFD lacks comprehensive information to compute these ratios. For Slovakia, we use dividend-price ratio data from the Bratislava Stock Exchange’s official website.<sup>35</sup> For Latvia, we calculate dividend-price ratios using data on total dividends paid by companies from Nasdaq Baltic and data on total market capitalization from GFD.<sup>36</sup> We use dividend-price ratios from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) for Portugal from 1934 to 1987.

We follow Anarkulova, Cederburg, and O’Doherty (2022) to fill the data gaps in GFD dividend-price ratio data for Chile from January 1967 to December 1970 and Czechoslovakia from April 1938 to March 1943. For Chile, we fill the data gap with a 7.0% yield based on the dividend yield observation in December 1966. The dividend yield in Czechoslovakia fluctuates between 1.4% and 2.6% in the three years before the break in the data, so we assume a 2.0% dividend yield for the missing observations. Table A.VI shows additional periods over which annual dividend-price ratio data are missing in the GFD database. We estimate dividend-price ratios for these periods with missing data using the methods described in the table.

## A.7 External validation tests

This section details the external validation tests for our stock and bond return data.

### A.7.1 Comparison of stock data from GFD and Jordà et al. (2019)

Anarkulova, Cederburg, and O’Doherty (2022) compare their data on stock returns from GFD with the stock returns from the overlapping periods in Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019). They find that the data from these two sources have very similar characteristics in terms of country-level average returns, standard deviations, and extreme returns. They also show that the return correlation across the two datasets exceeds 0.90 for nearly all countries. Given that our approach to constructing country-level stock returns closely follows the approach in Anarkulova, Cederburg, and O’Doherty (2022), these tests also provide external validation of our stock data.

### A.7.2 Comparison of bond data from GFD and Datastream

As described in Section A.4, we calculate bond returns using bond yield data from GFD and other sources. In this section, we perform an external validation exercise by comparing our bond returns with those from Datastream over the periods and countries for which they are available. This analysis serves both to ensure that our approach to converting bond yields to returns is empirically accurate and to assess whether our bond return data and the bond data from a popular alternative source exhibit common characteristics.

<sup>34</sup>The data are available at <https://www.ceicdata.com/en/indicator/united-kingdom/market-capitalization>.

<sup>35</sup>See <http://www.bsse.sk/%C5%A0tatistika/Mesa%C4%8Dn%C3%A1.aspx>.

<sup>36</sup>The dividend data for Latvia are available at <https://nasdaqbaltic.com/statistics/en/statistics>.

Table A.VII shows results from the external validation analysis. The table reports statistics for real returns. Our sample overlaps with Datastream for 27 countries. Datastream data begin in 1989 for several countries and more recently for others. The table reports the sample size, the arithmetic and geometric means, standard deviation, and minimum and maximum returns for our data, the corresponding statistics for Datastream data, and the correlation between our returns and those from Datastream.

Table A.VII indicates a close correspondence between our bond return data and those from Datastream. For nearly all countries, the means, standard deviations, and extreme returns are highly similar across the two data sources. The return correlations are above 0.90 for 24 of the 27 countries. Only Hungary, Mexico, and Singapore have correlations below 0.90. Of the 24 countries with high correlations, Greece is unique in Table A.VII as the only country with economically meaningful differences in the remaining summary statistics. We proceed to discuss these four exceptions.

Hungary and Singapore appear to be the simplest cases. We examine bond yields and returns across the two datasets. The GFDdatabase and Datastream bond yields differ, sometimes substantially, for these two countries. To reconcile the differences, we collect ten-year historical bond yield data from the Magyar Nemzeti Bank (the central bank of Hungary) and the Monetary Authority of Singapore.<sup>37</sup> For Hungary, the correlation in yield changes from the central bank data and our data is near one, whereas the correlation between yield changes from the central bank and Datastream is much lower. For Singapore, the GFDdatabase and Singaporean government data exactly match. The large deviations between Datastream and these other sources primarily occur in the first seven months of the sample, and the reported returns in Datastream imply changes in yields that are not reflected in the data from the Monetary Authority. Excluding the first seven months, the correlation between returns in our data and Datastream is 0.98. Our data appear reliable for these countries.

The bond yields for Mexico in our data and in Datastream are relatively similar. For several months in the sample, the reported Datastream return seems inconsistent with the reported yield change. For example, the reported yield increases by 0.08% in June 2015, but the reported return is 8.12%. We compare our calculated returns and the reported Datastream returns with the returns on the S&P/BMV Mexico Sovereign Bond Index in these months.<sup>38</sup> The S&P/BMV index tracks bonds with several maturities, and its duration is low compared with the other two series. Nonetheless, the returns from this index are much more consistent with our data versus Datastream. In June 2015, for example, the S&P/BMV index reports a return of  $-0.15\%$ , which is close to our return calculation of  $-0.10\%$  but far from the 8.12% reported return in Datastream. Given the consistency between the GFDdatabase and the S&P/BMV index, the deviations between our data and Datastream appear to be reporting errors for returns in Datastream.

The largest deviations in bond returns for Greece are related to the Greek bond default in 2012. As discussed in Section A.4.6, we calculate a bond return in March 2012 that accounts for the bond exchange and the associated haircut. Our return calculation, which reflects information from ten-year bond yields and the default, is  $-22.80\%$  in this month. This return differs substantially from the  $-4.16\%$  return reported by Datastream. Our study focuses on domestic debt, so we take the perspective of a hypothetical domestic investor. Participation rates in the exchange were higher among domestic investors compared with international investors [Zettelmeyer, Trebesch, and Gulati (2013)]. We do not have information on Datastream’s return calculation for this month, but the difference could arise from a different assumption about participation in the exchange. Late

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<sup>37</sup>See <https://www.mnb.hu/en/statistics/statistical-data-and-information/statistical-time-series/xi-money-and-capital-markets> and <https://eservices.mas.gov.sg/statistics/fdanet/BenchmarkPricesAndYields.aspx>.

<sup>38</sup>See <https://www.spglobal.com/spdji/en/indices/fixed-income/sp-bmv-mexico-sovereign-bond-index/>.

in 2012, Greece announced a voluntary bond buyback to be executed in December 2012, and the buyback led to an increase in market prices [Zettelmeyer, Trebesch, and Gulati (2013)]. We observe a 4.32% decrease in bond yield in December 2012 and calculate a return of 26.12%. Datastream reports a 3.21% decrease in bond yield and reports a return of 41.47%, such that the return is much larger than that implied by the yield change. Given that the buyback occurred at prevailing market prices, our view is that any effect of the buyback should be reflected in the change in yields. The return differences for these two months account for much of the difference in average returns for Greece in Table A.VII.



