

Belief Polarization and Investment^{*}

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Abstract

We study a canonical real option model where the decisions to acquire and subsequently abandon an asset are made sequentially by a group of agents with heterogeneous beliefs. Casting the decisions as a voting game, we show that inefficient underinvestment can occur when the group mediates disagreement through majority rule: Although each group member would acquire the option if they had post-acquisition control rights, the group votes against acquisition. We show that this inefficiency occurs when group members' beliefs are polarized in that they form two opposing factions with sufficiently large differences in beliefs. Given the pervasive nature of group decisions in the life of organizations and institutions, our theory is particularly relevant for the behavior of venture capitalists, financing syndicates, corporate boards, and committees at large.

Keywords: group decisions, dynamic voting, real investment.

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1 Introduction

Since a corporation is legally an individual decision-making entity, it is not surprising that the theory of corporate finance typically models a corporation as a single ‘manager’ or ‘entrepreneur’. Yet the vast majority of corporate decisions are group decisions where diverse views must be aggregated into a single corporate decision. Boards of directors, venture capital groups, financing syndicates, and committees, are obvious reminders of the ubiquitous nature of group decisions in all aspects of economic life. Despite the pervasiveness of group decisions within corporations, the finance literature has devoted little attention to the possible coordination frictions that can emerge in group decision making.

In this paper we address this gap by considering a canonical investment problem, the decisions to invest in and subsequently to abandon or continue a project. To this problem, extensively studied in the real options literature, we add a key variant: the investment and abandonment decisions are undertaken by a group of individuals rather than an “entrepreneur” or a “representative manager”. We further assume that group members have heterogeneous beliefs and that the group makes decisions based on majority rule voting.¹

We borrow from the political economy literature on voting by treating the real option problem as a dynamic *voting* game. We find that agents’ disagreement about the optimal management of the investment opportunity can be an important source of coordination friction within the group. Specifically, we focus on majority voting and identify pivotal voters as those whose votes can swing a decision one way or the other. We define *polarization* as the difference in beliefs held by pivotal voters in a sense to be made precise later in the paper. We then show that when group members’ beliefs are sufficiently *polarized*, the equilibrium of the voting game exhibits *inefficient underinvestment*: a project that is deemed valuable by each group member is nevertheless rejected by the group.

Our result that polarization can lead to investment inefficiency rests on the following four key assumptions. First, corporate decisions are made by a group that cannot avoid disagreement by having some group members abstain from the decision. Second, members

¹To focus explicitly on the coordination frictions that this governance rule entails, we explicitly abstract away from modelling informational asymmetries, learning and bargaining among group members.

of the group have heterogeneous beliefs about the appropriate management of the investment opportunity and, hence, about the value of the opportunity. Third, group members cannot solve their disagreement by trading votes. Fourth, disagreement about what the group should do is mediated by a majority vote: both the investment and the subsequent abandonment decision are undertaken when there is strict majority support.

Under these assumptions, we study the following real option problem. At an initial time, the firm has the opportunity to pay a fixed cost to invest in a project that generates a potentially infinite stream of future stochastic cash flows. At any point after the initial investment, however, the project may be abandoned and the asset reallocated to an alternative use with an associated certain value. The abandonment decision is a standard timing option. However, although each member of the group agrees on the volatility of the underlying cash flow and on the alternative value of the asset, they disagree on the expected growth rate of the cash flows. Hence, each member sees a different optimal time at which to abandon.

Intuitively, polarization and underinvestment due to majority voting are linked by the following tension. When evaluating an investment opportunity, all agents recognize that the project consists of a perpetual growing cash flow plus an abandonment or put option that allows the assets to be reallocated through liquidation. Agents with heterogeneous beliefs about cash flow growth rates will disagree about optimal abandonment timing. For a given observed cash flow a pessimist is more likely to abandon earlier than an optimist. Since this disagreement about abandonment is resolved through majority voting², a controlling or ‘pivotal’ member is defined as the member whose preferred timing is the earliest time that can generate a majority vote for abandonment. The DMG will adopt the optimal abandonment timing of the pivotal member.³ When non-pivotal members evaluate the initial investment, they recognize that, due to the political voting process, they will have to live with the abandonment decision of the pivotal member, a decision they see as suboptimal. To the non pivotal group, the lost value due to suboptimal timing increases with the difference

²In Section 5 we show that other governance mechanisms will also lead to inefficiency. We concentrate on majority voting as it is the most common scheme used in practice.

³In a group of 4 members where member 1 is the most pessimist and member 4 is the most optimist, the pivotal member for the abandonment decision is member 3.

in their beliefs relative to the pivotal belief. For small differences in beliefs there will be small differences in optimal option exercise and the overall attractiveness of the investment is not greatly impaired by this future political loss: they disagree with what will happen, but not enough to make the investment unattractive. As the divide between the beliefs of the pivotal and non-pivotal members of the DMG widens, the cost of perceived suboptimal abandonment increases. Eventually, the loss is sufficient to render the project unattractive at least for some members. At the initial investment stage, the proposal to invest can be defeated if the non-pivotal members of the *abandonment decision* are sufficiently numerous to prevent a majority vote in favor of investment. In this case the non-pivotal members of the abandonment decision become pivotal for the investment decision. *Underinvestment* occurs then when a project that is attractive to each member of the DMG is forgone by the DMG. This paper explores the conditions under which this switch in who is pivotal takes place and how this leads to inefficiency.

This intuition suggests three properties of underinvestment that we show are indeed general. First, underinvestment occurs when the pivotal voter for the abandonment decision is different from the pivotal voter for the investment decision. In turn this ‘switch’ in the pivotal voter happens because the group’s decision involves converting an agreed upon value (i.e. the investment amount) into a value that is not agreed upon (i.e. the risky cash flow) which can then be converted back to an agreed upon value (i.e. the liquidation value). Second, underinvestment is more likely to happen when beliefs are more polarized. When the beliefs are similar, although some members of the DMG see the group’s abandonment decision as suboptimal, the inefficiency is not sufficient to change their overall favorable view of the investment. As polarization increases, the perceived abandonment inefficiency grows to the point where a critical number of group members find the initial investment unattractive in the first place.

Third, in our simple single-dimensional problem, the size of the group is critical for the likelihood of investment inefficiency. In particular, with an odd number of members in the DMG, all with different beliefs, there will be a single pivotal player who effectively controls both the investment and abandonment decision. This result, which is a direct consequence

of the median voter theorem (Black, 1948), is consistent with the widely used rule of thumb in practice that calls for boards with an odd numbers of seats. As we will show however, this result is not robust to minor deviations in voting rules since underinvestment can occur with odd number of group members if some members have veto power. More importantly, as with the median voter theorem, the ‘odd number’ result does not hold when there is more than a single dimension to the decision problem.

In addition to characterizing investment inefficiency due to polarization, we highlight the role of polarization relative to risk. Increases in volatility in a simple timing option like ours means that abandoning a project and reallocating resources will be delayed to take advantage of the higher value of waiting. But resource reallocation opportunities are, however, delayed, not lost: abandonment and reallocation of assets is simply delayed until opportunities deteriorate further. In contrast, an increase in polarization leads to lost investment, not delayed abandonment. The projects are not undertaken in the first place so the abandonment decision is moot. We show that polarization within a group is an important economic force that interacts with volatility. In particular, underinvestment due to polarization can occur for any level of volatility. Moreover, high volatility might mitigate or exacerbate underinvestment, depending on the “moneyness” of the abandonment option.

The voting protocol is a key determinant of the investment inefficiency. We focus on majority voting, where underinvestment requires sufficient disagreement between the two agents whose beliefs straddle the median belief of the group. In contrast, with unanimity, underinvestment requires that the range of beliefs in the group is sufficiently dispersed. We also explore the implication of other voting protocols such as super-majority and majority with vetoers and show that even under these more general protocols, underinvestment requires some degree of polarization within the group.

Our work contributes to a relatively new literature on group decision making in financial economics. Garlappi, Giammarino, and Lazrak (2017) also show that inefficient underinvestment can occur in group decision making. However, their mechanism relies on agents learning about the value of a project overtime and on a utilitarian governance rule that aggregates individual beliefs for the purpose of decision making. In contrast, in our setting

underinvestment occurs even in the absence of learning and without a utilitarian aggregation rule. The only requirement is that decisions are made sequentially and that disagreements are mediated through voting.

Our paper is also related to a recent political economy literature on dynamic voting that rationalizes the emergence of inefficient gridlock and status quo. Dziuda and Loeper (2016) show that, in a dynamic voting game where preferences evolve over time and where the previous decision becomes the next status quo, inefficiency and deadlock can arise. Building on this intuition, Donaldson, Malenko, and Piacentino (2017) analyze and extend the problem of gridlock within corporate boards who select a CEO.

The key mechanism for obtaining gridlock in these papers is the change of agents' preferences overtime and the endogeneity of the status quo. Agents oppose current proposals even if they are perceived as better than the status quo. If a current proposals is accepted, it becomes the status quo for future decisions. Agents who hope for the arrival of their preferred proposal in the future will therefore block any current proposal. Although our result is similar in that it shows the possibility of an inefficient group choice, our mechanism differs along several dimensions. First we emphasize a new coordination friction due to polarization of beliefs between group members. Second, unlike the previous papers, we do not rely on changing preference. We therefore show that inefficiencies can also occur naturally in a standard real option framework where preferences do not change over time and where there is no role for the status quo.

