Municipal Capital Structure*

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Abstract

Municipalities provide critical infrastructure and essential services financed largely by debt and taxes. We define municipal capital structure as the debt-to-investment ratio and develop a model of municipal capital structure that rests on two primary economic forces; the elasticity of the tax base with respect to taxes and service levels, and municipal financial distress. We show how these forces interact to determine the optimal capital structure in a way that depends on the legal structure governing municipal financial distress, on whether or not states are allowed to use bankruptcy law, and on the pro—creditor leaning of the courts. In addition, we show that municipalities that, for either political or behavioral reasons, operate to ensure debt repayment may decrease overall welfare.

1 Introduction

The critical importance of well-functioning public infrastructure and the provision of essential services is undeniable. In the US context, state and local governments are the primary owners and operators of these systems and are responsible for the majority of their investment requirements.[1] These expenditures are expected to increase even further, since legacy investments in many jurisdictions are in need of renewal or repair, while at the same time new social, technical and ecological imperatives necessitate design, construction and operation of new projects.[2]

Funding infrastructure spending is ultimately the responsibility of taxpayers, current and future, who largely employ a combination of tax and debt financing. Despite the economic importance of financing infrastructure, there is a dearth of research into this important decision. In this paper, we theoretically model optimal investment and financing decisions of "municipal corporations," typically cities, that are granted the authority to own and operate infrastructure as well as the responsibility to pay for it. We show how the risks associated with exogenous fluctuations in the municipality's tax base and the sensitivity of the tax base to infrastructure quality and tax rates factor into investment and financing decisions. We also study how the municipalities' decisions are related to the legal structures that govern repayment and remedies available in financial distress. Our analysis in particular provides insights into the workings of Chapter 9 of the US Bankruptcy Code and demonstrates the consequences of state-by-state variation in how bankruptcy is accessed and applied.

[1]Tomer, Kane, and George (2021) estimate that state and local governments account for three quarters of annual spending on public infrastructure. The US BEA reports 2019 state and local fixed asset investment of $431 B (U.S. Bureau of Economic Analysis (2021)). If state and local government public infrastructure was considered an industry, it would have ranked second in 2019 investment only to US manufacturing ($555 B, Table 3.7).

[2]In terms of physical infrastructure, The American Society of Civil Engineers 2021 Report Card for American Infrastructure forecasts 2020-2029 investment needs of $5.9 T. Traditional infrastructure, such as transportation, is responsible for a large share of spending but highlighting its future importance, a special note on broadband is included in the report card. Tomer, Kane, and George (2021) also comment extensively on the need for infrastructure that enables resilient, smart cities. Beyond these capital needs comes calls for improved social infrastructure in terms of education, inclusion, and social justice, all requiring municipal investment.

The fiscal history of Detroit, prior to and including its 2013 bankruptcy, dramatically illustrates possible negative outcomes that should be recognized and factored into municipal investment planning and financing. Infrastructure assets typically provide services to particular geographies and are therefore exposed to local economic fluctuations. Shocks to large employers or correlated shocks affecting many can give rise to a cycle of depopulation, failure of infrastructure, inadequate city services, and an inability to raise sufficient funds through taxation. Financial distress among the "big three" automakers, precipitated by the Great Recession of 2007-2009, ultimately led to a financial crisis for the city of Detroit, and on June 14, 2013 the city presented a Proposal to Creditors asking to reschedule debt payments.[3] The city argued that its debt burden along with underlying economic factors placed Detroit in default of cash flow obligations to its creditors as well as default of its service obligation to its citizens. The proposal notes the population of the city had declined by 26% since 2000 and that property tax revenues had shrunk by 20% over the previous five years despite imposing the highest tax burden in Michigan. Directly highlighting the impact on essential city services, the police department had seen a dramatic decline in manpower resulting in slow response times, low case clearing rates, and a high crime rate.[4] A shocking number of streetlights did not work (40%). In terms of the city's responsibilities for education, only 9% of 8th graders were at minimal reading levels compared to a national average of 35%. Deterioration of infrastructure had also contributed to out-migration and abandonment of houses; between 2009-2013, there were 75,000 house mortgage foreclosures, and the report notes 78,000 vacant and blighted structures.[5]

From a corporate finance perspective the Detroit bankruptcy illustrates a number of important questions that we address. What explains the city's choice of debt financing levels? Since there is no tax advantage for municipalities, what is the benefit of debt relative

[3] City of Detroit Proposal for Creditors, June 14, 2013.

[4] The report notes police manpower had fallen by 40% over the previous 10 years, response times averaged 58 minutes vs. a national average of 11 minutes, case clearing rates were 8.7% vs. 35% for Pittsburgh, and the crime rate was 5 times the national average. 40% of streetlights did not work.

[5] For further details on the Detroit and its bankruptcy see also Gilson, Mugford, and Lobb (2020).

to tax financing? What are the rules of municipal bankruptcy, how do they recursively impact on investment and debt levels, and how do they affect economic efficiency? Should municipalities be required to structure their finances to avoid financial distress? Should municipalities be allowed to access bankruptcy law in addition to contract law? Our theory may be viewed as a model of municipal capital structure, defined as the ratio of debt to investment,[6] that addresses these questions.

In doing so, we add to the traditional capital structure literature by recognizing that the municipal corporation is fundamentally different from a public corporation. For instance, while the market value maximization objective of a public corporation is well defined, there is no clear equivalent objective for a municipal corporation. Moreover, there is essentially no liquidity for a share of municipal ownership: A citizen who helped pay for infrastructure is not able to monetize the value of the asset they helped build if they are required to move or if the value they see in the infrastructure drops.[7] In addition, our theory recognizes that the process by which municipal debt contracts are enforced is fundamentally different from public corporations due to the quasi-sovereign nature of the municipality.

Based on an utilitarian objective function that recognizes these factors, we identify benefits of municipal debt that derive from efficiently sharing, through debt payments, infrastructure costs over time and across states. A key assumption of our analysis is that the tax base is "tax and service" elastic: The propensity of citizens to leave a municipality rises if taxes are increased or infrastructure deteriorates.[8] To build intuition for why tax base

[6]The traditional debt/equity or debt/value measures are conceptually defined but practically of little value for a municipality since the value of the underlying public assets, providing non market externalities, is difficult to measure.

[7]For instance, the value of high quality schools may be high while a taxpayer's children attend but may drop when they become empty-nesters.

[8]The importance of "tax base elasticity" is reflected in the proposal presented in City of Detroit (2013) to its creditors where a key stated objective of the restructuring is to "Provide incentives (and eliminate disincentives) for businesses and residents to locate and/or remain in the City.

- The City cannot stabilize or pay creditors meaningful recoveries if it continues to shrink.
- Achieving this goal requires improvements in City services, particularly in the area of public safety and tax reform to reduce the cost of living in the City to more closely approximate costs of living in nearby areas.

More generally Tiebout (1956) argued that municipalities compete for citizens who 'vote with their feet' for elasticity matters, consider a municipality constructing irreversible infrastructure today that will benefit its citizens today and in some distant tomorrow. If the infrastructure is paid exclusively by levying high taxes today many citizens will leave (i.e., the tax base will decrease) thus necessitating higher taxes or lower service quality, both of which will induce even further emigration. In future years, conversely, the infrastructure will provide services that have already been paid for, allowing lower taxes and a population rebound. If instead the municipality mixes taxes and borrowing to put the infrastructure in place, the fluctuations in the tax burden and migration will be dampened as debt issuance today will reduce current taxes but debt repayment tomorrow will require higher taxes.

Although we assume all agents are risk neutral, we find that the city enjoys non-linear benefits from sharing tax revenue risk with debt holders. Concavity in municipality objective functions results from the tax/service elasticity of the tax base when welfare accounts for the number of people who enjoy public infrastructure, the quality of that infrastructure, and the taxes that must be levied to pay for the infrastructure. At the optimal financing structure, therefore, the city will smooth payment for infrastructure over time and across states of the world to equate marginal tax burdens.

Is the tax smoothing benefit of debt modified by the institutional environment in which municipal financial distress is resolved? Understanding municipal financial distress involves more than a reinterpretation of existing models, both because a municipality is fundamentally different from a public corporation, as discussed above, and because a municipality legally has a degree of sovereignty requiring a different legal apparatus to resolve financial distress. At the same time, Municipalities do not have the degree of sovereignty needed to repudiate its debt, as is central to the sovereign debt literature.