TABLE A.1: DEVELOPED COUNTRY SAMPLE PERIODS

The table shows developed countries, initial development dates, classification reasons for development, sample eligibility details, and sample coverage. The development year classifications are based on agricultural labor share or organizational membership in the Organisation for European Economic Co-operation (OECC) or the Organisation for Economic Co-operation and Development (OECD). Sample eligibility for a given developed country requires that the country has issued long-term government bonds. The sample period start date is the later of the sample eligibility date and the first date with return data for stocks, bonds, and bills.

Country	Development details		Sample eligibility details		Sample coverage		
	Year	Reason for classification	Year	Reason for delayed sample eligibility	Start date	End date	Coverage (%)
United Kingdom	1841	Agricultural labor share	1890	Sample for study starts in 1890	1890:01	2023:12	100.0
Netherlands	1849	Agricultural labor share	1890	Sample for study starts in 1890	1914:01	2023:12	82.1
Belgium	1856	Agricultural labor share	1890	Sample for study starts in 1890	1897:01	2023:12	94.8
France	1866	Agricultural labor share	1890	Sample for study starts in 1890	1890:01	2023:12	100.0
Norway	1875	Agricultural labor share	1890	Sample for study starts in 1890	1914:01	2023:12	82.1
Germany	1882	Agricultural labor share	1890	Sample for study starts in 1890	1890:01	2023:12	100.0
Denmark	1890	Agricultural labor share	1890	n/a	1890:01	2023:12	100.0
Switzerland	1890	Agricultural labor share	1890	n/a	1914:01	2023:12	82.1
United States	1890	Agricultural labor share	1890	n/a	1890:01	2023:12	100.0
Canada	1891	Agricultural labor share	1891	n/a	1891:01	2023:12	100.0
Argentina	1895	Agricultural labor share	1895	n/a	1947:02	1966:12	27.7
New Zealand	1896	Agricultural labor share	1896	n/a	1896:01	2023:12	100.0
Australia	1901	Agricultural labor share	1901	n/a	1901:01	2023:12	100.0
Sweden	1910	Agricultural labor share	1910	n/a	1910:01	2023:12	100.0
Austria	1920	Agricultural labor share	1920	n/a	1920:01	2023:12	100.0
Chile period I	1920	Agricultural labor share	1920	n/a	1927:01	1970:12	86.3
Greece	1920	Agricultural labor share	1920	n/a	1981:02	2023:12	41.3
Czechoslovakia	1921	Agricultural labor share	1921	n/a	1922:05	1945:05	94.5
Japan	1930	Agricultural labor share	1930	n/a	1930:01	2023:12	100.0
Portugal	1930	Agricultural labor share	1930	n/a	1934:01	2023:12	95.7
Italy	1931	Agricultural labor share	1931	n/a	1931:01	2023:12	100.0
Ireland	1936	Agricultural labor share	1936	n/a	1936:01	2023:12	100.0
Singapore	1947	Agricultural labor share	1998	Long-term bonds first available in 1998	1998:07	2023:12	100.0
Iceland	1948	OECC membership	1992	Long-term bonds first available in 1992	2002:01	2023:12	68.8
Luxembourg	1948	OECC membership	1948	n/a	1982:01	2023:12	55.3
Türkiye	1948	OECC membership	2010	Long-term bonds first available in 2010	2010:02	2023:12	100.0

(Continued on next page)

TABLE A.I (Continued)

Country	Development details		Sample eligibility details			Sample coverage	
	Year	Reason for classification	Year	Reason for delayed sample eligibility	Start date	End date	Coverage (%)
Spain	1959	OEEC membership	1959	n/a	1959:01	2023:12	100.0
Finland	1969	OECD membership	1969	n/a	1969:01	2023:12	100.0
Mexico	1994	OECD membership	2001	Long-term bonds first available in 2001	2001:08	2023:12	100.0
Czech Republic	1995	OECD membership	2000	Long-term bonds first available in 2000	2000:05	2023:12	100.0
Hungary	1996	OECD membership	1999	Long-term bonds first available in 1999	1999:02	2023:12	100.0
Poland	1996	OECD membership	1999	Long-term bonds first available in 1999	1999:06	2023:12	100.0
South Korea	1996	OECD membership	2000	Long-term bonds first available in 2000	2000:11	2023:12	100.0
Slovakia	2000	OECD membership	2000	n/a	2000:01	2023:12	100.0
Chile period II	2010	OECD membership	2010	n/a	2010:01	2023:12	100.0
Estonia	2010	OECD membership	—	No qualifying long-term bonds	—	—	—
Israel	2010	OECD membership	2010	n/a	2010:01	2023:12	100.0
Slovenia	2010	OECD membership	2010	n/a	2010:01	2023:12	100.0
Latvia	2016	OECD membership	2016	n/a	2016:01	2023:12	100.0
Lithuania	2018	OECD membership	2018	n/a	2018:01	2023:12	100.0
Colombia	2020	OECD membership	2020	n/a	2020:01	2023:12	100.0

TABLE A.II: DATA SOURCES

The table summarizes the data series used to compute returns for each developed country in the sample. For each country and asset class, the table reports the data series used to construct returns and the corresponding start and end dates. The data series symbols correspond to those in the GFDatabase from Global Financial Data. For stocks and bills, the table also indicates the data series type. Stock returns are based on either total return indexes (TRI) or combinations of price indexes and dividend-price ratios (PI/DP). Bill returns are based on short-term Treasury yields (TBY), central bank interest rates (CBR), interbank interest rates (IIR), one-year government bond yields (GBY-1), deposit interest rates (DIR), interest rates on advances (IRA), overnight interest rates (OIR), or time money rates (TMR). All bond returns correspond to returns on ten-year government bonds. We provide details on alternative data sources in the footnotes.