Our paper also contributes to the real option literature (Brennan and Schwartz, 1985; McDonald and Siegel, 1986; Dixit and Pindyck, 1994) which we extend to allow for the case in which real options are exercised by a group instead of an individual. To the best of our knowledge, ours is the first study to formally analyze the exercise of a real investment option by a group of agents with heterogeneous beliefs. Our analysis provide a general tractable framework to study dynamic investment decisions undertaken collectively by a group of agents. One of the message of the real option literature is to emphasize the impact of volatility on option delays. To that literature, our paper delivers the news insight that, in

presence of polarization, volatility could reinforce or mitigate the underinvestment problem that we highlight.

Our paper is also related to a literature in macroeconomics that investigates the effect of political uncertainty and polarization on aggregate investment. Most notably, Azzimonti (2011) builds a neoclassical growth model in which agents are polarized along a “political dimension”, captured by the size of the government. The model rationalize the empirical observation that, for a cross section of countries, greater polarization is typically associated with lower growth and lower private investment. The main mechanism for underinvestment in this model is however different from ours. While our model provides a micro-foundation for individual underinvestment, in Azzimonti (2011), underinvestment occurs due to political uncertainty. In particular, governments are shortsighted and tend to engage in overspending financed by distortionary taxation that crowds out private investment. Azzimonti (2018) provides corroborating evidence of this channel by constructing a partisan conflict index from textual analysis of newspaper articles.

Finally, our paper is also related to the theoretical literature on experimentation by groups. Strulovici (2010) studies the problem of how a group of individual agents with heterogeneous preferences experiments with new opportunities in a two-arm bandit framework. The main result of his analysis is that incentives for experimentations are always weaker when decision power is shared among group members as opposed to concentrated in the hands of a dictator. As in our model, the coordination friction imposed by the necessity to mediate across different beliefs, may give rise to underinvestment. In particular, the concept of “option value”, i.e., the ability of a decision maker to react to news, may be muted in a context with multiple decision makers.

Our paper proceeds as follows. In the next section we present the intuition of our theoretical results through a simple two person example of a continuous time, dynamic voting game. Section 3 provides a generalization of these results while Section 4 explores the quantitative implications of our results. We consider how our results would change under other voting rules in Section 5 and conclude the paper in Section 6.

2 An illustrative example

Consider a group of two agents, P (for pessimist) and O (for optimist), who form a “Decision Making Group” (DMG hereafter). At time zero the DMG must decide, on behalf of the corporation, whether or not to invest in a project that generates a potentially infinite uncertain stream of cash flows (*the investment decision*). Subsequent to this initial decision, the DMG must also decide whether and when to abandon the project and reallocate the resources to an alternative use with a fixed redemption value (*the abandonment decision*).

Technology. The investment opportunity requires I to implement and disappears if not implemented at $t = 0$. Upon investment, the project generates a random cash flow, X_t , per unit of time. The project may be abandoned at any time after the initial investment is made. Abandonment means that the assets are reallocated to an alternative use that has a safe value of D . Formally, the cash flow of the project X follows a geometric Brownian motion with volatility σ and drift μ_n , where n indexes the two members of the DMG, that is

$$dX_t = \mu_n X_t + \sigma X_t dW_{n,t}, \quad X_0 = x > 0, \quad \mu_P < \mu_O,$$

and where $W_{n,t}$ is a standard Brownian motion under agent n 's belief, $n \in \{P, O\}$. That is, P and O disagree on the underlying cash flow dynamics of the project.

DMG decisions. The investment and abandonment decisions are each determined by separate votes, each subject to strict majority rule (unanimity in a group with two members). The investment vote takes place at time zero while the abandonment vote can happen at *any* time afterwards. In particular, following the initial investment, any member of the DMG can, at any time, propose that the project be abandoned, thereby triggering a vote. Abandonment will take place at that point in time only if the majority, i.e., both parties, agree.

We assume that each member of the DMG anticipates how the subsequent abandonment decision will be made when voting on the initial investment decision. Figure 1 illustrates the decisions faced by the DMG.

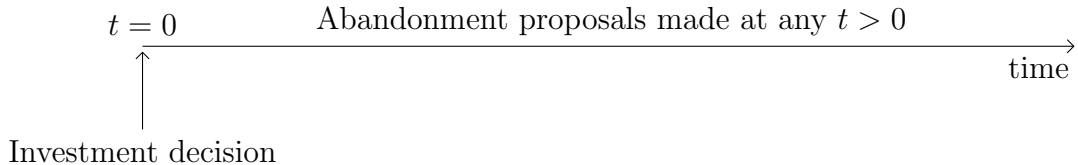


Figure 1: Model timeline

At time $t = 0$ the group decides whether to invest in a project. Investment will occur only if a strict majority of group members vote in favor. Abandonment proposals are made by any group members at any time after $t = 0$ and undertaken only if voted for by a strict majority of group.

DMG members. Agent P is the pessimist and agent O is the optimist, i.e., $\mu_P < \mu_O$. We assume that agents are risk-neutral and discount cash flow at the risk free rate r .⁴ We further assume that $\mu_O < r$. It is important to emphasize that although agents have different beliefs, they have the same information. In other terms investors agree on the nature of the information that they receive but they disagree on how to interpret it.

Individual Abandonment Decisions If investment is made at time zero, each DMG member n may choose when to propose abandonment. It is therefore helpful to examine when each DMG member would abandon if they were acting in autarky. In such a case, the abandonment decision is a well-known optimal stopping problem that involves choosing the stopping time τ_n that maximises the stream of cash flow X_t , given the redemption value $D > 0$. Formally, agent n solves

$$J_n^*(x) = \max_{\tau_n} \mathbb{E}^n \left[\int_0^{\tau_n} X_t e^{-rt} dt + D e^{-r\tau_n} \right], \quad X_0 = x. \quad (1)$$

⁴Equivalently, one can assume that markets are complete so that changes in X can be spanned by existing assets, in which case the agents' discount rate will include a risk premium proportional to the correlation between the changes in the replicating portfolio and the source of systematic risk, e.g., the market portfolio in a CAPM world.

Acting individually, the optimal abandonment time for agent n , is given by the stopping time

$$\tau_n^* = \inf \{t \geq 0 : X_t \leq X_n^*\},$$

where the time-invariant threshold X_n^* is given by (see, e.g., Karatzas and Shreve (2012))

$$X_n^* = \frac{m_n}{m_n + 1} D(r - \mu_n), \quad \text{with } m_n = \frac{(\mu_n - \frac{1}{2}\sigma^2) + \sqrt{(\mu_n - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}, \quad n \in \{P, O\}. \quad (2)$$

Using Equations (2), it can be shown that $m_P < m_O$ and that the pessimist agent has a strictly higher abandonment threshold than the optimist, that is, $X_P^* > X_O^*$. This implies that, when acting individually, P will abandon the project earlier than O : $\tau_P^* < \tau_O^*$.

In general, the value at time zero to investor n of abandoning at a generic threshold \hat{X} is given in the following proposition.

Proposition 1. *For any $x > 0$, the value associated to the policy $\tau_{\hat{X}} = \inf \{t \geq 0 : X_t \leq \hat{X}\}$ is given by*

$$J_n(x, \hat{X}) = \mathbb{E}^n \left[\int_0^{\tau_{\hat{X}}} X_t e^{-rt} dt + D e^{-r\tau_{\hat{X}}} \right] = \begin{cases} D, & \text{for } x \leq \hat{X} \\ \frac{x}{r - \mu_n} + \left(\frac{\hat{X}}{x}\right)^{m_n} \left[D - \frac{\hat{X}}{r - \mu_n} \right], & \text{for } x > \hat{X} \end{cases}, \quad (3)$$

The proposition shows that, prior to abandoning, the value function associated with a generic stopping time $\tau_{\hat{X}}$ has two terms. The first term is the value of the project if it was never abandoned. This is simply the value of receiving the perpetual cash flows X_t for a risk neutral agent with a discount rate r , that is, $x/(r - \mu_n)$. The second term is the value of the abandonment option exercised at hitting time $\tau_{\hat{X}}$. Exercising the abandonment option is equivalent to replacing the perpetual risky cash flow stream with the safe cash flow stream $rDdt$ from time $\tau_{\hat{X}}$ onward, thus

$$\left(\frac{\hat{X}}{x}\right)^{m_n} \left[D - \frac{\hat{X}}{r - \mu_n} \right] = E \left[\int_{\tau_{\hat{X}}}^{\infty} e^{-rt} (rD - X_t) dt \right].$$

Alternatively, one can think of $\left(\frac{\widehat{X}}{x}\right)^{m_n}$ as the price of an Arrow-Debreu security that pays one dollar the first time X_t hits \widehat{X} from above. We can therefore think of the value function $J(x, \widehat{X})$ as the value of a perpetuity $x/(r - \mu_n)$ plus an option with payoff $D - \frac{\widehat{X}}{r - \mu_n}$ received at the random time $\tau_{\widehat{X}}$.

We notice that $J(x, 0) = \frac{x}{r - \mu}$. This is due to the fact that the stopping policy is $\tau_0 = \infty$ almost surely and thus this implies that the option will never be exercised. We also notice that $J(x, x) = D$. In this case, the policy τ_x implies that the option will be exercised immediately generating the cash flow D .

