Does the Market Understand Time Variation in the Equity Premium?*

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December 2022

Abstract

We test whether the market has intertemporally consistent expectations about the log equity premium. We use option prices to estimate the expected future equity premium (the forward rate) and compare this to the equity premium estimated in the future (future spot rate). Forward rates are strong predictors of future spot rates, suggesting that the market qualitatively understands variation in the equity premium. The market does, however, make predictable and quantitatively significant forecast errors. To match the data, we propose a model in which an increase in the equity premium causes investors to overestimate the future equity premium.

Keywords: asset pricing, expected stock returns, rational expectations, diagnostic expectations, time-varying discount rates

JEL classification: G10, G12, G40

*We thank Pedro Bordalo, John Campbell, Fousseni Chabi-Yo, Nicola Gennaioli, Stefano Giglio, Ben Hébert, Martin Lettau, Ian Martin, Jonathan Parker, Andrei Shleifer, David Thesmar, and seminar participants at UC Berkeley Haas, Oxford University, Warwick Business School, and MIT Sloan for helpful comments. We are grateful to the Fama-Miller Center, the Asness Junior Faculty Fellowship, and the Fischer Black Doctoral Fellowship at the University of Chicago for financial support. Gandhi and Gormsen are at the University of Chicago, Booth School of Business (mihir.a.gandhi@chicagobooth.edu and niels.gormsen@chicagobooth.edu), and Lazarus is at the MIT Sloan School of Management (elazarus@mit.edu).

Why do stock prices move? At an accounting level, movements in stock prices must reflect changes in expected future cash flows, interest rates, or expected excess returns (equity premia), with empirical evidence suggesting that the majority of price movements reflect changes in equity premia. The traditional interpretation of this fact is that investors understand that equity premia vary over time, and that this variation arises because the return investors require for holding stocks varies. Periods with high prices, for instance, are periods where investors require low returns for holding stocks. An alternative behavioral view argues that investors do not understand variation in equity premia. Under this view, periods with high prices are periods where investors have irrationally high expectations about future prices, which eventually materializes in low realized returns and thus low observed equity premia (Shiller 1981, Bordalo, Gennaioli, LaPorta, and Shleifer 2022).

The literature has assessed the competing views by studying investors' expectations about the equity premium. Martin (2017) extracts equity-premium expectations from option prices and finds that these expectations are good predictors of realized excess returns. In contrast, a survey-based literature finds that investors' stated perceptions of the equity premium do not fully line up with the objective equity premium (Greenwood and Shleifer 2014, Nagel and Xu 2022b).

Surprisingly, though, the literature has mostly studied investors' expectations about the contemporaneous equity premium at a given date, as opposed to expectations about future equity premia. Stock prices, however, depend on the full term structure of future equity premia, so quantifying the impact of expectations on prices requires a study of expected future equity premia. Moreover, the term structure of future equity premia represents a powerful laboratory for identifying the dynamics of expectations and expectation errors, and it lends itself to new, and sharper, tests of the rationality of these expectations.

This paper introduces new methods to study the behavior of expectations about future equity premia. We use a sample of global equity options prices to extract the perceived equity premium in the eyes of the "representative agent," by which we mean an investor willing to hold the market portfolio. After extracting this perceived equity premium at multiple horizons, we can isolate and study the behavior of the expected future equity premium. We show that we can test the intertemporal consistency of expectations of the equity premium under significantly weaker identifying assumptions than those required for testing whether expected returns are themselves rational. In

addition, our approach allows us to study the dynamics of return expectations in much richer data — and with greater statistical power — than is usually attainable in the survey-based literature, as we can extract the perceived equity premium at various horizons in a sample that runs over 30 years and spans 20 stock market indexes.