Country	Stocks				Bonds				Bills			
	Series	Type	Start	End	Series	Start	End	Series	Type	Start	End	
Argentina	_IBGD, SYARGYM	PI/DP	1947:02	1966:12	③	1947:02	1966:12	IDARGD	CBR	1947:02	1966:12	
Australia	_AORDAD	TRI	1901:01	2023:12	IG AUS10D	1901:01	2023:12	②	TBY	1901:01	1920:06	
Austria	_ATXTRD	TRI	1920:01	2023:12	IG AUT10D	1920:01	2023:12	ITAUS3D	CBR	1920:07	1928:06	
								ITAUT3M	TBY	1928:07	2023:12	
								IDAUTD	CBR	1920:01	1959:12	
								ITAUT1D	TBY	1960:01	1990:12	
Belgium	_BCSHD	TRI	1897:01	2023:12	IG BEL10D	1897:01	2023:12	IGAUT1D	GBY-1	1991:01	2023:12	
								IDBELD	CBR	1897:01	1947:12	
								ITBEL3D	TBY	1948:01	2023:12	
Canada	_TRGSP TSE	TRI	1891:01	2023:12	IG CAN10D	1891:01	2023:12	+	DIR	1891:01	1901:12	
								IMCANMOM	OIR	1902:01	1913:12	
								*	IRA	1914:08	1934:02	
								ITCAN3D	TBY	1934:03	2023:12	
Chile period I	_IGPAD, SYCHLYM	PI/DP	1927:01	1970:12	④	1927:01	1929:09	IDCHLD	CBR	1927:01	1970:12	
					⑤	1929:10	1930:12					
								IGCHLCM				
					⑥	1956:03	1970:12					
Chile period II	_IPSAD	TRI	2010:01	2023:12	IGCHLIM	2010:01	2015:03	ITCHL3D	TBY	2010:01	2012:09	
					⑦	2015:04	2023:12	IDCHLD	CBR	2012:10	2023:12	
Colombia	TRCOLSTM	TRI	2020:01	2023:12	IGCOL10D	2020:01	2023:12	ITCOL3W	TBY	2020:01	2023:12	
Czech Republic	_PXTRD	TRI	2000:05	2023:12	IGCZE10D	2000:05	2023:12	ITCZE3D	TBY	2000:05	2017:02	
								IDCZED	CBR	2017:03	2023:12	
Czechoslovakia	CZINDEXM, SYCZEYM	PI/DP	1922:05	1937:11	④	1922:05	1927:01	IDCZED	CBR	1922:05	1945:05	
					⑤	1927:02	1944:06					

(Continued on next page)

TABLE A.II (Continued)

Country	Stocks				Bonds				Bills			
	Series	Type	Start	End	Series	Start	End	Series	Type	Start	End	
Denmark	_OMXCGID	TRI	1890:01	2023:12	IGDNK10D	1890:01	2023:12	IDDNKD	CBR	1890:01	1975:12	
Finland	_OMXHGID	TRI	1969:01	2023:12	IGFIN10D	1969:01	2023:12	ITDNK3D	TBY	1976:01	2023:12	
								IDFIN	CBR	1969:01	1996:08	
France	TRSBF250D	TRI	1890:01	2023:12	IGFRA10D	1890:01	2023:12	IGFIN1D	GBY-1	1996:09	2012:01	
								ITFIN1D	TBY	2012:02	2013:05	
								IGFIN1D	GBY-1	2013:06	2014:07	
								IDFIN	CBR	2014:08	2023:12	
								IDFRAD	CBR	1890:01	1930:12	
Germany	_CDAXD	TRI	1890:01	2023:12	IGDEU10D	1890:01	2023:12	ITFRA3D	TBY	1931:01	2023:12	
								IDDEUD	CBR	1890:01	1952:12	
Greece	_RETMD	TRI	1981:02	2023:12	IGGRC10D	1981:02	2014:01	ITDEU3D	TBY	1953:01	2023:12	
								ITGRC3D	TBY	1981:02	2023:12	
Hungary	_BUXD	TRI	1999:02	2023:12	IGGRC10D	2014:02	2014:02	ITHUN3D	TBY	1999:02	2023:12	
								ITISL3D	TBY	2002:01	2013:01	
Iceland	_OMXIPID, SYISLYM	PI/DP	2002:01	2002:06	IGHUN10D	1999:02	2023:12	IDISLD	CBR	2013:02	2023:12	
								ITDEU3D	TBY	1953:01	2023:12	
Ireland	_IVRTD	TRI	1936:01	2023:12	IGIRL10D	1936:01	2023:12	IDIRLD	CBR	1936:01	1969:11	
								ITIRL3M	TBY	1969:12	2008:12	
Israel	TRISRSTM	TRI	2010:01	2023:12	IGISR10IM	2010:01	2014:12	ITISR3D	TBY	2010:01	2023:12	
								IDITAD	CBR	1931:01	1939:12	
Italy	_BCIPRD	TRI	1931:01	2019:12	IGITA10D	1931:01	2023:12	ITITA3D	TBY	1940:01	2023:12	
								IDJPND	CBR	1930:01	1959:12	
Japan	_TOPXDVD	TRI	1930:01	2023:12	IGJPN10D	1930:01	2023:12	ITJPN3D	TBY	1960:01	2023:12	
								ITJPN3D	TBY	1960:01	2023:12	
Latvia	_OMXRGID	TRI	2016:01	2023:12	IGLTU10D	2016:01	2023:12	ITIRL3M	IIR	2009:01	2023:12	
								ITISR3D	TBY	2010:01	2023:12	
Lithuania	_OMXVGD	TRI	2018:01	2023:12	IGLTU10D	2018:01	2023:01	ITIRL3M	IIR	2016:01	2023:12	
								ITIRL3M	IIR	2018:01	2023:12	
Luxembourg	_LUXXD, SYLUXYM	PI/DP	1982:01	1984:12	IGLUX10D	1982:01	2023:03	IMLUXM	OIR	1982:01	1998:12	
								ITIRL3M	IIR	2016:01	2023:12	
Mexico	_LUXXR	TRI	1985:01	2016:11	IGLUX10D	1982:01	2023:03	ITMEX3D	TBY	2001:08	2023:12	
								ITMEX3D	TBY	2001:08	2023:12	
Mexico	_JRTD	TRI	2001:08	2023:12	IGMEX10D	2001:08	2023:12	ITMEX3D	TBY	2001:08	2023:12	
								ITMEX3D	TBY	2001:08	2023:12	

(Continued on next page)

TABLE A.II (Continued)