Figure 2 illustrates the values $J_P(\cdot, \widehat{X})$ and $J_O(\cdot, \widehat{X})$ as a function of the current cash flow x for different values of \widehat{X} . It is easy to show that the values defined in (3) are uniformly ranked, as illustrated in Figure 2:

$$J_P(x, \widehat{X}) \leq J_O(x, \widehat{X}), \text{ for all } \widehat{X} > 0$$

with the inequality being strict for $x < \widehat{X}$. Thus

$$J_P(x, X_O^*) < J_P(x, X_P^*) < J_O(x, X_P^*) < J_O(x, X_O^*), \text{ for all } x < X_O^* \quad (4)$$

At time $t = 0$ the investment cost is I . This cost is equally borne by both agents. Hence the NPV to agent n of acquiring the license and investing at the threshold \widehat{X} is

$$V_n(x, \widehat{X}) = J_n(x, \widehat{X}) - \frac{I}{2}. \quad (5)$$

Equilibrium investment.

Because both the decision to invest at time zero and the decision to abandon requires a majority (i.e. unanimity in a group with two agents), we note that this sequential voting problem may exhibit inefficient investment due to the fact that different members form different *decisive coalitions* for the investment vote relative to the abandonment vote.

To see this, consider the pessimist's voting expectations and assume that the current level of cash flow x is larger than X_P^* . The pessimist understands that, if X declined from the

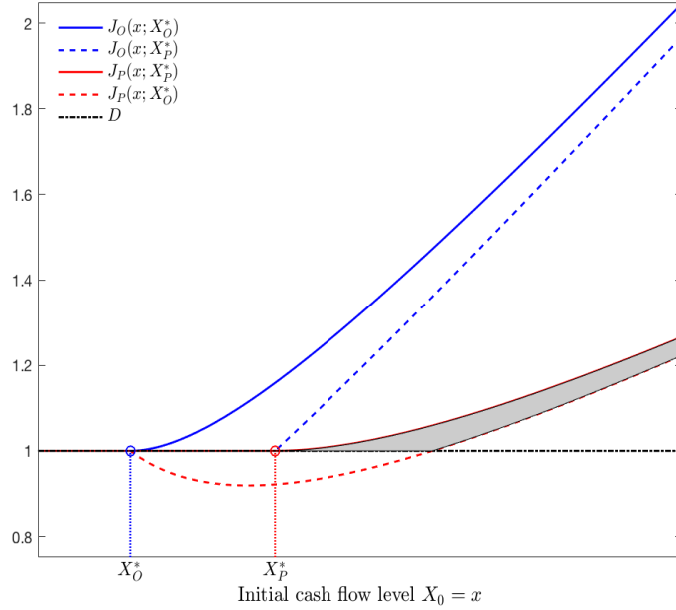


Figure 2: Value Functions

The figure depicts the value functions of $J_P(\cdot, X_P^*)$ (solid blue), $J_P(\cdot, X_O^*)$ (dashed blue), $J_O(\cdot, X_O^*)$ (dashed red), $J_O(\cdot, X_P^*)$ (solid red). Parameter values: $r = 0.05$, $\sigma = 0.3$, $D = 1$, $\mu_P = 0.01$, $\mu_O = 0.03$.

current level to X_P^* , the level at which the pessimist would optimally abandon, the optimist would *not* vote in favor of abandonment. The reason for this is that the optimist knows that, if she rejects abandonment at X_P^* and waits until cash flows fall to the lower level X_O^* , the pessimist will support her abandonment proposal. Although the pessimist would prefer to abandon sooner, he is still better off supporting abandonment as soon as the optimist is willing to do so. That is, the optimist is pivotal for the abandonment vote. As a result, the expected abandonment policy must be $\tau_O^* = \inf\{t : X_t \leq X_O^*\}$, that is the optimal policy for the optimist.

Anticipating this, the pessimist will understand that the time zero value of the subsequent exercise decision will be given by $J_P(x, X_O^*)$, the value of the investment under the pessimist's beliefs and assuming the optimists preferred exercise. From the ranking property (4), we have that

$$J_P(x, X_O^*) < J_P(x, X_P^*) < J_O(x, X_O^*) \text{ for all } x < X_O^*. \quad (6)$$

When $I/2 < J_P(x, X_O^*)$, both P and O will agree to invest, and there will be no underinvestment.

We can see from inequality (6), however, that there are alternative values of the initial investment cost I satisfying $J_P(x, X_O^*) < I/2 < J_P(x, X_P^*)$. In such a case, although both agents would agree to invest if they were able to unilaterally pick the subsequent abandonment timing, the investment will be rejected by the group. In fact, the decision to invest will be blocked by investor P who is pivotal for the investment vote. We refer to this situation where investment is blocked by an extreme (pessimistic) view as *underinvestment*. It is important to observe that underinvestment is inefficient: both group members would invest if they had unilateral control of the investment and abandonment decision. The pessimist would abandon at τ_P^* and he would invest because $I/2 < J_P(x, X_P^*)$. The optimist would abandon at τ_O^* and she would invest because $I/2 < J_O(x, X_O^*)$.

A compromise timing would facilitate investment but majority voting does not allow compromise. Any value I for which

$$J_P(x, X_O^*) < \frac{I}{2} < J_P(x, X_P^*) < J_O(x, X_O^*), \quad (7)$$

would give rise to underinvestment.

The shaded area in Figure 2 represents the underinvestment region. For a given level of cash flow value x , there is an interval of values for the investment cost $[\underline{I}(x), \bar{I}(x)]$ for which underinvestment occurs. These bounds are given by

$$\frac{\underline{I}(x)}{2} = J_P(x, X_O^*), \quad \frac{\bar{I}(x)}{2} = J_P(x, X_P^*).$$

When $I < \underline{I}(x)$, the investment cost is so low that the pessimist will agree to invest even if the optimist determines the abandonment decision. When $I > \bar{I}(x)$, the investment cost is so high that the pessimist will reject investment. That decision is, however, efficient since the pessimist would not invest even if she had full control of the abandonment decision as $I/2 > J_P(x, X_P^*)$.

Discussion. Both investors would benefit from giving all the DMG decision power to investor P for the investment decision. Under the latter assumption, the DMG will abandon at the threshold policy X_P^* , yielding the welfare $J_P(x, X_P^*)$ to investor P and $J_O(x, X_P^*)$ to investor O . Since

$$\frac{I}{2} < J_P(x, X_P^*) < J_O(x, X_P^*),$$

letting investor P rule the DMG for the abandonment decision Pareto improves the equilibrium outcome of a strict majority decision rule. To see this, note that under strict majority, underinvestment occurs and the investment is not undertaken resulting in a value of $I/2$ to both P and O . Hence giving decision power to P will eliminate underinvestment and generates a payoff that is greater than $I/2$ to *both* agents, a Pareto improvement.

At a broader level, the inefficient investment that we describe in this example requires a change of the identity of the *pivotal* voters for the investment decision and the abandonment decision. Being more pessimistic, it is agent P who is more reluctant to invest and thus he is pivotal for the investment decision. On the other hand, the more optimistic O is more reluctant to abandon and therefore she is pivotal for the abandonment decision. Since (i) P is pivotal in the investment decision, and (ii) he will lose his status as the pivotal player in the future abandonment vote, and (iii) since the beliefs of P and O are sufficiently different (polarized), P will vote against investment. When there is no disagreement between DMG members, both investors are pivotal as they are identical and, as a result, underinvestment cannot occur.

Similarly, if the decision rule is dictatorial, that is, when all the voting power is in the hand of agent O (resp. P), the pivotal individual remains agent O (resp. P) for both the investment and abandonment decision. There is therefore no threat of change of identity of the pivotal voter and as a result underinvestment does not occur.

It is also important to notice that the change of identity of the pivotal voter is only possible because we have a switch in the nature of the alternatives that DMG members vote for. The first vote (investment) is on a proposal to convert an agreed upon quantity (the

investment amount) into an asset about which there are differences in beliefs (the risky cash flow stream). Since the pessimistic investor P places a lower value on the risky cash flow, he is pivotal. The second vote (abandonment) is a proposal to convert the risky cash flow, about which there are differences in beliefs, to a risk free cash flow with an agreed upon value. As the optimistic investor O is more reluctant to abandon the risky cash flow, she will be pivotal. The mechanism that we describe would not occur for example in a real option model when an investment decision is followed by an expansion decision since the conversion is from a risky cash flow to a larger risky cash flow, both of which are subject to different beliefs.

Perhaps less intuitive is the fact that, as we formally prove in the next section, inefficient underinvestment cannot occur with a DMG that includes *three* or, in general, an odd number of members and is governed by a strict majority rule. In this case, the median belief member of the DMG must be the pivotal investor for the abandonment timing decision as his preferred timing is the earliest time that can generate a majority vote for abandoning. Two configurations are possible for the investment decision. In the first configuration, the median member votes for investment. As a consequence, the DMG invests and hence no underinvestment occurs. In fact all investors who are more optimistic than the median belief investor will in this configuration vote for investing, thereby forming a coalition leading the DMG to invest.

In the second configuration, the median belief member votes against investing. Then all investors who are more pessimistic than the median belief investor will also reject investing, which leads the group to forgo the investment. Although the DMG does not invest, this is not an inefficient outcome: The median belief member rejects the investment as it correctly anticipates the optimal abandonment policy. In this setting, investing will be suboptimal for the belief of the median voter. This result resembles the median voter theorem in static spatial voting models in one dimension. Our result is however different because we have a *dynamic* voting game. The dynamics of this voting game depart from the classical spatial voting game because the preferences towards current investment change in anticipation of the future abandonment vote.