From a legal perspective two bodies of law are involved in resolving financial distress for both municipal and public corporations; contract law and bankruptcy law. Contract law provides a process for assessing the legitimacy of a creditor's claim, determining a remedy and

the municipal service bundle they wish to acquire through their taxes. See Saltz and Capener (2016) for a survey.

employing the power of the state to enforce the remedy. Bankruptcy law is a mechanism that can impose a stay of contract law in order to allow the debtor to propose a reorganization.

For municipalities, both bodies of law are constrained by the quasi-sovereign nature of a municipality. In terms of contract law, the sovereign nature of municipalities means that property owned by the debtor cannot generally be seized nor can the court dictate operating decisions as it can for a public corporation.[9] Allowing either seizure or operating interference could be viewed as an imposition on the ability of elected representatives to govern as they see fit. As a result, the actions available to the creditor of a municipality under contract law are constrained. In terms of bankruptcy law, in the US this is governed by a federal law. However, allowing municipalities unencumbered access to federal law was, in the early incarnations of Chapter 9, seen as an infringement on a state's responsibility to govern the citizens of the state. As a result and unlike public corporations, a municipal debtor must have the permission of the state to utilize bankruptcy law.

Our work adds to the capital structure literature by recognizing the special nature of municipalities and the special rules around debt enforcement for municipalities. At the core of our model is the importance of net tax base migration to the riskiness of municipal debt combined with the specifics of bankruptcy law. Tiebout (1956) first introduced the idea of intercity competition for the tax base and the affect of this competition on intercity migration. This was followed by a large literature examining this force in detail (see the survey of Saltz and Capener (2016).). This literature does not consider the use and riskiness of municipal debt or the debt enforcement mechanism.

There are relatively few theoretical studies of municipal debt financing with default. Gordon, Guerrón-Quintana, et al. (2021) is the only work we are aware of that relates municipal financing and default to a somewhat endogenous tax base. While Gordon and Guerron provide greater detail on the migration decision, they employ a very simple default mechanism, similar to that employed in the soveriegn debt literature; municipalities can

[9] We realize that there are work around tactics; Detroit was not able to sell its art gallery but was able to monetize it. However, even when possible, seizure is difficult. See Skeel Jr (2015).

decide to repudiate their debt without making any payments and then reissuing debt after a delay. While intersting, this is not consistent with actual municipal bankruptcy and contract law since municipalities are not allowed to repudiate debt nor do creditors recieve zero in default. We explicitly identify the value of debt to municipalities and show how contract law and bankruptcy law allow courts to determine state contingent debt reorganizations and hence marginal economic efficiency.

Myers (2021) considers a model of municipal default that is the result of risky exogenous revenues. In his model there is no tax base migration, and the focus is on a game between government and taxpayers where governments realize overspending may generate a future tax payer bailout. In contrast the risk in our model is due to shocks in the tax base of the municipality and our focus is on bankruptcy laws as opposed to emergency bailouts.

In section 2 we review the relevant institutional details involved. Section 3 presents the analytical model that we use to capture this setting. We present basic results in Section 4 and conclude the paper in section 5.

2 Institutional Setting

There are two important institutions represented in our model, the municipality and the court. In this section we sketch out some of the essential features of these institutions and the assumptions we make to capture them.

2.1 Municipal Corporation

A municipal corporation is established to provide basic services to those who live within a particular geographic area. A municipal corporation is established through state or provincial incorporation that grants corporate status along with a municipal charter that defines the rights, responsibilities, and governance of the municipality. Clearly the political economy underlying municipalities is complex and interesting in many ways. To focus on the finance components, however, we greatly simplify by assuming that decisions are made by a benevolent *mayor* who has the power to invest in infrastructure, is able to compel citizens to pay taxes, and is able to establish municipal debt liabilities in order to finance municipal investment.

In reality, an active player in the governance of the municipality is the state that, in addition to granting corporate status, also monitors the municipality and has considerable power to intervene in the event of municipal fraud or mismanagement.[10] Indeed, an important decision of the state is whether or not it will allow a municipality to access the relevant part of the bankruptcy code to resolve financial distress. We further simplify our model by assuming there is no principal agent conflict between the mayor and the state, so that monitoring is not an issue. To study the gate keeping role of the state with respect to the bankruptcy code, we do not explicitly model the state as a strategic agent but instead consider games where the municipality can choose to apply for bankruptcy protection and games where they are prohibited from doing so.

2.1.1 Municipal Debt

Municipalities generally have the ability to issue two forms of debt, municipal bonds and pension liabilities. Pension liabilities are in practice very important and economically interesting.[11] In order to simplify our analysis, however, we assume that the municipality does not incur separate pension liabilities nor does it obtain separate pension assets. While this simplification eliminates an important financing consideration, it also allows us to more clearly set out the essence of the capital structure decision.

There are two main types of municipal bonds, revenue bonds and general obligation or GO bonds. Some municipal assets, such as toll bridges, generate cash flows that can be pledged in a debt contract. Such debt contracts are referred to as Revenue Bonds and, although

[10]See Moringiello (2017) for a detailed discussion of the states role in municipal bankruptcy.

[11]See Myers for an interesting analysis of municipal pension liabilities. Also, a more complete model of the role of pension debt in financial distress can be found in Carlson et al 2022. However, neither paper examines optimal infrastructure investment or capital structure decisions.

interesting, do not raise the novel issues that GO bonds do. Hence we only consider GO bonds as they are more distinct from the standard debt of public corporations. GO bonds are not backed by a particular revenue stream or asset and are often said to be backed by the 'good faith and credit' of the citizens of the municipality. Essentially, this implies that the bonds are backed by the potential taxes collected.

2.2 Financial Distress and the Courts

In common with public corporations, municipal financial distress can be evidenced by the inability of the debtor corporation to make required debt payments as they come due. In addition, however, municipalities may be 'service insolvent,' defined as ' a significant reduction in the availability of city services' (Gillette (2019)). This is in sharp contrast to public corporation financial distress where the quality of the product provided is not a consideration apart from its cash flow or regulatory implications.

The environment under which municipal restructuring takes place varies widely and includes the following[12]:

1. **Informal restructuring**, where all claimants to the municipality agree to alter the nature of their claims. For example, the city of Fitch Texas announced that it was unable to meet debt obligations due to what was later shown to be fraud. It subsequently announced a mutually agreed upon extension of its debt .

2. **State intervention:** The state may provide emergency funding, technical advice and appoint an emergency manager who has the power to make operating decisions and renegotiate the municipality's obligations.

3. **Contract court** where debt holders' petition the court to help them collect as much as possible from the creditor.[13] In sharp contrast to public corporations, the court

[12] For an excellent overview of the legal environment see Frost (2014).

[13] For public corporations this involves seizing and liquidating corporate assets. For municipalities, seizure is not generally available and only applies to assets that can be legally pledged. See Skeel Jr (2015).

is not able to interfere with the operations of the municipality. An aspect of debt collection law that is very distinct for municipalities is the limited ability to require the municipality to increase taxes. In a chapter 11 filing, although a judge is not able to require that the company increase prices for its products, it can appoint a trustee to do so. For municipalities, such interference in the operations of the municipality is considered a breach of the municipality's and the state's sovereignty. The court is able, however, to issue a *writ of mandamus* directing an officer of the city to increase taxes. The effectiveness of this is dampened by the fact that the officer need not comply with the writ if prohibited to by state law. Moreover, the officer to whom the writ is directed may also resign from the position, making the writ ineffective and requiring the issue of a new writ.

Despite the somewhat imperfect mechanism available to contract courts, we assume that the court has limited ability to set terms of the restructuring. Specifically, in our model, we will assume that the court is able to enforce a repayment amount that is the most that can be repaid while still meeting the minimum service requirement.

4. **Bankruptcy court:** In the US this involves Chapter 9 of the bankruptcy code as discussed below.

2.3 Chapter 9 versus Chapter 11

One of the primary purposes of Chapter 11 is to solve the so called "common pool" problem that arises due to economies of scope, where the value of assets are worth more together than they are separately. The common pool problem arises when various creditors can seize specific collateral through contract court despite the impact this would have on the value of other claims. Bankruptcy law is a solution to the problem in that it provides a stay of legal actions against a debtor, so that no assets can be seized, while a reorganization (chapter 11) or liquidation (chapter 7) is contemplated. Since municipal assets cannot generally be seized, however, the common pool problem is not an issue. Instead, chapter 9 is intended to facilitate an adjustment to the debt outstanding while balancing a responsibility to act in the best interests of the creditors with the responsibility of honoring the quasi-sovereign nature of the municipality and its obligation to provide services to its citizens.