Our main results are as follows. First, the representative investor *qualitatively* understands time variation in the equity premium. During the global financial crisis and the recent Covid crisis, for instance, investors understood not only that the equity premium was high, but also that it was expected to come back down quickly. Our second main finding, however, is that the representative investor appears to make predictable mistakes in expectations about the dynamics of the equity premium. We quantify the impact of these forecast errors on equity prices, finding that they explain a sizeable part of the observed volatility in stock prices. We then provide two alternative explanations for the behavior of forecast errors, focusing in turn on the potential role of the stochastic discount factor and imperfect belief formation.

To understand the nature of our analysis, let us start by defining the following notation for the expected log equity premium over n periods:

$$\mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n} - r_{t,t+n}^f].$$

If at time t we observe the expected n-period and $(n+1)$-period returns, we can back out the expected equity premium between these two periods, which we call the forward rate:

$$f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}].$$

After n periods, we can compare this forward rate to the realized one-period spot rate, $\mu_{t+n}^{(1)}$. Under the law of iterated expectations, the forward rate should be an unbiased predictor of the realized spot rate, which means that forecast errors,

$$\varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)},$$

should have an expected value of zero and be unpredictable by any time-t information.

These forecast errors can be estimated from option prices under mild conditions. For simplicity, we start by assuming a representative agent with log utility over returns on the stock market. In this case, we can directly estimate expected stock returns at different horizons from option prices, allowing us to estimate forward rates and spot rates at different points in time and thereby realized forecast errors. However, an important

contribution of our paper is to show that we can extract the forecast errors under more general conditions than log utility: under any stochastic discount factor (SDF), we can identify expected forecast errors down to a small risk premium term that can be managed empirically or through theory. Our estimates of forecast errors are thus useful for a wide range of specifications of preferences and the data generating process. Our analysis is most easily understood as a study of the expectations of the representative agent, but we emphasize that our methods extend to cases of heterogeneous agents and can be used to understand the expectations of any investor who is content to hold the market portfolio.

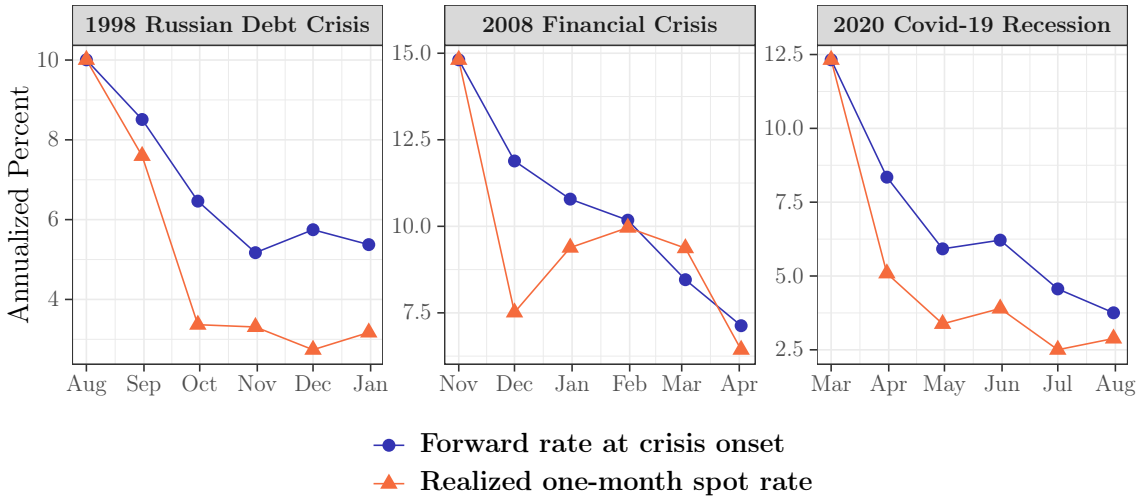
We estimate and study forward rates, spot rates, and forecast errors in a large, global sample of option prices. The sample runs from 1990 to 2021 and includes both the global financial crisis and the Covid-19 crisis. Our sample contains option prices on 20 different stock markets, including leading market indexes such as S&P 500, Euro Stoxx 50 (SX5E), FTSE 100, and Nikkei 225. The sample thus expands substantially on the U.S. sample that is often studied in the asset pricing literature.