3 A general model of investment decision by groups

3.1 The framework

Consider a filtered probability space $(\Omega, (\mathcal{F}_t)_{t \in \mathbb{R}}, \mathbb{F}, \mathbb{P}_1)$ where \mathbb{F} is the natural filtration of a Brownian motion $W_{1,t}$. We denote by \mathbb{E}_1 the expectation operator under \mathbb{P}_1 . A group $\mathcal{N} = \{1, \dots, n, \dots, N\}$ of N infinitively lived investors, a *decision making group* or DMG, collectively decides whether to invest in a project and subsequently when to abandon the project and receive a liquidation value D .

Technology and information. Investment is irreversible and the project cash flows X follows a geometric Brownian motion. agents agree on the volatility of the cash flow process but disagree on its expected growth. Specifically, investor 1 perceives the cash flow process as

$$dX_t = \mu_1 X_t + \sigma X_t dW_{1,t}, \quad X_0 = x > 0$$

where $\mu_1 \geq 0$ and $\sigma > 0$. agent 1's belief is captured by the probability measure \mathbb{P}_1 under which the value process is a geometric Brownian motion with drift μ_1 .

Agents $n = 2, \dots, N$ disagree with investor 1 on the value of the drift of the value process. Their perceived drift is μ_n , where, without loss of generality, we assume that

$$\mu_1 \leq \mu_2 \leq \dots \leq \mu_N, \tag{8}$$

that is, investors are ranked based on how optimistic they are with regard to the expected growth of the cash flow process X_t . agent's n belief, for each $n = 2, \dots, N$, is given by a probability measure \mathbb{P}_n , equivalent to \mathbb{P}_1 under which the cash flow process satisfies⁵

$$dX_t = \mu_n X_t + \sigma X_t dW_{n,t}, \quad X_0 = x > 0$$

⁵By the Girsanov Theorem (Karatzas and Shreve, 2012), the probability \mathbb{P}_n is formally defined as $\left(\frac{d\mathbb{P}_n}{d\mathbb{P}_1}\right)_{\mathcal{F}_t} = \xi_{n,t}$, where the process $\xi_{n,t}$ is the non-negative $(\mathbb{P}_1, \mathbb{F})$ -martingale defined by $d\xi_{n,t} = -\eta_n \xi_{n,t} dW_{1,t}$, $\xi_{n,0} = 1$, with $\eta_n = \frac{\mu_n - \mu_1}{\sigma}$.

where W^n is a standard Brownian motion under the probability \mathbb{P}_n and given by

$$W_{n,t} = W_{1,t} - \eta_n t,$$

where $\eta_n = \frac{\mu_n - \mu_1}{\sigma}$.

It is important to emphasize that although investors have different beliefs, they have the same information. This is mathematically captured by the fact that the Brownian filtration generated by the Brownian motion $W_{n,t}$ for each $n = 1, \dots, N$ is identical to the natural filtration of the commonly observed process X_t . In other terms investors agree on the nature of the information that they receive but they disagree on how to interpret it. Furthermore, note that disagreement about the drift is only possible if $\sigma > 0$. If $\sigma = 0$ then $X_t = xe^{\mu t}$ and therefore all agents can learn the drift μ by observing X_t and x .

Agents' preferences, payoffs and strategies . agents are risk neutral and discount future cash flows at the risk free rate r . We assume that

Assumption 1. *Agents are sufficiently impatient, that is,*

$$r > \mu_N.$$

Given condition (8), this assumption implies that for each investor, there are states of the world where it is optimal to exercise the abandonment option. Without this assumption, the patient investors will always oppose exercise of the abandonment option.

The DMG make two consecutive decisions. The first decision is the investment decision, taken at time 0. This decision consists of two alternatives. The first alternative (invest) is to collectively pay a cost I to acquire exclusive ownership of the cash flow and the right to exercise the abandonment opportunity at a future date. The second alternative is to reject the acquisition of the project.

If the members of the DMG agree to invest, at any time in the future, any DMG member can propose abandonment of the project and obtain the lump sum payoff D , resulting in a payoff of D/N to each group member.

If no proposal is offered at a given date or if a proposal is rejected by the DMG, the option remains unexercised and available for exercise in future dates.⁶

We assume that the timing strategies investor follow to propose abandonment are of the threshold type, that is,

Assumption 2. *agents submit a proposal to the group when the observed value of the cash flow is smaller than a threshold value $\hat{X} \in [0, \infty]$. The proposal date is the stopping time $\tau_{\hat{X}}$ at which the threshold \hat{X} is first reached, that is,*

$$\tau_{\hat{X}} = \inf\{t : X_t \leq \hat{X}\}.$$

This class of timing decisions is standard in the real option literature and natural, given the stationary structure of our game. This formulation greatly simplifies our equilibrium voting outcome and allows us to discuss our economic insight in a transparent way.

If a proposal $\hat{X} > 0$ is accepted by the DMG the value to investor $n = 1, \dots, N$ defined as

$$J_n(x, \hat{X}) = \frac{1}{N} \mathbb{E}^i \left[\int_0^{\tau_{\hat{X}}} X_t e^{-rt} dt + D e^{-r\tau_{\hat{X}}} \right]$$

is given by,

$$J_n(x, \hat{X}) = \begin{cases} \frac{1}{N} D, & \text{for } x \leq \hat{X} \\ \frac{1}{N} \left[\frac{x}{r-\mu_n} + \left(\frac{\hat{X}}{x} \right)^{m_n} \left[D - \frac{\hat{X}}{r-\mu_n} \right] \right], & \text{for } x > \hat{X} \end{cases}, \quad (9)$$

with m_n defined in equation (2).

It can be shown that the values J_n are ordered, that is,

$$J_n(x, \hat{X}) \leq J_{n+1}(x, \hat{X}) \text{ for all } (x, \hat{X}) \in (0, \infty)^2, \quad n = 1, \dots, N - 1. \quad (10)$$

⁶Note that it is possible to engage in “take-it-or-leave it options,” that is, an agent that propose a vote to abandon, could potentially threat the group to withdraw his support in the future if the proposal does not go through. Those threat will however not be credible because this will amount to sub-optimally give up an option to abandon at future dates.

If we rename investor P (resp. O) from section 2 investor 1 (resp. 2), Figure 2 where the dashed red curve represents $J_1(\cdot, \widehat{X})$ and the solid red curve represents $J_2(\cdot, \widehat{X})$ illustrate this inequality. The figure shows that the graph of the two functions are uniformly weakly ordered and thus inequality (10) is satisfied for $n = 1$.

It is also possible to prove that the function $J_n(x, \widehat{X})$ that maximized at

$$J_n(x, X_n^*) = \sup_{\widehat{X}} J_n(x, \widehat{X}),$$

where X_n^* is the bliss point for investor n defined in equation (2) with the substitution $i = n$. Since pessimistic investors will find it optimal to abandon at higher level of the cash flow levels, the sequence of bliss points satisfies

$$X_1^* > \dots > X_N^*. \quad (11)$$

In our baseline analysis we assume that at the current level of the project value x , no group member finds it optimal to propose an immediate exercise of the abandonment option. This amounts to insuring that each agent faces a non-trivial investment decision ruling out situation where the investment cost is so low that some investors find it optimal to buy a project and liquidate it immediately, an unlikely equilibrium opportunity.

Assumption 3. *We assume that at the current level $X_0 = x$ of the project value the investment option is out of the money for all investors, that is,*

$$x > X_1^*.$$

Lastly, we assume that investors' preferences and the DMG decision rules (described below) are common knowledge and that investors anticipate the future investment decisions of the DMG when expressing their preferences to the group at the licensing stage. This implies that, if the proposal \widehat{X} is adopted by the group, the value to investor n of the

project at time zero is given by

$$V_n(x, \hat{X}) = J_n(x, \hat{X}) - \frac{I}{N}. \quad (12)$$

At time zero each DMG member n decides whether to support the investment, if $V_n(x, \hat{X}) > 0$ or reject it, otherwise.

3.2 Decisions rules, equilibrium and inefficient underinvestment

The DMG makes decisions with simple majority voting at both the investment and abandonment stage. Formally,

Assumption 4. *The DMG make decisions based on a simple majority rule, that is, a decision is undertaken by the group if it is accepted by at least k members, with k the smallest integer larger than $N/2$.*

This assumption is motivated by the fact that majority voting is a pervasive collective decision process in small committees of board of directors and venture capital syndicates.

An equilibrium outcome for the DMG consists of a pair $s = (\lambda, \hat{X}) \in \{0, 1\} \times [0, \infty]$, where $\lambda = 1$ ($\lambda = 0$) denotes that the DMG undertakes (rejects) investment, and the variable \hat{X} denotes the threshold at which the DMG abandons the project. Equivalently, abandonment takes place at the stopping time $\tau_{\hat{X}}$. We adopt the convention that $\hat{X} = \infty$ means that the DMG never abandons. The strategy of forgoing the investment is denoted by $s_N \equiv (0, \infty)$. We denote by $\mathcal{S} \equiv \{0, 1\} \times [0, \infty]$ the set of possible strategies.

In order to state the equilibrium conditions, it is convenient to define the set of *feasible* thresholds \mathcal{X}_f , as the set of threshold \hat{X} such that a proposal to invest at time $\tau_{\hat{X}}$ generates a majority vote.

The following definition describes the equilibrium in our dynamic voting game:

Definition 1. *The strategy $s = (1, \hat{X})$ is an equilibrium outcome of the dynamic voting game if*

1. *The threshold \hat{X} is the largest feasible threshold: $\hat{X} = \sup_X \{X : X \in \mathcal{X}_f\}$.*

2. Conditional on $\widehat{X} \in \mathcal{X}_f$ being implemented in the second stage vote, the investment alternative $\lambda = 1$ generates a majority vote of the DMG against s_N .