Chapter 9 enhances the bargaining power of the creditor relative to Chapter 11 in a number of ways. As stated, municipal assets can not be seized or redeployed through the court. Moreover, while both Chapter 11 and 9 allow the creditor the exclusive right to present a proposal to the court, the exclusivity period in Chapter 11 is 90 days where as it is indefinite in Chapter 9 - creditors are never able to present a proposal to the court. In addition, unlike chapter 11, the court is not able to direct the activity of the debtor during the bankruptcy process and, hence, has less direct impact on the reorganization.

The court does, however, have two important controls in the case of Chapter 9; the ability to allow a petition to be heard by the court (admission control) and the ability to confirm a proposed reorganization (exit control). If the court does not allow a case to be heard or if it refuses to confirm a proposal, the case is adjudicated through contract law.

In terms of admission to the bankruptcy process, a municipality is considered eligible for chapter 9 if: a) it is insolvent, either because it is not able to make debt payments as they come due or it is not able to provide a minimum level of service to its citizens;[14] b) it has attempted to negotiate with its creditors but has failed to reach an agreement; and c) the state has given the municipality permission to file for chapter 9 protection. In terms of exit the court will confirm a proposal if a) it is feasible in that the proposal is expected to meet budget and minimum service constraints, and b) it is a 'good faith offer' that is in the 'best interests' of the creditors. The terms good faith and best interests are not given a precise meaning in law.[15]

Although all of these responses are in principle possible, what municipalities can actually do is governed by State law. For instance, while the federal bankruptcy code allows

[14]We provide precise model based definitions of these condition in section 3.
[15]We provide a model specific definition of these terms in section 3.

municipalities to petition the court under Chapter 9, it also states that this requires that the state first give its permission to do so. According to the Pew Charities Study, only 21 states provide blanket authorization to apply for bankruptcy protection, another 12 allow conditional or limited filing and 10 have an outright prohibition on filing for bankruptcy.

Similarly, there is considerable variation in state intervention and, in the event of intervention, in what the intervenor is allowed to alter. Table 1, based on Gao, Lee, and Murphy (2019) and Pew Charitable Trusts (2013) illustrates the differences across states.

3 Model

We assume the formation of a municipal corporation, which we will refer to simply as the municipality, created by state law and governed by a mayor.

3.1 Agents

The municipality in our model interacts with four groups of agents; citizens or the tax base (N), the mayor (M), a bond holder (B),[16] and a court (C). All agents are risk neutral and the discount rate is zero. Nature determines the only exogenous risk in our model by selecting a state contingent population shock ϵ^i, where $i \in \{+, -\}$ is the state of the world revealed to all parties at $t = 1$ and realized at $t = 2$. For convenience, assume $\epsilon^- < 0 < \epsilon^+$ and $|\epsilon^-| = |\epsilon^+|$. Let p denote the probability of $i = +$, hence, $p > .5$ implies a municipality that is expected to grow.

We examine decisions taken at three points in time, $t \in \{0, 1, 2\}$, spanning two periods. At $t = 0$ the municipality is founded through operating, investment and financing decisions, which attract an initial population to the city. At $t = 1$ information arrives about a population shock and, based on the information, renegotiation of the issued debt takes place, but

[16] We recognize that municipal debt is often widely held. We assume the existence of a distressed debt investor who acquires a sufficient toehold to justify representing all debt holders. Our bondholder can be thought of as the default insurer or some other large investor who internalizes the bargaining externalities available in financial distress.

Table 1: State Financial Distress Environments

		State Can Restructure		
State	Bankruptcy Authorization	Debt Contracts	Labor Contracts	Taxes
Alabama	Yes (bonds only)			
Alaska	No			
Arizona	Yes			
Arkansas	Yes			
California	Conditional			
Colorado	Limited			
Connecticut	Conditional	Yes	Yes	Yes
District of Columbia	?	No	No	No
Deleware	No			
Florida	Conditional	No	No	No
Georgia	No			
Hawaii	No			
Idaho	Yes	No	No	No
Iowa	No (with exception)			
Illinois	?	Yes	Yes	Yes
Kansas	No			
Kentucky	Conditional	Yes	No	No
Louisiana	Conditional			
Main	No	Yes	No	Yes
Maryland	No			
Massachusetts	No	Yes	No	Yes
Michigan	Conditional			
Minnesota	Yes	Yes	Yes	No
Mississippi	No			
Missouri	Yes			
Montana	Yes			
Nebraska	Yes			
Nevada	No	Yes	Yes	Yes
New Hampshire	No			
New Jersey	Conditional	Yes	No	Yes
New Mexico	No			
New York	Conditional	Yes	Yes	No
North Carolina	Conditional	Yes	No	Yes
North Dakota	No			
Ohio	Conditional	Yes	No	No
Oklahoma	No			
Oregon	Limited	Yes	No	No
Pennsylania	Conditional	Yes	Yes	Yes
Rhode Island	Conditional	Yes	No	Yes
South Carolina	Yes			
South Dakota	No			
Tennessee	No	Yes	No	Yes
Texas	Yes			
Utah	No			
Vermont	No			
Virginia	No			
Washington	Yes			
West Virginia	No			
Wisconsin	No			
Wyoming	No			

Notes: This table shows states in which access to Chapter 9 is permitted.

no operating decisions are made. Finally at $t = 2$ the court rules on any petitions presented to it, after which final operating decisions are made. The structure of our model is depicted in Figure 1.

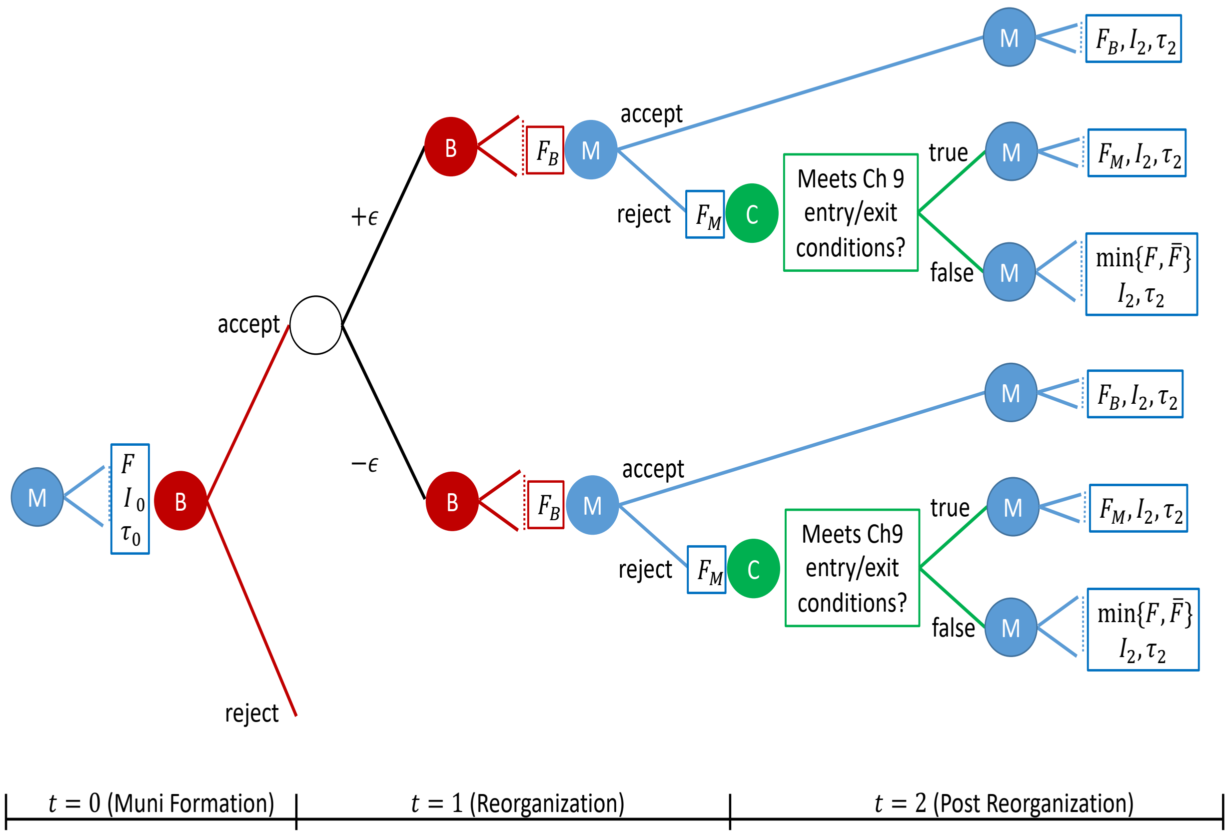

Figure 1: **Game Structure**

3.1.1 The Municipality

At $t = 0$ the municipality is founded with a governance structure empowering a mayor with investment and taxing authority.