Country	Stocks				Bonds				Bills			
	Series	Type	Start	End	Series	Start	End	Series	Type	Start	End	
Netherlands	_AAXGRD	TRI	1914:01	2023:12	IGNLD10D	1914:01	1944:08	IDNLD3D	CBR	1914:01	1940:12	
			②	1944:09	1945:12	ITNLD3D	TBY	1941:01	2023:12			
New Zealand	_NZGID	TRI	1896:01	2023:12	IGNLD10D	1946:01	2023:12	■	TBY	1896:01	1919:12	
			■	1896:01	2023:12	■	DIR	1920:01	1922:12			
Norway	_OSEAXD	TRI	1914:01	2023:12	IGNOR10D	1914:01	2023:12	IDNZLD	CBR	1923:01	1978:02	
			■	1914:01	2023:12	ITNZL3D	TBY	1978:03	2023:12			
Poland	_WIGD	TRI	1999:06	2023:12	IGNOR10D	1914:01	2023:12	IDNORD	CBR	1914:01	1941:11	
			⑦	1999:06	2023:12	IGPOL10D	1999:06	2023:03	ITNOR3D	TBY	1941:12	2023:12
Portugal	_IBTAD, ② _BVLGD	PI/DP	1934:01	1988:01	IGPRT10D	1934:01	1974:04	IDPRTD	CBR	1934:01	1988:12	
		TRI	1988:02	2023:12	②	1974:05	1975:12	ITPRT6D	TBY	1989:01	1999:01	
Singapore	_TFTFSTD _SAXD	TRI	1998:07	2023:12	IGSGP10D	1998:07	2023:12	IDPRTD	CBR	1999:02	2001:12	
			⑦	1998:07	2023:12	⑦	2023:07	2023:12	②	TBY	2002:01	2010:09
Slovakia	_SBITOPD, SYSVNYM _TRKORSTM	TRI	2000:01	2023:12	IGSVK10D	2000:01	2023:03	ITPRT6D	TBY	2010:10	2022:09	
			⑦	2000:01	2023:12	⑦	1998:07	2023:12	⑦	IIR	2022:10	2023:12
Slovenia	_BCNPR30	TRI	2010:01	2023:12	IGSGP10D	2010:01	2023:12	ITSVN3M	TBY	2010:01	2023:12	
			⑦	2010:01	2023:12	⑦	2010:01	2023:12	IGKORID	GBY-1	2000:11	2023:12
Spain	_OMXSBDGI	TRI	1959:01	2023:12	IGESP10D	1959:01	2023:12	IDESP	CBR	1959:01	1978:12	
			⑧	1959:01	2023:12	⑧	1959:01	2023:12	ITESP12D	TBY	1979:01	2023:12
Sweden	_SSHID	TRI	1910:01	2023:12	IGSWE10D	1910:01	2018:12	IDSWED	CBR	1910:01	1954:12	
			⑦	1910:01	2023:12	⑦	2019:01	2019:01	ITSWE3D	TBY	1955:01	2023:12
Switzerland	_TRRBILED	TRI	1914:01	2023:12	IGSWE10D	2019:02	2023:12	IDCHED	CBR	1914:01	1979:12	
			⑧	1914:01	2023:12	⑧	1942:01	1942:01	ITCHE3D	TBY	1980:01	2023:12
Türkiye		TRI	2010:02	2023:12	IGCHE10D	1942:02	2023:12	ITTUR3D	TBY	2010:02	2014:09	
			⑨	2010:02	2023:12	⑨	2010:02	2023:06	IGTUR1D	GBY-1	2014:10	2023:03
			2023:07	2023:12				⑩	TBY	2023:04	2023:12	

(Continued on next page)

TABLE A.II (Continued)

Country	Stocks			Bonds			Bills		
	Series	Type	Start End	Series	Start End	Series	Type	Start End	
United Kingdom	_TFTASD	TRI	1890:01 2023:12	IGGBR10D	1890:01 2023:12	IDGBRD	CBR	1890:01 1899:12	
United States	_SPXTRD	TRI	1890:01 2023:12	IGUSA10D	1890:01 2023:12	ITGBR3D	TBY	1900:01 2023:12	
						▲	TMR	1890:01 1914:10	
						IDUSAD	CBR	1914:11 1919:12	
						ITUSA3CMD	TBY	1920:01 2023:12	

**Footnotes:**

- ① Stock returns for Luxembourg for the period from 2016:12 to 2023:12 are from the Luxembourg Stock Exchange. See <https://www.bourse.lu/home>.
- ② Dividend-price ratios, bond returns, and bill returns for several countries are from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019).
- ③ Bond returns for Argentina for the period from 1947:02 to 1966:12 are based on data from the Central Bank of Argentina. See [http://www.bcra.gov.ar/PublicacionesEstadisticas/Boletin\\_estadistico.asp](http://www.bcra.gov.ar/PublicacionesEstadisticas/Boletin_estadistico.asp).
- ④ Bond returns for select periods in Chile and Czechoslovakia are based on London quotes from the International Center for Finance at Yale. See <https://som.yale.edu/faculty-research/our-centers-initiatives/international-center-finance/data/historical-financial-research-data/london-stock-exchange>.
- ⑤ Bond returns for select periods in Chile and Czechoslovakia are based on London quotes from the League of Nations reports.
- ⑥ Bond returns for Chile for the period from 1956:03 to 1970:12 are based on data from the Central Bank of Chile. See <https://repositoriodigital.bcentral.cl/xmlui/handle/20.500.12580/26/browse?type=dateissued>.
- ⑦ Bond and bill returns for several countries are based on data from Federal Reserve Economic Data (FRED) at the Federal Reserve Bank of St. Louis. See <https://fred.stlouisfed.org/>.
- ⑧ The bond return in Switzerland for 1942:01 is based on data from the Swiss National Bank ([https://www.snb.ch/de/i/about/stat/statpubdis/id/statpub\\_histz\\_arch#t2](https://www.snb.ch/de/i/about/stat/statpubdis/id/statpub_histz_arch#t2)). We also shift a portion of the series IGCH10D from GFD, which originally covers the period from 1942:01 to 1990:11, to cover the period from 1942:02 to 1990:12.
- ⑨ The bond return in Türkiye for the period from 2023:07 to 2023:12 are based on data from <http://www.worldgovernmentbonds.com/bond-historical-data/turkey/10-years/>.
- ⑩ Bill returns for Türkiye for the period from 2023:04 to 2023:12 are based on data from <https://www.worldgovernmentbonds.com/bond-historical-data/turkey/3-months/>.
- ✚ Bill returns for Canada for the period from 1891:01 to 1901:12 are based on interest rates on deposits in government savings banks. See p. 363 of <https://www66.statcan.gc.ca/eng/1901-eng.htm>.
- \* Bill returns for Canada for the period from 1914:08 to 1934:02 are based on data from Shearer and Clark (1984).
- Bill returns for New Zealand for the period from 1896:01 to 1922:12 are based on data from the annual New Zealand Official Year-book. See, e.g., [https://www3.stats.govt.nz/New\\_Zealand\\_Official\\_Yearbooks/1896/NZ0YB\\_1896.html](https://www3.stats.govt.nz/New_Zealand_Official_Yearbooks/1896/NZ0YB_1896.html).
- ▲ Bill returns for the United States for the period from 1890:01 to 1914:10 are based on data from Macaulay (1938).