The strategy $s = s_N \equiv (0, \infty)$ is an equilibrium outcome of the dynamic voting game if none of the strategies $(1, \widehat{X})$ with $\widehat{X} \in \mathcal{X}_f$ generates majority support of the DMG in an election against s_N .

As stated in our definition of the equilibrium we assume that the equilibrium investment strategy is in the set of feasible strategies \mathcal{X}_f . In this complete information voting game, we therefore assume that no proposal is made unless it is expected to generate a majority vote. We additionally assume that the equilibrium investment threshold is the largest element of the set of feasible strategies. This means that whoever moves first with a proposal that can generate majority will determine the abandonment policy of the DMG.⁷

At the investment stage, investors internalize their expectation on the abandonment policy in their decision of how to vote. Although the voting game is a complete information game, for strategic motives, investors could vote against investment even if investment is the alternative they would favor when acting individually.

3.3 Analysis of the voting game

We now solve for the equilibrium of the dynamic voting game and the resulting behavior of the DMG. To state our results, it is convenient to identify two particular group members: the *left pivot*, n_L , and the *right pivot*, n_R defined as follows.

Definition 2. *When the number of group members N is odd, the left pivot and the right pivot are the same individual*

$$n_L = n_R = \frac{N + 1}{2}$$

⁷Under Assumption 2, the investment voting game could equivalently be modeled as a static game where investors vote over alternative threshold policies at time $t = 0$ as opposed to proposing abandonment at time $t > 0$. Under this alternative view, the set of alternative is one dimensional (indexed by the thresholds \widehat{X}) and it can be shown that the individual preferences are single peaked. We can thus apply the median voting theorem delivering the existence of equilibria (see Black (1948), Black (1958)). This static interpretation of the investment voting game would give rise to multiple equilibria but our dynamic interpretation allows to select only one of them. In the context of a DMG with two members from Section 2, the set of equilibria in the static voting game is $\widehat{X} \in [X_O^*, X_P^*]$ but in the dynamic voting game the only equilibrium is $\widehat{X} = X_O^*$.

When the number of group members is even, the left and right pivot are defined by

$$n_L = \frac{N}{2}, \quad n_R = \frac{N}{2} + 1.$$

The next lemma characterizes the set of feasible investment thresholds

Lemma 1. *The set of feasible investment is given by*

$$\mathcal{X}_f = [0, X_{n_R}^*].$$

Proof: Because abandonment occurs only if a strict majority of members supports it, when n_R propose abandonment, all investors $n < n_R$ will support the abandonment proposal because their optimal abandonment threshold is larger than that of agent n_R . ■

The lemma implies that the bliss point threshold for the right pivot n_R , given by $X_{n_R}^*$ is the first threshold that generates a majority vote. The *decisive coalition* that is, the coalition that generate the outcome of the vote when the threshold $\hat{X} = X_{n_R}^*$ is proposed is given by $\{1, \dots, n_R\}$. The lemma also implies that a candidate equilibrium should be of the form $s = (1, X_{n_R}^*)$ or of the form $s = s_N$. The next proposition characterize the equilibrium outcome of the dynamic voting game.

Proposition 2. *The equilibrium outcome of the voting game is unique and consists of*

1. *Not investing, i.e., $s = s_N$, if*

$$J_{n_L}(x, X_{n_R}^*) \leq \frac{I}{N} \tag{13}$$

2. *Investing and abandoning at the threshold $X_{n_R}^*$, i.e., $s = (1, X_{n_R}^*)$, when*

$$J_{n_L}(x, X_{n_R}^*) > \frac{I}{N}. \tag{14}$$

The proposition implies that the preference of the left pivot is binding for the decision to invest. When condition (13), the coalition formed by group members from the set $\{1, \dots, n_L\}$

vote against investment. This is due to the inequality

$$J_1(x, X_{n_R}^*) < J_2(x, X_{n_R}^*) < \dots < J_{n_L}(x, X_{n_R}^*) < \frac{I}{N}$$

and, by definition of the left pivot in a majority vote, the coalition is able to block investment. When Condition (14) holds, the coalition formed by group members from the set $\{n_L, n_R, \dots, N\}$ vote for investment. This is an implication of

$$\frac{I}{N} < J_{n_L}(x, X_{n_R}^*) < J_{n_R}(x, X_{n_R}^*) < \dots < J_N(x, X_{n_R}^*)$$

and, by definition of the pivots, the coalition leads the DMG to invest.

The proposition also shows that the equilibrium outcome for a DMG with N investors is identical to the equilibrium outcome for a DMG formed with only members n_L and n_R , revealing the importance of the illustrative example from Section 2. In fact our results could also apply for DMG governed with alternative rules than majority as long as they give rise to two decisive members.

3.4 Underinvestment

In this section, we show that the voting equilibrium may generate an inefficient underinvestment. Formally,

Definition 3. *Inefficient underinvestment occurs when the equilibrium outcome is not to invest (s_N) and yet all investors prefer to invest if they were acting individually, that is,*

$$\frac{I}{N} < J_n(x, X_n^*) \quad \text{for } n = 1, \dots, N. \quad (15)$$

The decision to not invest is inefficient under condition (15). If the investors could commit to the most pessimistic abandonment threshold policy $\hat{X} = X_1^*$, investing would generate a unanimous vote. The inequality (15) and condition (10) imply

$$0 < J_1(x, X_1^*) - \frac{I}{N} < J_2(x, X_1^*) - \frac{I}{N} < \dots < J_N(x, X_1^*) - \frac{I}{N}.$$

Therefore, each investor get a positive payoff from investing and they will all vote for investing against s_N provided they can commit to the threshold X_1^* .

Notice that inefficient underinvestment cannot occur if the all the voting power is attributed to a single investor (a *dictator*) for both elections. In that case, the group will follow the dictator's preferred investment timing policy and condition (15) implies that the dictator will accept the licensing. Thus inefficient underinvestment cannot occur when the DMG is ruled by a self interested dictator.

Similarly, inefficient underinvestment cannot occur with a utilitarian planner who perceive a value process following a geometric Brownian motion with the average drift

$$\mu_{\text{pool}} = \frac{\sum_{n=1}^N \lambda_n \mu_n}{N}$$

with weights satisfying $\lambda_n \geq 0$ and

$$\sum_{n=1}^N \lambda_n = 1.$$

Such a planner will adopt the investment threshold X_{pool}^* defined by equation (2) with μ_{pool} replacing μ_n . Because $\mu_1 \leq \mu_{\text{pool}}$, we must have

$$J_1(x, X_1^*) \leq J_{\text{pool}}(x, X_{\text{pool}}^*).$$

This inequality implies that if all individual investors would invest if they were the sole owner of the project, the planner also decides to invest. Hence underinvestment does not occur when such a utilitarian planner is in charge of the DMG decisions at both round.

The next proposition provides a characterization of inefficient underinvestment.

Proposition 3. *If the number of group member is odd, inefficient underinvestment cannot occur in equilibrium. If the number of group members is even, inefficient underinvestment occurs if and only if*

$$J_{n_L}(x, X_{n_R}^*) < \frac{I}{N} < J_1(x, X_1^*). \quad (16)$$

The right inequality of (16) is in fact equivalent to (15).⁸ Since the DMG will adopt the abandonment threshold policy $X_{n_R}^*$ if the investment is undertaken, the left inequality of (16) shows that the left pivot n_L rejects investment. As a result, all investors who are more pessimistic than the left pivot, that is investors $\{1, \dots, n_R - 1\}$, will also reject investment. Therefore, condition (16) implies that inefficient underinvestment as described in Definition 3 occurs.

Underinvestment takes place because the investors $\{1, \dots, n_L\}$ dislike the threshold policy $X_{n_R}^*$. Inefficient underinvestment is an implication of strategic voting in the investment decision: investors anticipate the abandonment threshold taken by the DMG and when some of them dislike the anticipated threshold, they vote against investment. When the group of investors who dislike the anticipated threshold form half of the group, they preclude those who favor investing, that is the investors $\{n_R, \dots, N\}$, to form a majority leading the DMG to reject investment.

Proposition 3 also shows that the underinvestment problem we highlight does not occur with an odd number of voters. The reason is that the left pivot coincides then with the right pivot giving rise to a single median voter $n_m = n_R = n_L$. If investment is undertaken, the DMG follows the median voter abandonment threshold $X_{n_m}^*$. Even though many pessimistic investors may dislike the anticipated abandonment strategy $X_{n_m}^*$, they will not be able to block investing since those in favor of investment, the investors $\{n_m, \dots, N\}$, form a majority.

Another useful intuition to understand the impossibility of inefficient underinvestment with an odd number of investors is to realize that the median voter n_m acts as a dictator of the DMG. This observation is reminiscent of the median voter theorem from the canonical spatial voting models. In fact, for both elections, if an alternative is taken by the DMG it must be that the median voter adhere to that alternative: With ranked single peaked preferences and an odd number of voters, any majority should include the median voter.

⁸To see this recall that condition (10) implies

$$J_1(x, X_1^*) < J_2(x, X_1^*) < J_2(x, X_2^*) < J_3(x, X_2^*) < J_3(x, X_3^*) < \dots < J_N(x, X_N^*).$$

Therefore, $J_1(x, X_1^*) = \min_n J_n(x, X_n^*)$ and the equivalence of the right inequality in (16) with condition (15) obtains.

Therefore, the DMG choice will not change if the median voter was the dictator of the DMG. We know however that inefficient underinvestment cannot take place if the DMG is ruled by a dictator. The reason is that if all group members would invest if they act individually, the dictator will not reject licensing because the dictator is by definition acting individually.