At $t = 0$ the mayor installs municicial infrastructure with an investment of I_0. Let the replacement cost of municipal infrastructure at $t = 0$ be denoted by A_0 and assume $A_0 = I_0$. Let $A_1 = A_0$ and

$$A_2^i = (1 - \delta)A_0 + I_2^i$$

where δ is exogenous depreciation and $I_2^i \geq -(1-\delta)A_1$ is incremental investment ($I_2^i \geq 0$), or disinvestment ($I_2^i < 0$). We further assume that disinvestment generates a positive cash flow to the municipality of $-I_2^i$ but also involves a deadweight decomissioning cost of γI_2^i. Hence, the dead weight cost of disinvestment is $(\Gamma(I_2^i) \times I_2^i)$ where

$$\Gamma(I_2^i) = \begin{cases} 0 & \text{if } I_2^i \geq 0 \\ \gamma & \text{if } I_2^i < 0. \end{cases}$$

The parameter γ can be thought of as the degree of partial irreversibility of the infrastructure.[17]

The municipal charter also allows the mayor to impose a tax on each resident[18] at $t = 0$ and $t = 2$ of $\tau_0 \geq 0$ and $\tau_2^i \geq 0$, respectively.

3.1.2 Citizens/tax base

The municipality's residents enjoy utility from unmodelled private consumption as well as the consumption of the modelled public infrastructure.We assume each person's utility from consumption of infrastructure, net of the tax disutility, is additively separable from private consumption and is given by,

$$u = q - \tau$$

where

$$q = \beta \times A, \quad \beta > 0$$

is the service each individual enjoys from the infrastructure. Each resident of the municipality must either pay taxes or move to another municipality and does so based on whether or not u_0 and u_2^i, net of taxes, meet some unmodelled heterogeneous participation constraint.

[17] We have considered the case of irreversible investment and omit it from our discussion here simply to focus on our main issues.

[18] This includes all taxes under the municipality's control. For example, municipalities are able to impose some or all of property tax, sales tax, income tax, hotel taxes, etc., sometimes with self imposed restrictions. We treat these as one form of taxation.

Incorporating these factors, we model the tax base at $t = 0$ as

$$N_0 = a + bq_0 - c\tau_0. \tag{1}$$

Similarly, the tax base at $t = 2$ is

$$N_2^i = a + bq_2^i - c\tau_2^i + \epsilon^i. \tag{2}$$

Therefore, the aggregate tax revenue collected at $t = 0$ and $t = 2$ is $\tau_0 N_0$, and $\tau_2^i N_2^i$, respectively.

3.1.3 General Obligation Bonds

The mayor's authority also allows her to determine investment amounts and the extent to which the investment is financed by GO bonds, as described in section (2.1.1).

Debt contracts are characterized by a promised single contractual face value $\tilde{F}_t$ payable at time $t = 2$. The contractual amount begins at a value of $\tilde{F}_0 = F$ when the municpality issues the bond. Between $t = 0$ and $t = 2$ $\tilde{F}_t$ evolves through renegotiation and the court process, as described below. The final value of $\tilde{F}_2$ is enforced by the court and results in a payment to the bondholders of $\tilde{D}_2 = \tilde{F}_2$. This equality reflects the assumption that, at the maturity of the bond, the court ensures that the final contractual amount, $\tilde{F}_2$ is actually collected from the municipality.

3.1.4 Bond Holder

The bond holder is assumed to be competitive in the sense of having unlimited funds and being willing to acquire any asset that provides at least an expected return of zero.

At $t = 0$, as stated in the previous section, the bond holder is offered a bond with face value F and an asking price of D_0 and either accepts or rejects the offer. In equilibrium, the contract will be accepted and

$$D_0 = E_0(D_2^i).$$

At $t = 1$ the debt holder rationally anticipates how the debt enforcement game will be played and proposes a new face value of F_B[19] that maximizes $E_1(D_2^i)$, based on rational anticipation of the debt enforcement game.

At $t = 2$ D_2^i is received from the municipality and no further actions are taken by the bond holder.

3.1.5 The Mayor

Objective

The mayor maximizes the sum of the citizens' single period utility flows. The first period welfare flow is defined by

$$W_0(\tau_0, q_0) = N_0(q_0 - \tau_0) = (a + bq_0 - c\tau_0)(q_0 - \tau_0), \tag{3}$$

while the state-contingent welfare at time $t = 2$ is given by

$$W_2(\tau_2^i, q_2^i, \epsilon^i) = N_2^i(q_2^i - \tau_2^i) = (a + \epsilon^i + bq_2^i - c\tau_2^i)(q_2^i - \tau_2^i). \tag{4}$$

At $t = 0$ the mayor maximizies

$$V_0 = W_0(\tau_0, q_0) + E_0(W_2(\tau_2^i, q_2^i, \epsilon^i)) \tag{5}$$

while at $t = 2$ the mayor maximizes

$$V_2^i = W_2(\tau_2^i, q_2^i, \tilde{\epsilon}). \tag{6}$$

[19]No change in the bond would involve an offer of $F_B = F$.

Actions

At $t = 0$ the mayor determines the size of the initial investment, I_0, and finances this with debt and taxes. To debt finance, the mayor offers a debt contract with face value F to bondholders at a price of D_0. If the offer is rejected, the game ends. If the offer is accepted it becomes the contractually owed amount $\tilde{F}_0$, the mayor constructs the infrastructure and imposes a per person tax rate of τ_0 on all citizens, thereby raising aggregate tax revenue of $N_0\tau_0$.

At $t = 1$ the mayor must respond to the bondholder's proposal of F_B. We assume the mayor must either accept the offer or file a Chapter 9 petition asking the court to adjust the promised debt payment to F_M.

At $t = 2$ the court rules on any petitions that have been filed and the mayor makes the debt payment D_2^i that the court mandates.[20] The mayor then selects I_2^i and τ_2^i honoring the mandated payment.

Contraints

The mayor's choices at $t = 0$ and $t = 2$ are constrained by an exogenously imposed *minimum service* requirement, effectively a lower bound on q, denoted by q_L, with an implied lower bound $A_0, A_2^i \geq q_L/\beta$. Alternatively, the constraints can be cast in terms of minimum investment amounts with a $t = 0$ constraint of

$$I_0 \geq \frac{q_L}{\beta} \tag{7}$$

and a $t = 2$ constraint of

$$I_2^i \geq \frac{q_L}{\beta} - (1-\delta)A_0. \tag{8}$$

The mayor must satisfy budget constraints at both $t = 0$ and $t = 2$. The $t = 0$ budget constraint is

$$D_0 + N_0\tau_0 = I_0, \tag{9}$$

[20]The court process that determines D_2^i is set out in section 3.2

while the $t = 2$ budget constraint is

$$N_2\tau_2 = (1 - \Gamma(I_2))I_2 + D_2. \tag{10}$$

3.2 Debt Enforcement

Debt enforcement begins at $t = 1$ with the revelation of ϵ^i, which is a shock to the tax base that will take place at $t = 2$. We assume that, based on the information about ϵ^i, the bond holder proposes an adjustment of the face value from F to F_B. The mayor moves next by either accepting the adjustment, in which case a new contract replaces the existing contract, i.e., $\tilde{F}_1 = F_B$, or rejecting the proposal by filing a petition with the court to confirm a new contract with a face value of F_M.

If at $t = 2$ the mayor has accepted B's proposed adjustment, then $\tilde{F}_2 = F_B$. Alternatively, the mayor has rejected the offer and filed a petition with the court to confirm that $\tilde{F} = F_M$. The judge first makes an admission decision, which either allows the petition to be adjudicated under Bankruptcy Law, or dismisses the petition. If dismissed, the contract is adjudicated under contract law. If admitted, the proposed face value adjustment F_M is considered under bankruptcy law. The judge's exit decision is either to confirm the proposed adjustment or to reject it and impose the adjustment consistent with contract law. More specifically the court follows the following admission and exit decisions.