TABLE A.III: MULTI-MONTH STOCK RETURNS

The table reports periods of multi-month stock returns associated with exchange closures and details our approach to converting each return to a series of monthly returns. For each multi-month return observation, the table reports the number of months, the start and end dates of the period, the nominal and real net stock market returns earned over the period, and the adjustment method. For adjustment method 1, we use alternative data sources from GFD (e.g., black market trading data) to fill in a complete series of monthly returns. For adjustment method 2, we assign the full multi-month real return to the first month of the period and assign zero real returns to the remaining months. The cases marked with a **+** are discussed in Section A.2.1. Panels A and B show events corresponding to World War I and World War II, respectively, Panel C shows periods with revolutions, Panel D shows financial and banking crises, and Panel E shows labor strikes.

Country	Months	Start date	End date	Nominal return (%)	Real return (%)	Adjustment
Panel A: World War I						
Australia	6	1914:08	1915:01	-0.45	-0.39	Method 1
Belgium	52	1914:08	1918:11	25.12	-55.91	Method 2
Canada	7	1914:08	1915:02	1.38	-3.59	Method 1
Denmark	4	1914:08	1914:11	0.72	-0.27	Method 1
France	6	1914:08	1915:01	-10.89	-27.54	Method 1
Germany	42	1914:08	1918:01	20.03	-38.87	Method 1
Netherlands	7	1914:08	1915:02	-1.23	-3.50	Method 1
Norway	3	1914:08	1914:10	-3.80	-4.36	Method 2
Sweden	4	1914:08	1914:11	-5.91	-8.96	Method 2
Switzerland	24	1914:08	1916:07	0.17	-18.71	<b>+</b>
United Kingdom	6	1914:08	1915:01	0.11	-2.94	Method 1
United States	5	1914:08	1914:12	-2.14	-3.11	Method 1
Panel B: World War II						
Austria	2	1938:04	1938:05	6.01	5.64	Method 2
Austria	113	1939:07	1948:11	315.61	-16.90	<b>+</b>
Belgium	5	1940:06	1940:10	22.38	12.54	Method 2
Belgium	11	1944:08	1945:06	-0.29	-17.08	Method 2
Czechoslovakia	16	1938:10	1940:01	32.23	16.91	Method 2
Czechoslovakia	4	1942:01	1942:04	20.66	12.32	Method 2
Denmark	2	1940:05	1940:06	-7.64	-10.67	Method 2
France	2	1939:09	1939:10	-2.96	0.53	Method 1
France	10	1940:06	1941:03	94.57	75.61	Method 2
Germany	67	1943:01	1948:07	-87.62	-91.10	Method 2
Japan	45	1945:09	1949:05	449.38	-87.15	Method 1
Netherlands	5	1940:05	1940:09	20.63	15.21	Method 2
Netherlands	21	1944:09	1946:05	-14.33	-33.15	Method 2
Norway	2	1940:04	1940:05	-2.07	-3.52	Method 1
Switzerland	2	1940:06	1940:07	-3.57	-5.11	Method 1
Panel C: Revolution						
Czechoslovakia	26	1943:04	1945:05	-90.00	-88.59	<b>+</b>
Portugal	35	1974:05	1977:03	-80.40	-89.24	Method 2

*(Continued on next page)*

TABLE A.III (Continued)

Country	Months	Start date	End date	Nominal return (%)	Real return (%)	Adjustment
Panel D: Financial or banking crisis						
Austria	2	1931:10	1931:11	6.25	5.60	Method 2
Germany	2	1931:08	1931:09	-24.58	-23.01	Method 2
Germany	7	1931:10	1932:04	-8.22	1.78	Method 2
Greece	2	2015:07	2015:08	-21.53	-20.13	Method 2
Panel E: Labor strike						
France	2	1974:04	1974:05	-6.17	-8.76	Method 1
France	2	1979:03	1979:04	12.79	10.69	Method 1

TABLE A.IV: BOND RETURN SMOOTHING

The table summarizes periods over which we are missing bond yield data. In each case, we use the country-level yield data from before and after the missing observations to produce a series of constant monthly returns across the period noted in the table. For each period with missing bond data, the table reports the country, the number of missing observations, and the start and end dates of the period.

Country	Months	Start date	End date
Argentina	4	1948:08	1948:11
	11	1949:01	1949:11
	11	1950:01	1950:11
	11	1951:01	1951:11
	11	1952:01	1952:11
	11	1953:01	1953:11
	11	1954:01	1954:11
	24	1955:01	1956:12
	1	1958:02	1958:02
	1	1958:08	1958:08
	1	1959:05	1959:05
	1	1959:08	1959:08
	Belgium	3	1940:05
Czechoslovakia	15	1938:10	1939:12
Finland	1	1991:06	1991:06
Germany	8	1931:08	1932:03
	25	1943:12	1945:12
Greece	44	1989:01	1992:08
Netherlands	2	1940:05	1940:06
	3	1944:09	1944:11
	11	1945:01	1945:11
Portugal	7	1974:05	1974:11
	11	1975:01	1975:11
Switzerland	1	2014:02	2014:02
	5	1914:08	1914:12



TABLE A.V: DIVIDEND-PRICE RATIO DATA

The table shows dividend-price ratio data for each country in the sample. The data are annual, and the data series symbols correspond to those in the GFDdatabase from Global Financial Data. We provide details on alternative sources in Section A.6.