4 Quantitative implications

4.1 Impact of polarization

Notice that in the underinvestment condition (16) only three agents matter: the two pivots, n_L and n_R , and the most pessimistic agent 1. In particular, the equilibrium outcome would not change if the optimistic agent were to become even more optimistic. This result is due to the fact that voting does not allow to express the intensity of preferences. With a majority vote where each person can only cast one vote, the DMG ignores the intensity of preferences over alternative.

Figure 3 describes the underinvestment conditions for a group of $N = 2$ agent. Note that in this case, the left pivot is agent P (pessimist) and the right pivot is agent O (optimist).⁹ It follows that for a two-agent group, condition (16) will always be satisfied as long as the two agents have different beliefs about the growth rate of X_t . The light gray-shaded area in between the curves $J_P(x; X_O^*)$ and $J_P(x, X_P^*)$ represents the combination of current cash flow x and investment cost I for which there is underinvestment. Intuitively, for any given level of x , $I/2$ falls within this region then agent P , aware that the optimal abandonment threshold is X_O^* , will find unattractive to invest in the project and therefore will fail to support it. Because investment requires strict majority, the project will not be funded, even if, under the optimal abandonment policies, each agent would agree to fund it.¹⁰

⁹Note that abandonment X_P^* of the left pivot, agent P , is located to the *right* of the abandonment X_P^* of the right pivot O . The labeling of “left” and “right” pivot refers to the ranking of their assessment of the cash flow growth rate, $\mu_P < \mu_O$.

¹⁰Note that we do not include values of investment falling below the abandonment value D . These values are not economically relevant because they would be generating a “money pump”: if $I < D$ then the optimal strategy would be to invest and immediately abandon.

Figure 3 also illustrate the effect of polarization of beliefs on underinvestment. We consider the case in which agent O entertains an even more optimistic forecast of cash flow growth. This is reflected in a lower abandonment threshold $X_O^{*'}$. As the figure shows, such a lower threshold further lowers the value that agent P attaches to a project. Overall the effect is to enlarge the underinvestment region by adding the darker-shaded area.

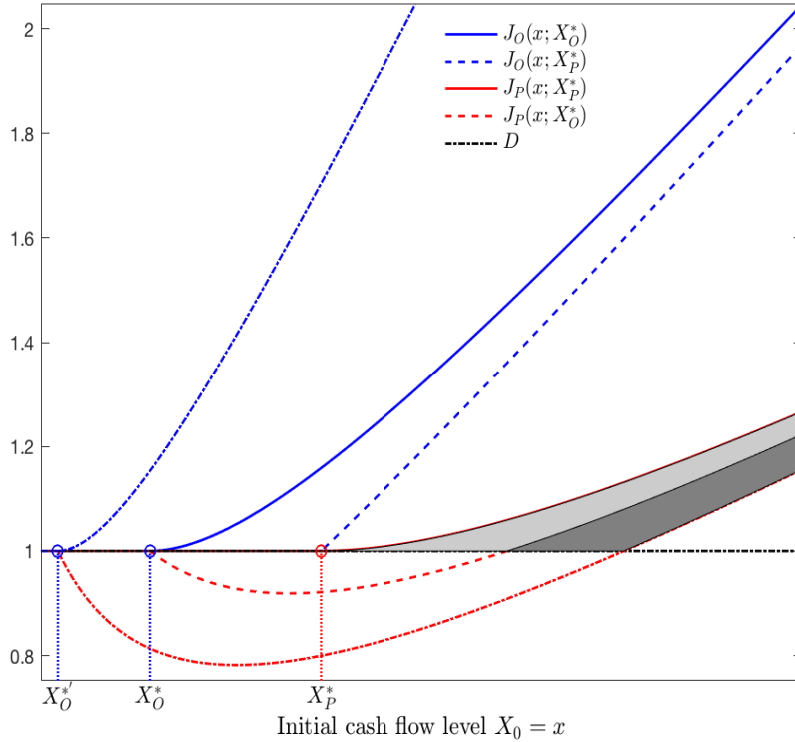


Figure 3: Belief polarization and underinvestment in a two-agent group.

The figure reports the value function of both agents as a function of the initial cash flow x under different abandonment policies. Parameter values: $r = 0.05$, $\sigma = 0.3$, $D = 1$, $\mu_P = 0.01$, $\mu_O = 0.03$, $\mu_{O'} = 0.039$.

Figure 4 describes the underinvestment conditions for a group of $N = 4$ agent. Agent 2 and 3 are, respectively, the right and left pivot. The shaded area represents the set of cash flow and investment cost (x, I) for which the underinvestment condition (16) occurs. From the figure we can infer that as the right and left pivot assessment of the drift of X_t diverge, the underinvestment region increases. For example, as agent 3 becomes more optimistic, the

value $J_2(x, X_3^*)$ deteriorates thus causing the gray-area to enlarge. Similarly for the case in which agent 2 were to become more pessimistic.

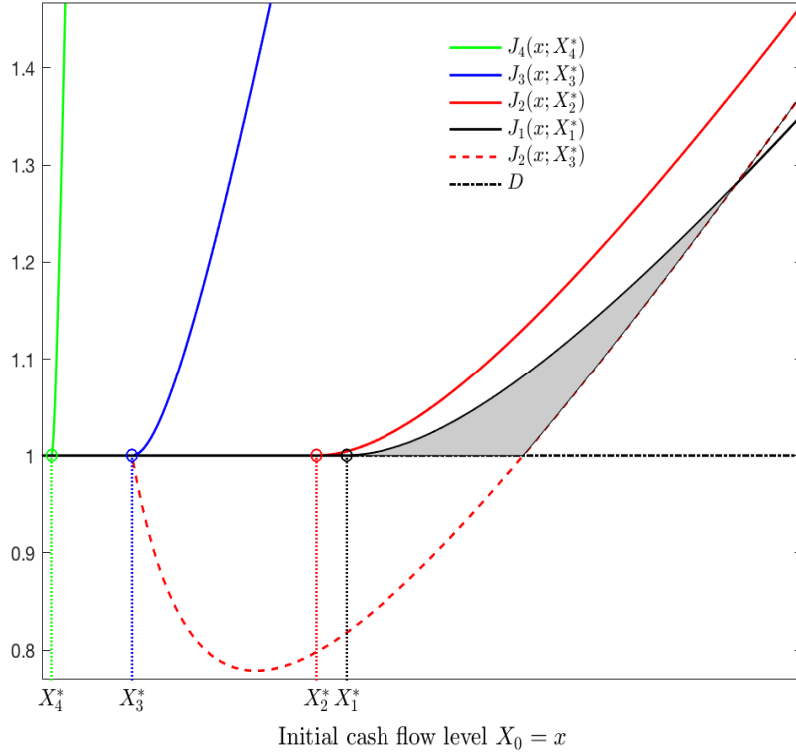


Figure 4: Belief polarization and underinvestment in a four-agent group.

The figure reports the value of the four agents as a function of the initial cash flow x for different abandonment policies. Parameter values: $r = 0.05$, $\sigma = 0.3$, $D = 1$, $\mu_1 = 0.01$, $\mu_2 = 0.015$, $\mu_3 = 0.04$, $\mu_4 = 0.049$.

The figures also makes it clear that polarization—that is, the divergence between agent 2 and agent 3’s beliefs about the expected growth of the project value—is a necessary requirement for underinvestment to occur. To see this, suppose that agent 2’s belief μ_2 increases towards μ_3 . As μ_2 increases, the distance between X_2^* and X_3^* in the figure shrinks, implying that there is a smaller region of cash flow and investment costs for which condition (16) is satisfied. Similarly, as agent 3’s beliefs decrease to towards μ_2 , X_3^* the underinvestment region will disappear.

4.2 Project uncertainty and underinvestment

The investment and abandonment decisions analyzed in the previous sections are the workhorse framework of the real option approach to project valuations. A common and well understood insight from that literature is that volatility makes the option to continue receiving cash flows more valuable and consequently delays the exercise of the abandonment option. In this section we investigate the effect of volatility on inefficient underinvestment. The main conclusion is that if polarization is sufficiently strong, then inefficient underinvestment is always possible, independent of the level of volatility. This result is important in that it highlights a unique channel through which underinvestment can occur, that is, the polarization of beliefs.

As in the standard real option setting, high volatility will increase the option value of abandoning (a put option) and lower the abandonment threshold for each player. However, when beliefs are sufficiently polarized, inefficient underinvestment can occur *at any level of volatility*. It is important to emphasize that inefficient underinvestment is conceptually different from “delayed” investment. In our setting the agent do not face a timing decision when deciding to invest I and therefore the problem is different from the standard investment timing in the real option literature.

We illustrate the effect of uncertainty on underinvestment in the context of a four-agent groups analyzed in the previous sections. Figure 5 reports, for a range of values of the initial cash flow x , the *minimum* amount of disagreement, between agent 2 and 3, that would lead to the occurrence of underinvestment. Formally, we are reporting, for different value of volatility the difference $\mu_3 - \bar{\mu}_2$ where $\bar{\mu}_2$ is such that

$$J_2^{\bar{\mu}_2}(x; X_3^*) = J_1(x; X_1^*),$$

and where $J_2^{\bar{\mu}_2}(x; X_3^*)$ is the value function of an agent with beliefs $\bar{\mu}_2$, assessed under agent 3’s threshold X_3^* . Geometrically, $\bar{\mu}_2$ represents the beliefs for which the curve $J_2(x; X_3^*)$ in Figure 4 intersects $J_1(x; X_1^*)$ on the right-most side of the graph. When $\mu_2 < \bar{\mu}_2$, the gap in beliefs between agent 2 and agent 3 is sufficiently wide. We then have $J_2^{\mu_2}(x; X_3^*) < J_1(x; X_1^*)$

and underinvestment can occur at the cash flow level x , for some values of I . When $\mu_2 \geq \bar{\mu}_2$, the dispersion in beliefs between agent 2 and agent 3 is narrow and we have $J_2^{\mu_2}(x; X_3^*) \geq J_1(x; X_1^*)$. As a result, underinvestment cannot occur at the cash flow level x .