3.2.1 Admission condition details

The court uses bankruptcy law to consider the mayor's proposal if it finds that the municipality is *insolvent*,[21] that is if there is no tax rate that would allow repayment of $\tilde{F}_1$ as well as achievement of the minimum service level. Accordingly, to make a ruling the court first computes the maximum payment that could be made to an outside claimant in the current

[21] In law a distinction is made between cash flow and service insolvency. Economically, these are not separable. cite a reference to this

state (i.e. either the growth and decline state):[22]

$$\bar{F}^{+} = \max_{I,\tau} \tau(a + \epsilon^{+} + b(A_0(1-\delta) + I) - c\tau) - (1 - \Gamma(I))I \tag{11}$$

$$\bar{F}^{-} = \max_{I,\tau} \tau(a + \epsilon^{-} + b(A_0(1-\delta) + I) - c\tau) - (1 - \Gamma(I))I \tag{12}$$

both subject to

$$I \geq A_L - (1-\delta)A_0.$$

In state i, the court will rule that the firm is insolvent and therefore consider the proposal under bankruptcy law if $\tilde{F}_1 \geq \bar{F}^i$.

Given the enforcement rules we have adopted, we can, without loss of generality, require that $F, F_B, F_M \leq \bar{F}^{+}$.[23] Hence, bankruptcy law is only relevant for a city in decline.

3.2.2 Exit condition details

If the petition is considered under bankruptcy law, the proposed contract is confirmed if the court rules that the proposal, F_M, is feasible and is made in "good faith." It is feasible if the municipality is able to pay F_M and provide a quality level of at least q_L. A contract is considered to be made in good faith if it provides a minimum acceptable payment as determined by the court.

While we are not aware of a theoretical basis that determines a good faith offer, we assume the judge uses a weighted average of the mayor's best possible contract and the bondholders' best possible contract. The best outcome the mayor can hope for is that the new face value would be 0. The best outcome the bondholder could expect is $\bar{F}^i$. To satisfy the exit condition, the court will therefore confirm any F_M satisfying

$$F_M \geq \pi \bar{F}^i \tag{13}$$

[22]See Appendix A for closed form solutions to the following optimizations.

[23]We discuss this further in Appendix B

where $\pi \leq 1$ is exogenous and represents the degree to which debtor interests are factored into the court's good faith requirement.

If the court rejects M's petition, it then applies contract law to the dispute by requiring that $\tilde{F}_2 = \bar{F}^i$.

3.2.3 Debt Enforcement Summary

In summary, contract enforcement will result in:

- $\tilde{F}_2 = F_B$, if B's offer is accepted by the mayor.
- $\tilde{F}_2 = F_M$, if B's offer is rejected by the mayor and the court confirms the mayor's proposal under bankruptcy law.
- $\tilde{F}_2 = \min\{F, \bar{F}^i\}$ if the court rejects the mayor's petition and uses contract law to resolve the dispute.
- In all cases, the court ensures that the municipality pays the bondholder $D_2^i = \tilde{F}_2$.

3.3 Equilibrium

We examine subgame perfect equilbria by solving the game recursively, and we focus only on pure strategies. Beginning with the final decision, the mayor honours the court's determination and accordingly makes payment $D_2^i = \tilde{F}_2$. Constrained by the repayment obligation, the mayor optimally selects I_2^i and τ_2^i.

Prior to the mayor's final choices the court acts as a strategic dummy that follows the rules set out above. It turns out that, although there are many possible equilibrium offer/counter offer strategies, due to the assumed behavior of the court and the fact that all agents have full information, for a given enforcement structure (i.e. q_L and π), all strategies will lead to the same D_2^i. Consequently, for each enforcement structure we will only discuss one set of equilibrium strategies.

Prior to the court's rulings, the mayor either accepts B's offer or proposes F_M. Consider first the equilibrium if the mayor rejects F_B and counters with F_M. If the municipality is solvent the court will not allow the case to be heard under bankruptcy law and will enforce $\tilde{F}_2 = F$ under contract law. Hence, if solvent, the mayor will offer $F_M = F$. If insolvent, for any $F_M < \pi \bar{F}_2^i$ the court will reject the petition and, under contract law, impose $\min\{F, \bar{F}^i\} \geq \pi \bar{F}_2^i$. Since the welfare of the municipality is decreasing in D_2^i, the mayor will offer $F_M = \pi \bar{F}_2^i$.

Next consider the strategy of accepting F_B. As we have just seen, if solvent, rejecting leads to a payment of $D_2^i = F$. Hence, the mayor will only accept $F_B \leq F$. If insolvent, the mayor realizes rejection leads to $\pi \bar{F}_2^i$ and hence will only accept an offer of $F_B \leq \pi \bar{F}_2^i$.

Now consider B's offer of F_B, based on the knowledge of ϵ^i. Understanding the mayor and the court's responses, B will maximize $E_1(D_2^i)$ by offering $F_B = F$ to a solvent municipality and $\pi \bar{F}_2^i$ to an insolvent mayor as all other offers would be rejected.

At $t = 0$ B must either accept or reject the mayor's debt offer of $\tilde{F}_0 = F$ at a price of D_0. B will accept this offer if

$$D_0 \leq E_0(D_2^i).$$

It is clear that, for any $\tilde{F}_0 = F$ the mayor will set $D_0 = E_0(D_2^i)$.

Finally, the game begins with the mayor selecting $\tilde{F}_0, I_0$, and τ_0 in order to maximize (5), based on rational expectation of all the above

In Appendix C we set out necessary parameter restrictions to ensure existence of an equilbrium. Our analytic analysis in Appendix D provides exptessions for τ_0, τ_2^i, F and D_0/I_0 *for any investment policy.* However, we requrie numerical methods to solve for the full problem, including the optimal investment policy. We provide analytical details on the equilibrium choices of the players in Appendix D and characterize these choices numerically in section 4.

4 Model Solutions

In order to provide a benchmark for our analysis, we begin in section 4.1 by characterizing the municipality under the assumption that the Mayor has access to complete capital markets. That is, rather than solving the recursive game above, the mayor selects state contingent values of I_t and τ_t subject to the constraint that total *expected* tax revenue is equal to total expected infrastructure expenditures. This solution implicitly involves pure securities that allow funds to be transferred from one state to the other in satisfying the budget constraint.

Using the complete markets solution as a benchmark we then proceed to consider the more realistic case where the mayor is only able to issue regular debt contracts. In section (4.2) we examine the case where mayors have access to bankruptcy law followed in section (4.3) where they do not. Finally in section (4.4) we consider the possibility that, for political or career reasons, the mayor chooses to finance infrastructure with safe debt only.

Our numerical model is based on parameter assumptions contained in Table 2.

Table 2: Model Parameters

Parameter	Value	Description
p	0.9	Probability of $+\epsilon$
ϵ	25.0	Economic shock
a	100.0	Population base
b	1.0	Quality sensitivity
c	100.0	Tax sensitivity
β	0.1	Public good utility (per unit q)
γ	0.5	Decomissioning cost (%)
δ	0.1	Public good depreciation
q_L	2.0	Minimum standard of public good
π	0.5	Bondholder recovery (% of $\overline{F}$)

Notes: This table reports the model parameters used in the numerical solutions.

4.1 The Mayor's First Best Solution

The mayor's problem in the complete markets case is given by equation (14) subject to the budget constraint (15) and the minimum service constraints (7) and (8).

$$\max_{\{I_0, I_2^+, I_2^-, \tau_0, \tau_2^+, \tau_2^-\}} V_0 = W_0(q_0, \tau_0) + pW_2(I_2^+, \tau_2^+, +\epsilon) + (1-p)W_2(I_2^-, \tau_2^-, -\epsilon) \tag{14}$$

s.t.

$$N_0\tau_0 + pN_1^+\tau_2^+ + (1-p)N_1^-\tau_2^- = I_0 + pI_2^+ + (1-p)I_2^-, \tag{15}$$

where $N_2^+ = a + \epsilon + bq_2^+ - c\tau_t^+$ and $N_2^- = a + \epsilon + bq_2^- - c\tau_t^-$.

The solution to this problem requires the solution to a system of six non-linear first order conditions and the budget and minimum qualilty constraints. Although we are not able to analytically solve for all six choice variables, we are able to obtain insight into the general municipal capital structure decision by fixing I_0, I_2^+ and I_2^-, which effectively also fixes q_0, q_2^+, and q_2^-, and optimally picking τ_0, τ_2^+ and τ_2^-. This delivers what might be thought of as the municipal debt Euler equations.[24]

$$\frac{\text{MTR}_2^+}{\text{MTR}_0} = \frac{q_2^+}{q_0} \tag{16}$$

$$\frac{\text{MTR}_2^-}{\text{MTR}_0} = \frac{q_2^-}{q_0} \tag{17}$$

$$\frac{\text{MTR}_2^+}{\text{MTR}_2^-} = \frac{q_2^+}{q_2^-} \tag{18}$$

where $MTR_i = \frac{d}{d\tau_i}N_i\tau_i$.