Country	Series	Start year	End year
Argentina	SYARGYM	1947	1966
Australia	SYAUSYM	1901	2023
Austria	SYAUTYM	1920	2023
Belgium	SYBELYM	1897	2023
Canada	SYCANYTM	1891	2023
Chile period I	SYCHLYM	1927	1970
Chile period II	SYCHLYM	2010	2023
Colombia	SYCOLYM	2020	2023
Czech Republic	SYCZEYM	2000	2023
Czechoslovakia	SYCZEYM	1922	1945
Denmark	SYDNKYM	1890	2023
Finland	SYFINYM	1969	2023
France	SYFRAYM	1890	2023
Germany	SYDEUYM	1890	2023
Greece	SYGRCYM	1981	2023
Hungary	SYHUNYM	1999	2023
Iceland	SYISLYM	2002	2023
Ireland	SYIRLYM	1936	2023
Israel	SYISRYM	2010	2023
Italy	SYITAYM	1931	2023
Japan	SYJPNYM	1930	2023
Latvia	See table caption	2016	2023
Lithuania	SYLTUYM	2018	2023
Luxembourg	SYLUXYM	1982	2023
Mexico	SYMEXYM	2001	2023
Netherlands	SYNLDYAM	1914	2023
New Zealand	SYNZLYM	1896	2023
Norway	SYNORYM	1914	2023
Poland	SYPOLYM	1999	2023
Portugal	See table caption	1934	1987
	SYPRTYM	1988	2023
Singapore	SYSGPYM	1998	2023
Slovakia	See table caption	2000	2023
Slovenia	SYSVNYM	2010	2023
South Korea	SYKORYM	2000	2023
Spain	SYESPYM	1959	2023
Sweden	SYSWEYM	1910	2023
Switzerland	SYCHEYM	1914	2023
Türkiye	SYTURYM	2010	2023
United Kingdom	_DFTASD	1890	2023
United States	SYUSAYM	1890	2023

TABLE A.VI: DATA GAPS FOR DIVIDEND-PRICE RATIOS

The table shows periods over which we are missing annual dividend-price ratio data. For each period, the table reports the country, the number of missing annual observations, the start and end dates for the period, and the method used to estimate the missing ratios. For estimation method 1, we infer annual dividend-price ratios using total return index and price index data. For estimation method 2, we fill in missing dividend-price ratio using the last non-missing dividend-price ratio. For estimation method 3, we use the next year's dividend-price ratio.

Country	Years	Start year	End year	Method
Denmark	31	1938	1968	Method 1
France	1	1940	1940	Method 2
Germany	7	1944	1950	Method 2
Iceland	1	2001	2001	Method 3
	17	2007	2023	Method 1
Italy	1	1945	1945	Method 1
Japan	4	1945	1948	Method 2
Luxembourg	1	1981	1981	Method 3
	29	1995	2023	Method 1
Mexico	1	2014	2014	Method 1
Singapore	1	2021	2021	Method 1
Switzerland	4	1914	1917	Method 2

TABLE A.VII: EXTERNAL VALIDATION TEST RESULTS

The table reports summary statistics for monthly real net bond returns for each developed country with a return sample that overlaps with the sample from Datastream. For each country, the table shows the number of sample months. The table also shows the following summary statistics for our sample and for the Datastream sample: the arithmetic average return ( $\bar{R}_a$ ), the geometric average return ( $\bar{R}_g$ ), the standard deviation of return (SD), the minimum (Min) and the maximum (Max) return, and the correlation between the return samples (Corr).

Country	Months	Our data						Datastream					
		$\bar{R}_a$ (%)	$\bar{R}_g$ (%)	SD (%)	Min (%)	Max (%)	Corr	$\bar{R}_a$ (%)	$\bar{R}_g$ (%)	SD (%)	Min (%)	Max (%)	Corr
Australia	420	0.39	0.36	2.19	-5.63	6.80	0.40	0.38	2.22	-6.75	7.84	0.97	
Austria	420	0.21	0.19	1.85	-7.31	5.99	0.26	0.25	1.67	-7.10	4.99	0.96	
Belgium	414	0.27	0.25	1.94	-6.49	7.47	0.33	0.31	1.92	-7.48	8.33	0.98	
Canada	420	0.33	0.31	2.02	-6.33	6.47	0.34	0.32	1.99	-6.11	6.36	0.97	
Czech Republic	284	0.11	0.08	2.45	-8.47	9.62	0.17	0.15	2.35	-8.49	8.76	0.94	
Denmark	419	0.31	0.29	2.11	-7.05	6.40	0.34	0.32	2.00	-7.02	5.83	0.98	
Finland	388	0.34	0.32	2.16	-7.16	8.59	0.38	0.36	1.96	-7.55	8.01	0.97	
France	420	0.31	0.29	1.92	-7.15	5.86	0.35	0.34	1.81	-5.85	5.19	0.97	
Germany	420	0.23	0.21	1.86	-6.84	5.31	0.26	0.24	1.78	-6.93	5.08	0.97	
Greece	297	0.33	0.14	5.95	-30.84	26.45	0.54	0.28	7.08	-41.35	41.84	0.93	
Hungary	299	0.23	0.17	3.49	-9.83	12.42	0.24	0.18	3.41	-16.31	15.34	0.86	
Ireland	420	0.34	0.30	2.62	-14.79	15.45	0.35	0.32	2.42	-14.16	14.54	0.97	
Italy	393	0.42	0.38	2.51	-8.03	10.26	0.44	0.41	2.38	-11.27	8.62	0.94	
Japan	420	0.19	0.18	1.45	-7.76	5.30	0.24	0.23	1.33	-6.15	5.19	0.95	
Mexico	162	0.13	0.09	2.60	-7.25	5.76	0.12	0.09	2.38	-7.18	7.95	0.87	
Netherlands	420	0.22	0.20	1.93	-8.54	6.71	0.27	0.25	1.83	-8.22	5.21	0.97	
New Zealand	393	0.38	0.36	2.10	-8.40	7.28	0.40	0.38	2.04	-7.11	6.93	0.95	
Norway	373	0.29	0.27	1.96	-6.24	6.01	0.28	0.27	1.85	-6.48	5.50	0.97	
Poland	276	0.32	0.28	2.85	-11.06	12.51	0.31	0.27	2.92	-11.71	13.92	0.94	
Portugal	365	0.40	0.35	3.18	-13.23	14.98	0.45	0.40	3.16	-14.84	20.14	0.95	
Singapore	180	-0.01	-0.03	1.89	-6.13	4.94	0.15	0.13	1.90	-4.66	11.07	0.88	
South Korea	141	0.12	0.10	1.84	-4.75	6.05	0.14	0.12	1.72	-3.90	5.88	0.98	
Spain	397	0.37	0.34	2.48	-9.92	9.47	0.42	0.40	2.38	-9.76	9.65	0.96	
Sweden	420	0.35	0.32	2.30	-7.82	6.77	0.38	0.36	2.12	-8.48	5.62	0.98	
Switzerland	420	0.19	0.18	1.65	-4.86	5.88	0.23	0.21	1.53	-4.43	5.06	0.94	
United Kingdom	420	0.23	0.20	2.27	-10.62	8.23	0.24	0.22	2.16	-10.79	7.03	0.98	
United States	420	0.24	0.22	2.23	-7.34	11.71	0.23	0.20	2.20	-6.24	11.69	0.98	