Figure 5 shows two important points. First, each of the curves reported are increasing in the cash flow x . This means that for any given level of volatility, a higher level of initial cash flow requires a higher belief discrepancy between agent 2 and agent 3. To understand this result, consider the region of high cash flow level in Figure 4 and recall that for underinvestment to occur, it must be that agent 2 value when using the suboptimal group policy (red dashed curve) must fall below agent 1's optimal value (black curve). At that region, the abandonment put option is out of the money and, for all agents, the option component of the investment value is very small compared to the perpetuity component. As the cash flow level x grows, the option value for each agent asymptotically converges to the value of the perpetuity given by $x/(r - \mu_i)$ for agent n . As can be visualized in Figure 4, this implies that the discrepancy between the optimal value of agent 2 (red curve) and the optimal value of agent 1 (black curve) increases as the cash flow level x grows. This means that for high level of cash flow x , in order to offset the wedge between the optimal value of agent 1 and the optimal value of agent 2, using the suboptimal policy X_3^* must drastically penalize agent 2's value. Only extreme difference of opinions between agent 2 and agent 3 can penalize agent 2 value and give rise to underinvestment.

Second, for low level of cash flows, the disagreement needed to generate underinvestment is *higher* when underlying volatility is high. In other words, the effect of volatility on underinvestment is not uniform across different level of cash flow. There are two forces at play. The first force operates through the "signal-to-noise" ratio. Intuitively, a difference of one percent in the growth rate of cash flow is much less relevant in a high volatility environment than in a small volatility environment. So for the disagreement of two agents to matter in a high volatile environment, their difference of opinion need to be sufficiently polarized. This explains why the minimal belief spread required for underinvestment under high volatility (red line) is highest for low level of cash flows.

However, as the cash flow increases, the relationship between the two lines in the figure gets reversed. In the example of the figure, the difference of beliefs required for underinvestment to occur is actually *higher* in the low volatility environment. In general, the high level of cash flow dampens the “signal-to-noise” effect and reduces the difference between the belief spreads across the two curves. In summary, Figure 5 shows that underinvestment can occur at any level of volatility and that volatility mitigates underinvestment—i.e., higher belief discrepancies are needed for underinvestment to occur—when the option to abandon is in the money (low x) and exacerbates underinvestment—i.e., lower belief discrepancies are needed for underinvestment to occur—when the option to abandon is out of the money (large x).

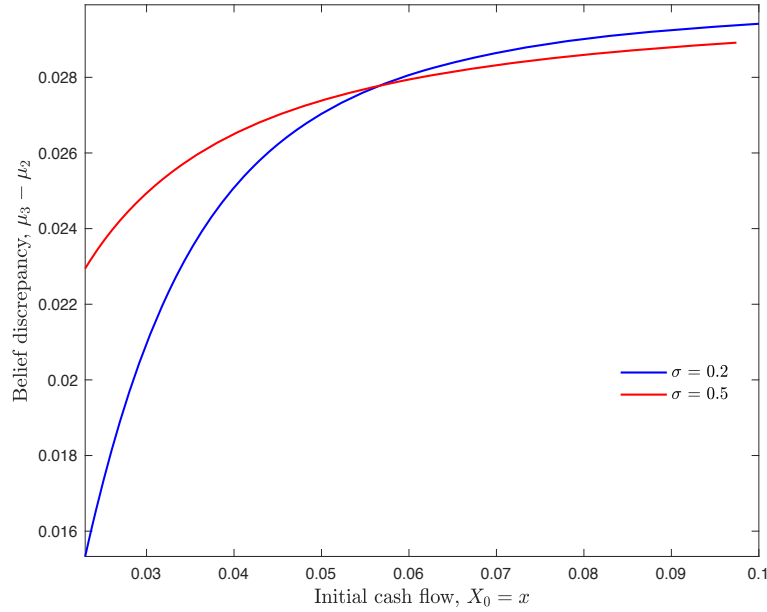


Figure 5: Uncertainty and underinvestment in a four-agent group.

The figure reports the minimum discrepancy between agent 2 and agent 3’s beliefs, $\mu_3 - \mu_2$ that lead to the existence of underinvestment, given an initial level of cash flow $X_0 = x$. Each curve corresponds to a different level of underlying volatility. Parameter values: $r = 0.05$, $D = 1$, $\mu_1 = 0.01$, $\mu_3 = 0.04$, $\mu_4 = 0.049$.

5 Other governance rules

In this section, we discuss whether the underinvestment problem we highlight occurs under different governance rules within a DMG. In Section 5.1 we consider groups in which agents have veto power and groups who mediate disagreement using super-majority rules. In Section 5.2 we consider the possibility of bargaining between disagreeing agents within the group.

5.1 More general voting protocols

As noted earlier, there cannot be inefficient underinvestment if the group delegates all the decision power to a single agent—a “dictatorial” governing rule. Unanimity is an alternative governance rule that we will also explore. Dictatorship and unanimity are polar cases of voting protocols within the class of *non-collegial* governance rules, that is, rules in which some members of the DMG have veto power, *the vetoers*. Majority and super-majority are voting protocols within the class of *collegial*, that is rules in which no group member has veto power. In the next two subsections we explore whether the inefficient underinvestment problem can be mitigated by changing the voting rule. The main result from this analysis is that the inefficiency will either be unaffected or worsened by the alternative voting procedures that we explore.

5.1.1 Voting rules

Given a group of $\mathcal{N} = \{1, \dots, N\}$, a voting rule is defined as a set of *decisive coalitions* $\mathcal{D} \subseteq 2^{\mathcal{N}}/\{\emptyset\}$ (Austen-Smith and Banks, 1999). If a proposal is made and all members of a decisive coalition $\mathcal{C} \in \mathcal{D}$ votes to accept (reject) it, then the DMG accepts (rejects) the proposal. Therefore, defining the voting rules is equivalent to fixing the set \mathcal{D} of decisive coalitions.

The next definition formally characterizes a majority governance rule in which some group member have veto power.

Definition 4. A governance rule is called *majority with vetoers* if and only if there is a set of investors $\mathcal{V} = \{v_1, v_2, \dots, v_m\}$ with $m \leq N$ and $v_1 < v_2 < \dots < v_m$ such that

$$\mathcal{D} = \left\{ \mathcal{C} \subseteq \mathcal{N} : |\mathcal{C}| > \frac{N}{2} \text{ and } \mathcal{V} \subseteq \mathcal{C} \right\}$$

Majority rule with a single vetoer captures the idea that the DMG gives special veto power to a single member, such as the chair of the board of directors or the founder of the corporation. Unanimity is a polar example of such rule, where every agent has veto power, that is $\mathcal{V} = \mathcal{N}$ and hence the set of decisive coalitions is the singleton $\mathcal{D} = \{\mathcal{N}\}$.

A DMG ruled by a dictator $n_d \in \mathcal{N}$ is defined by the set

$$\mathcal{D} = \{\mathcal{C} \subseteq \mathcal{N} : n_d \in \mathcal{C}\}$$

The next definition formally characterizes the super-majority or quota non-collegial rule.

Definition 5. A governance rule is called *super-majority or quota* if there exist q with $\frac{N}{2} + 1 < q < N$ such that

$$\mathcal{D}_q = \{\mathcal{C} \subseteq \mathcal{N} : |\mathcal{C}| \geq q\}.$$

5.1.2 Unanimity rule

When the DMG make decisions based on Unanimity, the following proposition holds

Proposition 4. Assume that the DMG utilizes the unanimity governance rule. Inefficient underinvestment occurs if and only if

$$J_1(x, X_N^*) < \frac{I}{N} < J_1(x, X_1^*). \quad (17)$$

With unanimity rules, the DMG will abandon only when all investors agree to abandon. Thus the DMG will abandon at the lowest threshold X_N^* . Anticipating this, the more pessimistic investor 1 will vote against investment if and only if the left inequality of (17) holds. With the unanimity rule, the DMG will thus not invest since investor 1 votes against investment. This result is very similar to the underinvestment with majority that we describe

in Proposition 3. The equivalence can be made clearer by observing that with a unanimity rule, the left pivot is investor 1 and the right pivot is investor N . With this observation in mind, it is possible to realize that the underinvestment region described in Figure 4 with four investors, will expand under unanimity relative to majority. This means that the underinvestment problem is worsened under unanimity.

5.1.3 Majority rule with vetoers

We assume in this section that DMG makes decisions based on a majority rule with vetoers. The following proposition characterizes the equilibrium outcome relative to the equilibrium outcome in the absence of vetoers.

Proposition 5. *Assume that the DMG utilizes the majority with vetoers governance rule.*

If the vetoer v_m is not more optimistic than the right pivot, $v_m \leq n_R$, then the equilibrium outcome is not altered and inefficient underinvestment occurs if and only if the number of investors in the DMG is even and condition (16) is satisfied.