These Euler equations illustrate the trade off inherent in a municipality's capital structure decision and the fundamental difference between the capital structure decision of a public corporation relative to a municipal corporation. Public corporation capital structure theory shows that a given level of real investment is financed in a way that balances the marginal

[24] See appendix (D) for details.

tax advantage of debt against the marginal bankruptcy costs. For a municipality, also taking investment as fixed, we see a fundamentally different trade off where debt and the tax rates are used to balance the marginal tax revenues over time and across states. That is, municipalities have an interior capital structure even under the assumption that there is no tax advantage or bankruptcy costs associated with debt.

Table (3) contains a numerical solution for all choice variables in the complete markets case.[25] At $t = 0$ the mayor puts in place infrastruture with a replacement value of 63.3, a quality level of $\beta A_0 = 6.3$, and sets per capita taxes at .33. With this tax rate and infrastructure quality, 72.9 residents move to the municipality. With this population and tax rate, total tax revenue at $t = 0$ is $N_0 \times \tau_0 = 72.9 \times .33 = 24.1$. The remainder of the infrastructure investment is debt financed. In a complete market the mayor sells 39.6 pure security claims to the growth state (which occurs with p=.9) and 32.9 pure security claims in the decline state ((1-p)=.1). These sales generate debt proceeds at $t = 0$ of $(.9) * (39.6) + (.1) * (32.9) = 38.9$. So, tax receipts (24.4) plus Arrow-Debreu securities sales proceeds (38.9) equals investment (63.3). The municipalities 'capital strucuture' i.e. D/I, is .61.

Table 3: Model Solution: Complete contracts

	t = 0	t = 2	
ϵ	0	25	-25
N	72.89	83.1	44.74
q	6.33	5.69	2
τ	0.33	0.48	0.32
I	63.26	0	-36.93
D	38.89	-39.55	-32.9
$\bar{F}$		58.79	33.29
W	436.7	433.55	75.04
V	834.39	0	0

Notes: This table summarizes optimized values of the endogenous variables when the municipality has the ability to write debt contracts with state-contingent repayments.

[25]See the Appendix for details on our numerical solutions.

In the complete markets setting there is no role for the bankruptcy court as only contract court is required to enforce the pure security. Hence, there are no actions of interest at $t = 1$.

At $t = 2$ the decision of the mayor will depend on whether the city has grown ($\epsilon+$) or has declined ($\epsilon-$). Consider the growth state first. Exogenously 25 people move to the city which, without what we might describe as 'endogenous migration,' would leave the municapilty with a population of 97.9. The mayor must decide on any additional infrastructure investment and derive a tax rate, τ_2 that raises enough funds to repay the pure security payment of 39.6 plus any incremental infrastructure investment, I_2. It turns out that the mayor finds it optimal not to augment the infrastructure, allowing it to depreciate to 56.9, with a quality level of 5.69. To finance repayment of the debt, recognizing the consequence of the decline in the quality level and the required tax rate, requires that the tax rate increase to .48. With the lower service quality, the higher tax rate and the exogenous growth, the resulting population of the municipality will be 83.1, 14.8 lower than the 97.9 that would result without endogenous migration.

Next consider the population decline state where the required pure security payment is 32.92. Interestingly, in order to make this payment, the mayor liquidates as much of the city's infrastructure as possible, that is she liquidates to the point where the minimum service constraint binds. The mayor selects a tax rate of $\tau_2 = .32$, much less than the tax rate in the growth state. Despite the low tax rate, however, the decline in infrastructure quality *exacerbates* out–migration: The exogenous decline in population of 25 would have resulted in a population of 47.9 but actions to repay the debt result in further out –migration of 3.2 and a final population of 44.7.

Summarizing qualitatively, with pure securities the mayor invests in initial infrastructure that will provide for the needs of the current population and the expected future growth. Included in the optimal plan, however, is a severe retrenchment in the unlikely event of a negative shock such as a major employer moving out of town.

The numerical example illustrates that at an optimum, pure securities allow sufficient financing flexibility to equalize marginal tax revenues, both across $t = 2$ states and over time. Since $q_2^+ = q_2^- = (1-\delta)q_0$, it can be shown that

$$\frac{\text{MTR}_2^+}{\text{MTR}_0} = \frac{q_2^+}{q_0} = .90$$
$$\frac{\text{MTR}_2^-}{\text{MTR}_0} = \frac{q_2^-}{q_0} = .32$$
$$\frac{\text{MTR}_2^+}{\text{MTR}_2^-} = \frac{q_2^+}{q_2^-} = 2.9.$$

4.2 Non-contingent (standard) Debt

We now consider the Mayor's optimal choice of investment and financing when pure securities are not available and only risky debt is issued. We begin with a formal statement of the problem.

$$\max_{\{I_0, I_2^+, I_2^-, \tau_0, \tau_2^+, \tau_2^- F\}} V_0 = W_0(q_0, \tau_0) + pW_2(I_2^+, \tau_2^+, +\epsilon) + (1-p)W_2(I_2^-, \tau_2^-, -\epsilon) \tag{19}$$

s.t.

$$D_0 + N_0\tau_0 = I_0$$
$$N_2^+\tau_2^+ = I_2^+ + D_2^+$$
$$N_2^-\tau_2^- = I_2^- + D_2^-.$$

We examine the solution to the mayor's problem in the following three special cases: when chapter 9 is available to municipalities, when chapter 9 is not available to municipalities, and when mayors decide to avoid financial distress by issuing safe debt. These cases differ in how renegotiation is resolved when a municipality is in decline.

4.2.1 Chapter 9

Recall from section (3.3) that chapter 9 is available only to municipalities in decline since a growth city would be legally solvent and therefore not eligible for bankruptcy protection. In such a case, a contract court would require repayment of the originally promised amount F. For a city in decline, chapter 9 provides the mayor with greater bargaining power in that the court is willing to confirm an offer $F_M \geq \pi \bar{F}^-$. Recognizing this, in equilibrium at $t = 1$, bondholders make the following repayment proposals

$$F_B = \begin{cases} F & \text{if } \tilde{\epsilon} = +\epsilon; \\ \pi \bar{F}^- & \text{if } \tilde{\epsilon} = -\epsilon. \end{cases} \tag{20}$$

The Mayor accepts the bondholder proposal, thereby determining the date $t = 2$ state-contingent bondholder payments $D_2^+ = F$ and $D_2^- = \pi \bar{F}^-$. The Mayor's optimization then becomes a standard non-linear, constrained optimization that we solve using standard numerical techniques.[26]

The impact of chapter 9 is perhaps clearest when we compare the fate of a municipality in decline using chapter 9 relative to the pure securities case. As is evident in Table 4, the biggest differences between chapter 9 and the complete market case is that the mayor will repay much less during financial distress when Chapter 9 is available; second period payments to bondholders with chapter 9 is roughly 1/2 of the amount that is paid if pure securities were allowed (i.e. 16.27 instead of 32.9). As a result of not repaying as much to bond holders, tax rates in the declining city are about 2/3 of what they would be in the pure securities case (.21 instead of .32), the infrastructure quality would be 2.5 times higher (4.73 versus 2) and the population would be 33% higher (59.11 versus 44.74).

[26]See the Appendix for further details.

Table 4: Model Solution: Risky debt with bankruptcy courts

	t = 0	t = 2	
ϵ	0	25	-25
N	72.52	82.77	59.11
q	6.16	5.55	4.73
τ	0.34	0.48	0.21
I	61.61	0	-8.18
D	37.22	-39.54	-16.27
$\bar{F}$		58.05	32.55
W	422.44	419.42	267.27
V	826.64	0	0

Notes: This table summarizes optimized values of the endogenous variables when the municipality can access the bankruptcy courts.

4.3 Contract Court

As indicated in table (reftable1), not all states allow municipalities to file under chapter 9. To model the case where the state does not allow chapter 9 we simply impose the condition $\pi = 1$. Recall the π is the extent to which the court leans towards creditors in determining what a 'good faith' offer is. Moreover, the only advantage of Chapter 9 is that it allows the court to confirm a proposal that does not provide creditors with the maximum they could obtain from the reorganization (i.e. $\pi < 1$), so by setting $\pi = 1$ we eliminate the only role for chapter 9.