## B Social Security benefits calculations

We compute Social Security benefit payments using the formulas effective in 2022. Given that Social Security formulas are inflation-indexed, all of our calculations use real 2022 dollars. As such, the amount subject to Social Security taxes for each year in the investor's working life is the minimum of annual income and \$147,000, which is the maximum taxable earnings. The average indexed monthly earnings (AIME) for each investor is the average of their highest 35 years of taxed earnings divided by 12. An investor who retires at the normal retirement age of 67 has a personal Social Security benefit according to a formula that sums 90% of AIME up to \$1,024, 32% of AIME between \$1,024 and \$6,172, and 15% of AIME in excess of \$6,172. This retirement benefit is reduced by  $(5/9)\%$  per month of early retirement up to 36 months and further reduced by  $(5/12)\%$  per month of early retirement between ages 62 (the earliest allowed retirement age) and 64 (normal retirement age minus 36 months). The retirement benefit increases by  $(2/3)\%$  per month of late retirement between ages 67 and 70 (the latest allowed retirement age).

Spouses may be eligible for additional benefits under the Social Security system. Some investors may optimally choose to take spousal and/or survivor benefits. When both members of the couple are living, each spouse has the option to take half of their spouse's personal retirement benefit as a spousal benefit in lieu of taking their own retirement benefit. This option becomes useful if one of the two household members earns substantially more than the other during their working years. The spousal benefit is reduced by  $(25/36)\%$  per month of early retirement up to 36 months and further reduced by  $(5/12)\%$  per month between ages 62 and 64. Upon the death of one spouse, the surviving spouse qualifies for a survivor benefit. The potential survivor benefit is the personal benefit of the deceased spouse if the surviving spouse is full retirement age. If not, the survivor benefit is reduced by up to 28.5% (if taken before age 60). The surviving spouse may take the larger of their personal retirement benefit and their survivor benefit.

Finally, the SSA administers the Supplemental Security Income (SSI) program, which provides payments to retirees (and certain other individuals) who have little income from other sources. SSI payments are reduced by earnings, and the maximum monthly benefit amounts in 2022 are \$1,261 for couples and \$841 for singles. We impose minimum monthly consumption levels of \$1,261 for couples and \$841 for singles to reflect SSI payments for investors who earn little enough during their working years that their retirement income from Social Security and savings falls below these levels. This modeling choice reflects the social safety net and avoids issues with computing utility when consumption levels are zero or low.

## C Supplemental results

This appendix contains supplemental empirical results.

### C.1 Household longevity

Table C.I summarizes the distribution of age at death in years conditional on survival to age 25 based on the SSA data and our simulation procedure described in Section 4. The table reports the mean, standard deviation, and distributional percentiles for age at death for the household, the female, and the male. The statistics for the household correspond to the age of the last survivor from the couple at death. The mean age of the last survivor at death is 87.6 years, and the median age is 88.9 years. There is, however, considerable uncertainty over longevity outcomes. The 5th percentile of age at death for the couple is 70.8 years, and the 95th percentile is 100.0 years. This uncertainty is an important feature to consider in assessing the ability of investment strategies to fund consumption through retirement. The last column of Table C.I reports the likelihood that a given investor type dies before reaching retirement age. There is a 19.5% (11.9%) chance that the male (female) dies before age 65, and there is a 2.3% chance that neither member of the couple survives into the retirement period.

### C.2 Conditional strategy performance

In Section 5.2.3, we show that the optimal fixed-weight strategy outperforms the four benchmark strategies in preserving capital during the retirement period (i.e., the optimal strategy leads to the lowest probability of financial ruin under the 4% withdrawal rule). The outperformance of the all-equity strategy in capital preservation during retirement challenges the traditional view that investors should diversify into bonds as they age. In this section, we further characterize this result by examining ruin probabilities conditional on couple and market outcomes. Figure C.1 plots conditional ruin probabilities for three benchmark strategies and the optimal strategy (bills, with their exceedingly high ruin probabilities, are omitted to enhance the readability of the figure). In each panel, we divide the 1,000,000 simulation draws into quintiles based on an outcome and plot the ruin probability within each quintile.

Panel A of Figure C.1 examines ruin probabilities in relation to couple longevity. Short-lived couples are unlikely to exhaust their savings, and the ruin probabilities range from 0.3% (optimal strategy) to 3.3% (domestic stocks). Differences across strategies are the most stark for long-lived couples. The optimal strategy produces a 14.9% probability of ruin, which is more than twice as high as the unconditional 7.0% probability. The other strategies fare much worse with high longevity, with ruin probabilities of 29.4% for domestic stocks, 32.2% for the balanced portfolio, and 41.2% for the TDF. These results emphasize that continuing to generate wealth throughout retirement is crucial when investors may have a very long retirement period. The poor unconditional performance of the TDF, which invests little in stocks during retirement, is partially attributable to its struggle to preserve capital for couples who live long lives.

Panel B conditions on the cumulative real domestic stock return in retirement. In the worst quintile of realized returns, the optimal strategy has a ruin probability of 18.9%, which far exceeds the unconditional probability. However, this all-equity strategy is still the safest when domestic stocks do poorly. The domestic stock strategy is hardest hit, naturally, with a 52.4% probability of ruin. The balanced portfolio and TDF also have high ruin probabilities of 45.9% and 39.6%. When real returns on domestic stocks are poor over long investment periods, bonds and bills also tend to have poor real returns. Thus, the QDIAs provide little shelter during the storm.

Panel C of Figure C.1 examines the role of inflation. If realized inflation during retirement is low, the strategies perform relatively well with ruin probabilities ranging from 1.1% (optimal)

to 6.0% (domestic stocks). If high inflation hits, the optimal strategy has a ruin probability of 15.3% versus 38.9% for domestic stocks, 51.1% for the balanced strategy, and 62.8% for the TDF. The low correlation between inflation and international stock returns over long horizons (as shown in Table I) implies that international stocks provide crucial diversification benefits in inflationary periods.

The benefits of international diversification depend on the correlation between returns on domestic stocks and international stocks. This correlation varies over time [e.g., Longin and Solnik (2001)], so the value of international diversification could also vary. Panel D studies strategy performance conditional on the realized correlation between domestic and international stocks during the couple's retirement period. The ruin probability of the optimal strategy is stable across the quintiles, ranging from 5.3% to 7.9%. Other strategies actually have more dependence, with higher ruin probabilities when the realized domestic-international correlation is low. The low correlations seem to be proxying for worse economic times and wars when domestic markets do relatively poorly, and high correlations tend to line up with better asset class returns. Whatever the underlying causes may be, the results in Panel D assuage concerns that a high correlation between domestic and international stocks will invalidate the optimal, all-equity strategy.