We start our empirical analysis by focusing on our estimated spot and forward rates, which correspond to those perceived by the representative agent only in the log-utility case. Spot rates estimated under this assumption are known to fluctuate substantially over time and increase in bad times like the global financial crisis and the Covid-19 crisis (Martin 2017). As the novel aspect of our analysis, we find that forward rates are good predictors of realized spot rates. In a simple regression of realized spot rates on forward rates, we obtain a slope coefficient close to 0.7 once accounting for measurement error. The R^2 is 20% at the 6-month horizon, implying substantial predictive power of forward rates over the future equity premium.

To illustrate the predictive power of forward rates over spot rates, Figure 1 shows forward rates and realized spot rates in the U.S. during three crises: the 1998 Russian debt crisis, the 2008 global financial crisis, and the early-2020 Covid crisis. The blue circles show one-month forward rates maturing $n = 0$, 1, 2, 3, 4, and 5 months from the first plotted date t (which, in all cases, is soon after the crisis onset). The orange triangles show ex post realized one-month spot rates as of months t, $t + 1$, ..., $t + 5$. In all cases, the spot equity premium (the first point) increases substantially as of the crisis onset. Forward rates also increase, but the forward curve is strongly downward sloping, suggesting that investors expect significant mean reversion of spot rates in subsequent months. Going forward, the spot rate indeed decreases substantially as the crisis recedes

Figure 1
U.S. Forward Rates and Realized Spot Rates in Three Crises

This figure plots the forward curve $f_t^{(0)}, f_t^{(1)}, \cdots, f_t^{(5)}$ (blue) and the corresponding ex post spot rates $\mu_t^{(1)}, \mu_{t+1}^{(1)}, \cdots, \mu_{t+5}^{(1)}$ (orange) in the U.S. as of three dates: August 31, 1998 for the Russian debt crisis, November 28, 2008 for the financial crisis, and March 31, 2020 for the Covid-19 recession.



in all cases. These patterns are consistent with investors correctly expecting the equity premium to decrease quickly following these crises.

In all cases, however, forward rates are too high at the peak of the crisis relative to future realized spot rates. This suggests that while investors understood that the equity premium would decrease in the future, they appear to have underestimated the speed of mean reversion. One interpretation of this finding is that the rebound following the crisis was driven by news, like policy shocks, that were unexpected ex ante. Another interpretation is that investors systematically overreact to high spot rates observed during the crises and mistakenly believe that spot rates are going to stay elevated by more and for longer than what one should rationally expect.[1]

We analyze such patterns of systematic overreaction more formally in time-series and panel regressions. We first run predictive regressions of realized spot rates on past forward rates, and similarly of forecast errors on past forward rates. Both sets of results confirm the dynamics discussed above, namely that high forward or spot rates today are associated with forward rates that are higher than the future realized spot rate. We next change the predictor variable from the forward rate to the three-month update in the forward rate, in the spirit of Coibion and Gorodnichenko (2015). We similarly find

[1]Alternatively, it is in principle possible to rationalize these patterns as arising from discount-rate risk pricing; we discuss this alternative interpretation further below.

that upward revisions in the forward rates are associated with realized spot rates that are lower than expected. The predictable forecast errors are quantitatively substantial in explaining variation in price levels, particularly during crises, when elevated equity-premium expectations contribute meaningfully to the observed stock-price drops.

We next investigate the conditions under which these patterns can be rationalized through the behavior of the stochastic discount factor. When going beyond the log-utility assumption, our estimates of forward and spot rates from option prices contain a risk premium that is potentially large. But importantly for our analysis, the effect on forecast errors is modest. In particular, for the forecast errors, which are spot rates minus forward rates, the risk premium term on the forward rate largely cancels out with the risk premium term on the spot rate, with the remaining risk correction being an order of magnitude smaller than the well-known correction from Martin (2017). Our baseline estimate of forecast errors is thus a useful measure of true forecast errors for a wide range of specifications of the stochastic discount factor. However, the remaining risk premium correction nonetheless implies that for large departures from log utility, the stochastic discount factor can imply risk-price dynamics under which the apparent forecast errors can be rationalized.