Hence in this case the following describes the equilibrium offer by the bond holder, which is accepted by the mayor in enforced by the court.

$$F_B = \begin{cases} F & \text{if } \tilde{\epsilon} = +\epsilon; \\ \bar{F}^- & \text{if } \tilde{\epsilon} = -\epsilon. \end{cases} \tag{21}$$

As seen in Table 5, contracts enforced by contract law provide results that are very close to the first best. Indeed, it is possible to search for a value of π that will generate the first best outcome. For the parameters we consider, the first best will obtain when $\pi = .89$ and, in the case of contract court, $\pi = 1$, hence the outcomes are quite similar. In general,

Table 5: Model Solution: Risky debt with no bankruptcy courts

	t = 0	t = 2	
ϵ	0	25	-25
N	72.89	83.09	44.27
q	6.33	5.69	2
τ	0.33	0.48	0.33
I	63.27	0	-36.94
D	38.9	-39.56	-32.96
$\bar{F}$		58.79	33.29
W	436.79	433.56	74.05
V	834.4	0	0

Notes: This table summarizes optimized values of the endogenous variables when the municipality cannot access the bankruptcy courts.

however, we see that prohibiting the use of chapter 9 weakens the bargaining power of the municipality in financial distress with the attendant result that financial distress is worse for the municipality that it would be if chapter 9 were allowed.

The difference between contract court and chapter 9 is also evident in the municipal yields. Under chapter 9 the yield on municipal debt is 6.2%, while under contract court the yield is 1.7%. Hence, our theory predicts that states that ban chapter 9 will have municipalities that issue debt with lower yields than municipalities that allow chapter 9 filings.

4.4 Safe Debt

It may be that for behavioural reasons the mayor simply does not want to default. This may reflect the career concerns of an elected official who feels that being re elected after default would be impossible or a directive issued to the mayor by the state.

In the context of our model, a mayor will only have a self imposed maximum face value of $F < F^i$. With this restriction, repayment is feasible and the firm is solvent. Moreover, even if chapter 9 is available to the municipality and the municipality filed a petition for a reorganization, the petition would be denied since the firm is solvent and hence not eligible (see section (3.2).

Table 6 provides the numerical results and it is clear that financing with safe debt has significant economic costs. Relative to the complete contracts case, the population is much more volatile, with the initial population being lower, the population in the growth state being higher and the population in the decline state being lower. Similarly, the tax rate is 20% higher at $t = 0$ and at $t = 2$ if the city declines, but it is 25% lower if the city experiences an expansion. Perhaps most importantly, the infrastructure quality is lower initially and in the expansion state but is the same in the decline state where, as with the complete contracts case, the minimum service constraint binds.

The general conclusion is that avoiding risky municipal debt generates the lowest bond yields but increases real volatility and reduces welfare.

Table 6: Model Solution: Safe debt

	0	25	
ϵ	66.1	94.45	39.28
N	5.63	5.45	2
q	0.4	0.36	0.38
τ	56.27	3.86	-30.64
I	30.14	-30.14	-30.14
D		55.64	30.14
$\bar{F}$	345.84	480.76	63.75
W	784.9	0	0
V	0	0	0

Notes: This table summarizes optimized values of the endogenous variables when the municipality must issue safe debt.

5 Conclusion

In this study we examine the use of debt financing by a municipality, focusing on the amount of new investment financed by debt relative to taxes. We define the debt/tax choice as the municipal corporation's capital structure and show the forces that determine a municipality's capital structure. The determinants of a municipality's s capital structure are very different from those of a public corporation. The long established trade-off between interest deductibility and bankruptcy costs that characterize public optimality are replaced by the trade-off of marginal tax revenues that are dependent on net migration to the city. Our model also captures the very different and complex bankruptcy process faced by municipal bond holders and we use this characterization to show how contract law and bankruptcy law combine to modify the marginal tax revenues across time and states in a way that alters welfare.

We also numerically examine our model and show that, if complete contracts existed, municipalities would invest in initial infrastructure that services both the initial residents as well as the expected growth. To do so, however, the mayor expects that, in the case of an unlikely decline in population, infrastructure quality and population will decline. Chapter 9 imposes a higher level on both infrastructure and population in the event of default, than would obtain with complete contracts. Finally, mayors who, for behavioural reasons, issue safe debt induce lower infrastructure quality and population with more volatile taxes.

A Solution to Programs (11) and (12)

This appendix provides details of our solution methods and, where possible, analytic results.

B Proof that $F < \bar{F}^+$ $F, F_B, F_M < \bar{F}^+$ is not restrictive/binding

Contrary to our assumption, suppose that the mayor considers F or $F_M > \bar{F}^+$. The quantities F, F_B, F_M are contractual amounts that could end up being $\tilde{F}_1$ and hence being adjudicated by the court.

C Parameter Restrictions

We require that the initial quality be chosen from the interval $[q_L, q_{UB}]$ where q_{UB} is the largest of the smallest roots of the quadratics in q

$$\big(a+bq\big)^2 + p\big(a+\epsilon+bq_L\big)^2 + (1-p)\big(a-\epsilon+bq_L\big)^2 - \frac{4c}{\beta}\Big(\big(\delta+\gamma(1-\delta)\big)q + (1-\gamma)q_L\Big) \quad (22)$$

or

$$\big(a+bq\big)^2 + p\big(a+\epsilon+bq_L\big)^2 + (1-p)\big(a-\epsilon+bq_L\big)^2 - \frac{4c}{\beta}\big(\delta q + q_L\big). \quad (23)$$

To ensures existence of q_{UB} we further require that the parameters satisfy

$$(a-\epsilon)^2 + (c-ab\beta)^2 - a^2(1+2b^2\beta^2) \geq 0. \quad (24)$$

D Solution Details

Analytic Solution for $\bar{F}$

In order to solve for the maximal payment available to bondholders in a default state, we begin by establishing the maximal payment for an arbitrary quality level at $t = 2$ and in the state $\tilde{\epsilon} = -\epsilon$:

$$\max_{\tau} N_2^- \tau = (a - \epsilon + bq - c\tau)\tau. \tag{25}$$

It is straightforward to show that the conditionally optimal tax rate is $\tau^*(q) = \frac{a-\epsilon+bq}{2c}$, yielding maximal tax revenues of

$$R_2^-(q) = \frac{(a - \epsilon + bq)^2}{4c}. \tag{26}$$

To determine $\overline{F}$ we must additionally determine the optimal level of q by solving for the maximal net-of-investment tax revenues

$$\max_{q} \frac{(a - \epsilon + bq)^2}{4c} - \frac{q}{\beta}. \tag{27}$$

Within the relevant range $q \in [q_L, q_{UB}]$ this objective is decreasing in q, hence the solution to bondholders' maximal request for payment, optimization problems (11) and (12) is given by

$$\bar{F} = \frac{(a - \epsilon + bq_B)^2}{4c} + (1 - \gamma)\frac{(1 - \delta)q_0 - q_B}{\beta} \tag{28}$$

where

$$q_B = \max\{q_L, (1 - \delta)q_0\} \tag{29}$$

when investment is irreversible and $q_B = q_L$ when investment is reversible.

Solution to the Base Case Optimization

To illustrate our solution method in all cases we begin with a detailed description of our solution methodology in the case where investment is irreversible. We restate the Mayor's optimization (5) in this special case

$$\max_{\{I_0, I_2^+, I_2^-, \tau_0, \tau_2^+, \tau_2^- F\}} V_0 = W_0(q_0, \tau_0) + pW_2(I_2^+, \tau_2^+, +\epsilon) + (1-p)W_2(I_2^-, \tau_2^-, -\epsilon) \tag{30}$$

s.t.

$$pF + (1-p)F^* + N_0\tau_0 = I_0$$
$$N_2^+\tau_2^+ = (1 - \gamma \mathbb{1}_{I_2^+ < 0})I_2^+ + F$$
$$N_2^-\tau_2^- = (1 - \gamma \mathbb{1}_{I_2^- < 0})I_2^- + F^*,$$

where $\mathbb{1}_{I<0}$ is an indicator for negative investment. Equation (28) shows that F^* is a function of q_0 and, therefore, not a distinct choice variable in the problem.