### **C.3 Endogenous retirement timing**

In Section 5.4.2, we consider the optimal fixed-weight asset allocation policy for households under endogenous retirement timing. Table C.II reports the estimated optimal retirement ages as a function of the household's current earnings, current retirement wealth, and expected Social Security benefit.

TABLE C.I: DISTRIBUTION OF AGE AT DEATH

The table summarizes the distribution of age at death in years conditional on survival to age 25 based on the actuarial life tables from the SSA. For each investor type (i.e., heterosexual couple, female, or male), the table reports the mean, standard deviation, and distribution percentiles of the age at death. The statistics for the couple correspond to the age of the last survivor at death. The last column in the table shows the likelihood of death prior to reaching retirement age at 65.

	Moments		Percentiles										$\mathbb{E}[\mathbb{1}\{T_{max} \leq 480\}]$		
	Mean	StDev	1%	5%	10%	25%	50%	75%	90%	95%	99%				
Investor															
Couple	87.6	9.1	59.3	70.8	76.1	83.0	88.9	93.8	97.7	100.0	104.5	0.023			
Female	81.9	13.9	36.1	54.1	62.7	75.5	84.9	91.5	96.3	98.9	103.8	0.119			
Male	77.2	15.3	30.6	46.0	55.9	68.9	80.6	88.3	93.5	96.2	101.1	0.195			

TABLE C.II: OPTIMAL RETIREMENT AGES

The table reports optimal retirement ages conditional on retirement balance level, income level, and Social Security level. At age 62, couples are divided into terciles in each of the three dimensions with independent sorts. The table shows the optimal retirement age for each of the 27 resulting couple types.

Income level	Social Security level		
	Low Social Security	Mid Social Security	High Social Security
Panel A: Low retirement account balance			
Low income	70	69	67
Mid income	70	69	67
High income	70	70	68
Panel B: Mid retirement account balance			
Low income	69	66	63
Mid income	69	67	65
High income	69	68	67
Panel C: High retirement account balance			
Low income	63	62	62
Mid income	65	64	62
High income	67	66	65



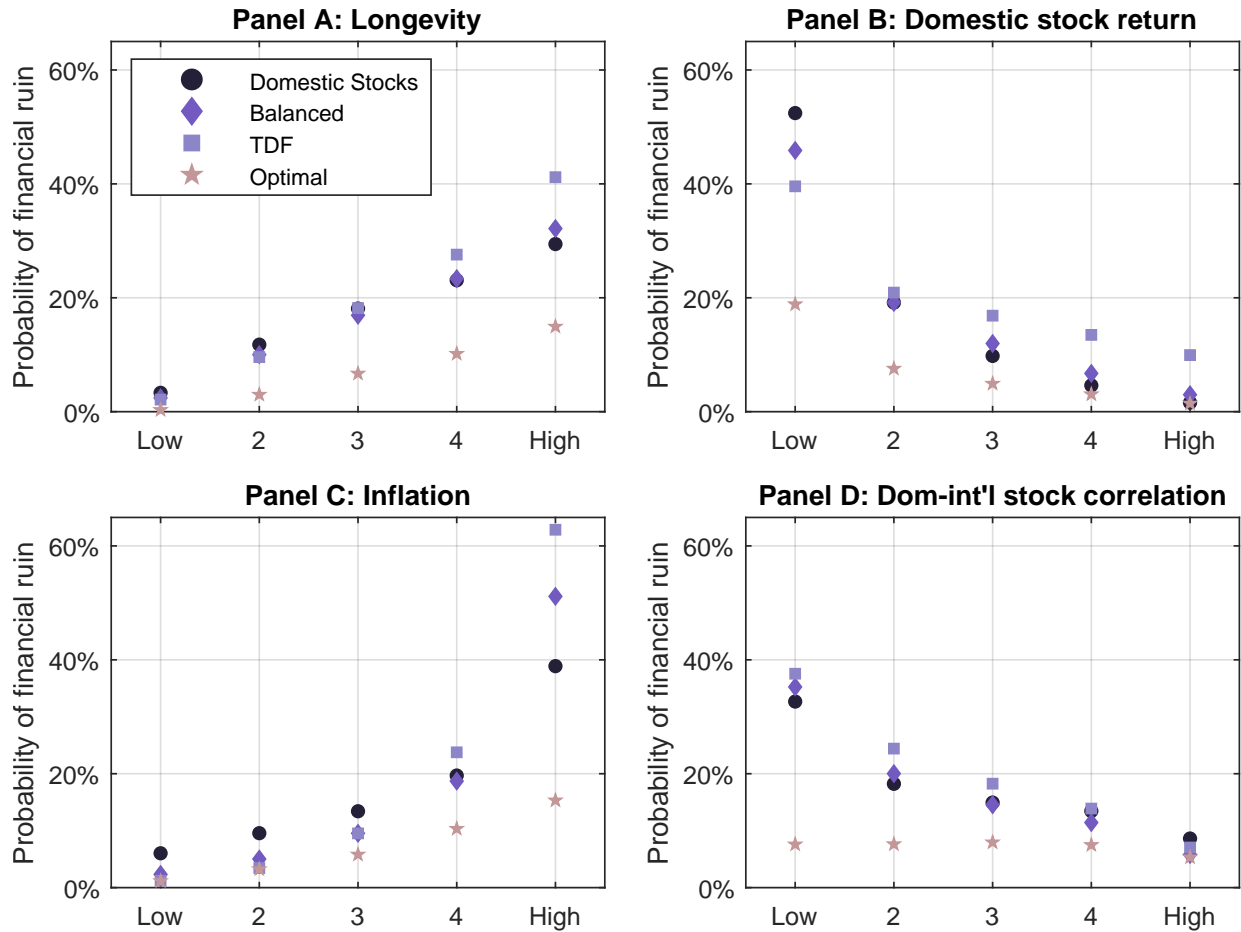


FIGURE C.1. CONDITIONAL RUIN PROBABILITIES. The figure shows the probability of financial ruin conditional on quintile outcomes of household longevity (Panel A), realized returns for domestic stocks (Panel B), realized inflation (Panel C), and realized correlation between real returns for domestic stocks and international stocks (Panel D) across 1,000,000 bootstrap simulations for households adopting various asset allocation strategies. The ruin probabilities in Panels B, C, and D condition on realizations during the retirement period.