Concretely, in order to rationalize the behavior of forecast errors, the stochastic discount factor must feature a highly volatile price of risk on shocks to the equity premium. In bad times, investors must be highly averse to upward shocks to the equity premium; in good times, investors must be close to indifferent toward such shocks, and the relevant SDF-related covariance must in fact switch signs. The notion that the price of risk increases in bad times is consistent with previous research (Campbell and Cochrane 1999), but the pace at which the price of risk must change, and the range of values it must take, are properties that appear difficult to reconcile with standard models. The goal of this analysis, however, is to provide useful disciplining tools for any such model to match the behavior of the term structure of equity premia.

Alternatively, we show that one can account for the observed forecast errors using a simple calibrated model featuring excessively volatile expectations of future equity premia. Our simple model builds on the diagnostic belief models of Bordalo, Gennaioli, and Shleifer (2018) and Bordalo et al. (2019). Beliefs are such that the representative agent puts too much weight on the most recent change in the equity premium when forming expectations about the future equity risk premium, leading to the type of overreaction discussed above. We calibrate the model under log utility with the estimates

from Bordalo, Gennaioli, and Shleifer (2018). The resulting model produces expectation errors that are statistically close to the ones we observe in the data along most dimensions. As such, the model appears to capture the dynamics of expectations well over this period. We emphasize, however, that while the agents in this model have irrational expectations, the agents understand the direction in which the future equity risk premium is going to evolve: when the equity premium is high today, the agents understand that the premium will be lower in the future, and vice versa. Expected returns accordingly *increase* when lagged returns are low, and vice versa. Such dynamics are necessary to explain the data under the law of one price, unless one is willing to assume highly non-standard preferences. Our model is accordingly not an unqualified success for all possible notions of overreaction.

After discussing our relation to previous literature immediately below, the remainder of the paper proceeds as follows. Section 1 sets up notation and provides theoretical results on the term structure of log equity premia and forecast errors. Section 2 describes our data and how we map from the theory to this empirical setting. Section 3 provides our main empirical results. Section 4 then discusses conditions under which observed forecast errors might arise in a rational framework; by contrast, Section 5 presents a simple model of expectation errors that is also capable of generating the observed forecast-error patterns. Section 6 concludes. An Online Appendix contains theoretical proofs (Appendix A) and additional technical details and robustness results (Appendices B–D).

Related Literature

A large and growing literature studies the behavior of investor expectations through survey data (e.g., Adam and Nagel 2022, Greenwood and Shleifer 2014). This literature often finds that investors exhibit less than fully rational expectations in their survey responses, though the behavior of expectations varies across settings. Greenwood and Shleifer (2014) find that retail and some institutional investors' expectations are cyclically biased (in that they think that expected returns are high when they are in fact low); Nagel and Xu (2022b), by contrast, argue that investors generally have less cyclical variation in the their expectations than the data suggests ex post. Dahlquist and Ibert (2021), Boutros et al. (2020), and Gormsen and Huber (2022), meanwhile, study groups of sophisticated investors and generally find that they are closer to rational in their understanding of the time variation in expected returns.

As expectations appear to vary substantially across different groups of investors, it is unclear ex ante how these groups interact in equilibrium. One advantage of our approach is that we directly study the expectations expressed in equilibrium behavior. We are, in effect, assessing the expectations of the marginal investor, or equivalently of any investor willing to hold the market portfolio at given prices ("Mr. Market" in Martin and Papadimitriou 2022); we do not, however, observe expectations for any particular trader over time. Our analysis also expands on the literature by considering the intertemporal dynamics of expectations – how investors expect future expected returns to evolve – which the survey data used in past work cannot speak to easily.