Substituting for the appropriate functions and conditional on $\tilde{\epsilon} = -\epsilon$, the Mayor's $t = 2$ subproblem is

$$\max_{\{I,\tau\}} \big(a - \epsilon + bq - c\tau\big)\big(q - \tau\big) \tag{31}$$

s.t.

$$\big(a - \epsilon + bq - c\tau\big)\tau - (1 - \gamma \mathbb{1}_{I<0})I - \pi \frac{\big(a - \epsilon + bq_B\big)^2}{4c} = 0$$
$$q - \big((1-\delta)q_0 + \beta I\big) = 0.$$

Substituting for the tax rate that satisfies the budget constraint yields

$$\tau = \frac{1}{c}\Big(\frac{a - \epsilon + bq}{2} - \phi_2\Big) \tag{32}$$

where

$$\phi_2^2 = \frac{\left(a-\epsilon+bq\right)^2}{4} - c\left[\frac{(1-\gamma\mathbb{1}_{I_2^+<0})(q-(1-\delta)q_0)}{\beta} + \pi\frac{\left(a-\epsilon+bq_B\right)^2}{4c}\right]. \tag{33}$$

A similar strategy allows elimination of τ_0 and τ_2^+ from the optimization. The first-order conditions of the Lagrangian of problem (30) produce the following equations for the tax rates:

$$\tau_0 = \frac{1}{c}\left(\frac{a+bq_0}{2} - \phi q_0\right)$$
$$\tau_2^+ = \frac{1}{c}\left(\frac{a+\epsilon+bq_2^+}{2} - \phi q_2^+\right)$$

where

$$\phi^2 = \left(\frac{\left(a+bq_0\right)^2 + p\left(a+\epsilon+bq_2^+\right)^2 + (1-p)\pi\left(a-\epsilon+bq_B\right)^2}{4} - c\frac{q_0 + p\left(1-\Gamma(I_2^+)\right)\left(q_2^+-(1-\delta)q_0\right)}{\beta}\right)\Big/\left(q_0^2 + pq_2^{+2}\right). \tag{34}$$

A final substitution produces the "concentrated" objective that we solve numerically

$$\max_{q_0,q_2^+,q_2^-} \frac{(a+bq_0)q_0 + p(a+\epsilon+bq_2^+)q_2^+ + (1-p)(a-\epsilon+bq_2^-)q_2^-}{2} - \frac{q_0 + p\left(1-\Gamma\right)\left(q_2^+-(1-\delta)q_0\right) + (1-p)\left(1-\Gamma\right)\left(q_2^--(1-\delta)q_0\right)}{\beta} + \phi\sqrt{q_0^2 + pq_2^{+2}} + (1-p)\phi_2 q_2^-. \tag{35}$$

Analogous arguments yield the following form of the objective for the first-best case:

$$\max_{q_0,q_2^+,q_2^-} \frac{(a+bq_0)q_0 + p(a+\epsilon+bq_2^+)q_2^+ + (1-p)(a-\epsilon+bq_2^-)q_2^-}{2} \\ - \frac{q_0 + p\big(1-\Gamma(I_2^+)\big)\big(q_2^+ - (1-\delta)q_0\big) + (1-p)\big(1-\Gamma(I_2^-)\big)\big(q_2^- - (1-\delta)q_0\big)}{\beta} \\ + \phi_{fb}\sqrt{q_0^2 + pq_2^{+2} + (1-p)q_2^{-2}}, \quad (36)$$

where

$$\phi_{fb}^2 = \Bigg(\frac{\big(a+bq_0\big)^2 + p\big(a+\epsilon+bq_2^+\big)^2 + (1-p)\big(a-\epsilon+bq_2^-\big)^2}{4} \\ - \frac{c}{\beta}\Big[q_0 + p\big(1-\Gamma(I_2^+)\big)\big(q_2^+ - (1-\delta)q_0\big) \\ + (1-p)\big(1-\Gamma(I_2^-)\big)\big(q_2^- - (1-\delta)q_0\big)\Big]\Bigg) \Bigg/ \Big(q_0^2 + pq_2^{+2} + (1-p)q_2^{-2}\Big). \quad (37)$$

Debt-to-Investment

For any levels of investment, the debt-to-investment ratio is given by the equation

$$\frac{D_0}{I_0} = 1 - \frac{\beta}{cq_0}\left(\frac{(a+bq_0)^2}{4} - \phi^2 q_0^2\right). \quad (38)$$

Expanding terms, this formula becomes

$$\frac{D_0}{I_0} = \Bigg[p\Bigg\{(1-\delta) + (1-\alpha)\Big[\frac{\beta a}{2c}\Big(\alpha b + \frac{(1+\alpha)a}{2q_0}\Big) - \alpha\Big] + \frac{\beta\epsilon}{2c}\Big(\alpha b + \frac{a+\epsilon/2}{q_0}\Big)\Bigg\} \\ + (1-p)\frac{\beta\pi\big(a-\epsilon+bq_B\big)^2}{4cq_0}\Bigg] \Bigg/ (1+p\alpha^2) \quad (39)$$

where $\alpha = q_2^+/q_0$ and q_B is defined in equation (29)

In the further special case with no uncertainty ($p = 1$), no depreciation, and no investment at date $t = 2$, this equation becomes

$$\frac{D_0}{I_0} = \frac{1}{2} + \frac{\beta\epsilon}{4c}\Big(b + \frac{a + \epsilon/2}{q_0}\Big). \tag{40}$$

Optimality of Safe Debt

We begin by eliminating τ_2^i from the time $t = 2$ welfare function $W_2(\tau_2^i, q_2^i, \tilde{\epsilon})$, given any q_2^i and D_2^i, using the budget constraint

$$\big(a + \tilde{\epsilon} + bq_2^i - c\tau_2^i\big)\tau_2^i - \frac{q_2^i - (1-\delta)q_0}{\beta} - D_2^i = 0. \tag{41}$$

The leftmost zero of this conditional quadratic equation in τ_2^i is given by

$$\tau_2^i = \frac{1}{c}\Big[\frac{a + \tilde{\epsilon} + bq_2^i}{2} - \phi\Big] \tag{42}$$

where

$$\phi_2^i = \sqrt{\rho_2^i - cD_2^i} \tag{43}$$

and

$$\rho_2^i \equiv \frac{\big(a + \tilde{\epsilon} + bq_2^i\big)^2}{4} - c\frac{q_2^i - (1-\delta)q_0}{\beta} \tag{44}$$

This substitution produces the following formula for $t = 2$ expected welfare

$$\begin{aligned} E_0(W_2(\tau_2, q_2, \tilde{\epsilon})) = p\Big(&\frac{a + \epsilon + bq_2^+}{2}q_2^+ - \frac{q_2^+ - (1-\delta)q_0}{\beta} - D_2^+ - \phi_2^+ q_2^+\Big) \\ &+ (1-p)\Big(\frac{a - \epsilon + bq_2^-}{2}q_2^- - \frac{q_2^- - (1-\delta)q_0}{\beta} - D_2^- - \phi_2^- q_2^-\Big) \end{aligned} \tag{45}$$

We now consider the choice of debt repayments at $t = 2$ given any values of the choice variables (q_0, q_2^+, q_2^-) and subject to $D_0 = pD_2^+ + (1-p)D_2^-$. The solution to optimization equation (45) is identical to that of

$$\min_{D_2^+, D_2^-} \omega\phi_2^+ + (1-\omega)\phi_2^- \tag{46}$$

where $\omega = \frac{pq_2^+}{pq_2^+ + (1-p)q_2^-}$. Equation (43 defining ϕ_2^i is convex in D_2^i, hence if

$$\rho_2^+ - cD_0 > \rho_2^+ - cD_2^+ > \rho_2^- - cD_2^- > \rho_2^- - cD_0 \tag{47}$$

then

$$\left(p\sqrt{\rho_2^+ - cD_0} + (1-p)\sqrt{\rho_2^+ - cD_0}\right) > \left(p\sqrt{\rho_2^+ - cD_2^+} + (1-p)\sqrt{\rho_2^+ - cD_2^+}\right) \tag{48}$$

This equation shows that when raising debt proceeds of D_0, safe debt results in higher welfare costs, hence lower welfare, than risky debt.

The inequalities (47) are typically satisfied under the parameterizations we consider. In words, the condition requires that conditional on a positive economic shock $\tilde{\epsilon} = +\epsilon$: 1) Tax revenues net of investment is higher, and; 2) Debt repayments are higher. Drawing parallels to the fundamentals of choice under uncertainty, when risky debt repayments give rise to mean-preserving variance reductions in welfare, increasing welfare in the $\tilde{\epsilon} = -\epsilon$ state and decreasing welfare in the $\tilde{\epsilon} = +\epsilon$ state, our concave $t = 2$ expected welfare function increases relative to the higher variance safe debt repayments.

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