If the vetoer v_m is more optimistic than the right pivot, $v_m > n_R$, irrespective of whether the number of investors in the DMG is even or odd, inefficient underinvestment occurs if and only if

$$J_{n_L}(x, X_{v_m}^*) < \frac{I}{N} < J_1(x, X_1^*). \quad (18)$$

In presence of vetoers the first abandonment proposal that will be accepted by the group must include a majority of investors *and* all vetoers. Therefore the investment threshold that the DMG applies should be either the right pivot n_R or the vetoer v_m . If the vetoer v_m is (weakly) less optimistic than the right pivot, then the DMG will apply the threshold policy $X_{n_R}^*$. The group will not invest under condition (16) in the exact same way as it occurs in the absence of vetoers: half of the investors will block the investment. If the vetoer v_m is more optimistic than the right pivot then the group will delay the investment (relative to a DMG with no vetoer) and apply the threshold $X_{v_m}^*$. Similar to the no vetoer case, the underinvestment condition is given by (18).

It is important to observe that condition (18) is less stringent than condition (16) as a result, when DMG utilizes a majority governance rule, the underinvestment region is larger in presence of vetoers than in the absence of vetoers. This is due to the fact that $X_{n_R}^* < X_{v_m}^*$ and hence $J_{n_L}(x, X_{v_m}^*) < J_{n_L}(x, X_{n_R}^*)$. This can be seen more easily in Figure 4 which describe the underinvestment for a group of four investors. Assuming a majority rule with the vetoer $\mathcal{V} = \{4\}$, the DMG will then apply the threshold policy X_4^* . We see then that introducing the vetoer 4 to a majority governance rule has the effect of increasing the shaded underinvestment area describing the the interval of investment costs that give rise to inefficient underinvestment. Similarly to the unanimity rule, introducing a veto rule to a simple majority rule can worsen the inefficient underinvestment problem.

The above results also implies that the presence of veto power can generate underinvestment also in a group with an odd number of agents. To see this, consider, for example, the case of a three-agent group. According to our analysis above, in the absence of veto power, the group will behave according to the preference of the median group member, agent 2. If however veto powers rests in the hand of agent 3, then investment will occur at the threshold X_3^* and therefore the underinvestment condition is $J_2(x, X_3^*) < J_1(x, X_1^*)$. In general underinvestment will occur in a odd-numbered group if veto power is in the hands of agents that are more optimistic than the median agent.

5.1.4 Non collegial rules: super-majority

Proposition 6. *Assume the DMG follows a super-majority rule with q being an integer satisfying $N/2 + 1 < q < N$. Inefficient underinvestment happens when*

$$J_{N-q+1}(x, X_q^*) < \frac{I}{N} < J_1(x, X_1^*). \quad (19)$$

With a super-majority rule, underinvestment will occur independently of whether the number of group members is odd or even. This is because the investment policy adopted by the DMG is X_q^* . For the super-majority rule, the left pivot becomes the member $n_R = N - q + 1$ and the right pivot is the member $n_L = q$. Super-majority creates a natural wedge between

the left and the right pivot without requiring an even number of investors. Notice that condition (19) is naturally less stringent than (16) because $X_q^* > X_{n_R}^*$ and $J_{N-q+1}(x, X_q^*) < J_{n_L}(x, X_q^*) < J_{n_L}(x, X_{n_R}^*)$. We thus conclude that the inefficient underinvestment problem is worsened under a super-majority rule.

5.2 Bargaining in presence of disagreement

So far in the paper, we assumed that, under majority voting, the DMG voting outcome is a tie as can be the case in a DMG with an even number of investors, the DMG does not invest or does not abandon. This implies that not investing and not shutting down are the (exogenous) the status quo for our voting game.

In this section, we consider without loss of generality the case of a two-agent DMG (P and O) in which disagreeing investors engage in bargaining. Specifically,

Assumption 5. *When the two DMG members disagree on the investment threshold a bargaining game ensues in which agent O succeeds with probability ρ .*

The above assumption capture the implications of bargaining without modeling the details of the negotiations. The assumption states that the outcome desired by the optimistic voter is obtained with an exogenous independent probability ρ . The parameter ρ represents the bargaining power of the most optimistic agent who desires a late abandonment policy. When $\rho = 1$, we obtain the case described in the illustrative example of Section 2.

The following proposition shows that, as long as the parameter $\rho \neq 0$, underinvestment can occur.

Proposition 7. *Assume $x > X_P^*$ and that Assumption 5 holds with $\rho \neq 0$. If*

$$(1 - \rho)J_P(x, X_P^*) + \rho J_P(x, X_O^*) < \frac{I}{2} < J_P(x, X_P^*)$$

then inefficient under-investment occurs with probability $1 - \rho$.

When $\rho = 1$ the underinvestment is most severe, as can be seen from Figure 3. As $\rho \rightarrow 0$, agent P becomes the dictator and therefore no underinvestment will be possible. Although the underinvestment area is reduced, underinvestment problem is still an issue in this setting.

6 Conclusion

In this paper, we have examined coordination frictions and consequent investment inefficiencies that arise when a group instead of an individual makes corporate decisions. More specifically, we examine the acquisition and subsequent management of a real option by a DMG whose members have heterogeneous beliefs. We show that in such a setting the DMG may reject an investment opportunity on behalf of the corporation even though each member of the DMG sees the opportunity as valuable. This happens when views of group member are polarized. In the dynamic setting we analyze, inefficiencies arises when the composition of the decisive coalition within the DMG changes over the life of the project.

We also characterize and contrast the role of polarization and volatility in investment inefficiency. Volatility is an important determinant of the value and management of a real option. Increases in volatility can lead to a delay in investment as the value of waiting increases. Such delays, however, are optimal and do not constitute inefficiencies. In contrast, polarization of beliefs leads to inefficiencies in that investments are not just delayed but lost altogether. Our result apply generally to groups that mediate conflicts according to either majority or super-majority voting rules and to groups in which agents have veto power. While descriptive of actual final decision making in groups, in reality votes are cast in the context of dynamic pre-vote interactions. Developing theories that are more descriptive of the political economy of corporate finance will be the subject of future research.

A Appendix: Proofs

Lemma 2. For any $\widehat{X} < x$, the Laplace transform of the hitting time $\tau_{\widehat{X}}$ is given for any $r > 0$ by

$$E[e^{-r\tau_{\widehat{X}}}] = \left(\frac{\widehat{X}}{x}\right)^m$$

with

$$m = \frac{(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2r}}{\sigma^2}.$$

Proof: See for example, Karatzas and Shreve (3.5.C). ■

Proof of Proposition 1

From equation (1), we see that $J(x, \tau_{\widehat{X}})$ has two terms.

From Lemma 2, the second term is given by

$$E(De^{-r\tau_{\widehat{X}}}) = D \left(\frac{\widehat{X}}{x}\right)^m.$$

To calculate the first term, an integration by part yields

$$X_{\tau_{\widehat{X}}}e^{-r\tau_{\widehat{X}}} - x = (\mu - r) \int_0^{\tau_{\widehat{X}}} e^{-rt} X_t dt + \int_0^{\tau_{\widehat{X}}} \sigma X_t dW_t^1.$$

Taking the expectation of the above equation gives

$$E \left[\int_0^{\tau_{\widehat{X}}} e^{-rt} X_t dt \right] = \frac{1}{r - \mu} [x - E(X_{\tau_{\widehat{X}}} e^{-r\tau_{\widehat{X}}})]$$

We define the probability measure Q with the density

$$\left(\frac{dQ}{dP}\right)_{\mathcal{F}_t} = e^{-\frac{\sigma^2}{2}t + \sigma W_t^1}.$$

Girsanov theorem shows that the process W^Q defined by

$$W_t^Q = W_t^1 - \sigma t$$

is a Brownian motion under Q . Under the probability Q , the process X is also a geometric Brownian motion following

$$dX_t = (\mu + \sigma^2)X_t + \sigma X_t dW_t^Q, \quad X_0 = x > 0$$

We then have

$$\begin{aligned} E(X_{\tau_{\hat{x}}} e^{-r\tau_{\hat{x}}}) &= x E(e^{-\frac{\sigma^2}{2}\tau_{\hat{x}} + \sigma W_{\tau_{\hat{x}}}^1} e^{-(r-\mu)\tau_{\hat{x}}}) \\ &= x E_Q(e^{-(r-\mu)\tau_{\hat{x}}}) \end{aligned}$$

From Lemma 2, we then get

$$E(X_{\tau_{\hat{x}}} e^{-r\tau_{\hat{x}}}) = x \left(\frac{\hat{X}}{x} \right)^{\frac{(\mu + \frac{1}{2}\sigma^2) + \sqrt{(\mu + \frac{1}{2}\sigma^2)^2 + 2\sigma^2(r-\mu)}}{\sigma^2}}$$

It can be checked that

$$\frac{(\mu + \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} = m + 1$$

and thus

$$E(X_{\tau_{\hat{x}}} e^{-r\tau_{\hat{x}}}) = x \left(\frac{\hat{X}}{x} \right)^{m+1}.$$

To sum up, we have

$$\begin{aligned} J(x, \tau_{\hat{X}}) &= \frac{1}{r - \mu} \left[x - x \left(\frac{\hat{X}}{x} \right)^{m+1} \right] + D \left(\frac{\hat{X}}{x} \right)^m \\ &= \frac{x}{r - \mu} + \left(\frac{\hat{X}}{x} \right)^m \left[D - \frac{\hat{X}}{r - \mu} \right] \end{aligned}$$

■

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