A separate literature also considers the behavior of expectations as inferred from option prices. For example, Polkovnichenko and Zhao (2013) and Ross (2015) provide different frameworks for estimating physical beliefs (and belief distortions) from options (see also Borovička, Hansen, and Scheinkman 2016); Martin (2017) studies option-implied expected returns under a covariance condition we build on; and Augenblick and Lazarus (2022) derive volatility bounds for fixed-maturity index options. We differ in our focus on the term structure of expected returns. Our paper is more closely related to Stein (1989) and Giglio and Kelly (2018), who document excess volatility in long-maturity relative to short-maturity derivatives prices. Those papers' key assumption is that cash flows follow an autoregressive process; we instead consider restrictions on the behavior of the SDF. This allows us to consider the term structure of expected equity returns, whereas Stein and Giglio and Kelly consider equity only indirectly through the term structure of implied volatility.

Our results also relate to the literature on the equity term structure (Binsbergen and Koijen 2017, Binsbergen, Brandt, and Koijen 2012), which studies how expected returns vary across equity dividend claims with different maturities. Our object of analysis is conceptually different: we focus on the expected return on the market as a whole (rather than on individual dividend claims), and ask how these aggregate expected returns vary by horizon. However, the simple model we propose to explain the behavior of forecast errors also speaks to the behavior of the equity term structure, as discussed in Section 5.3. Finally, in comparing forward and realized spot rates, our analysis appears similar in spirit to tests of the expectations hypothesis (EH) for the fixed-income term structure (e.g., Fama and Bliss 1987, Campbell and Shiller 1991). Aside from focusing on equity, our methodology is geared towards estimating physical expectations (and particularly forecast errors) for future expected returns; by contrast, the forward and

spot rates used in fixed-income EH tests embed risk premia by design, and violations of the EH in that setting are equivalent to bond-return predictability.[2]

1 Identifying Equity Premia and Forecast Errors

We begin with a theoretical analysis of the term structure of log equity risk premia. After setting up notation for spot rates, forward rates, and forecast errors, we then present our main results on forecast-error identification.

1.1 Notation

We start by generalizing our notation slightly relative to the introduction. Continue to define the spot rate as the time-t log equity premium between period t and $t+n$:

$$\mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n} - r_{t,t+n}^f],$$

where $r_{t,t+n} = \ln(R_{t,t+n})$ is the log return on the market portfolio from t to $t+n$, and $r_{t,t+n}^f$ is similarly the log risk-free return.[3] For any horizon n, this expected excess return can be written as the one-period risk premium plus a series of one-period forward rates:

$$\mu_t^{(n)} = \mu_t^{(1)} + \sum_{i=1}^{n-1} f_t^{(i,1)},$$

where forward rates are now defined as

$$f_t^{(n,m)} = \mu_t^{(n+m)} - \mu_t^{(n)} = \mathbb{E}_t\left[\mu_{t+n}^{(m)}\right]. \tag{1}$$

We refer to $f_t^{(n,m)}$ as the $n \times m$ forward rate.

We define forecast errors as the difference between realized spot rates and ex ante forward rates,

$$\varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_t^{(n,m)}.$$

[2]That said, bond-return predictability may arise from the dynamics of physical expectations, as in Cieslak (2018), Brooks, Katz, and Lustig (2019), and Farmer, Nakamura, and Steinsson (2021); our analysis relates more closely to this line of work.

[3] For $n > 1$, we set $r_{t,t+n}^f = \sum_{i=0}^{n} r_{t+i,t+i+1}^f$, so compounded one-period risk-free rates (and market returns) are used consistently in calculating excess returns for all horizons. As this distinction does not affect empirical implementation (see (5)), we often elide it in what follows.

Under the law of iterated expectations, the time-t conditional expectation of forecast errors is zero:

$$\mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] = 0.$$

1.2 Identification

We start with the following identity from the law of one price, introduced by Gao and Martin (2021):

$$\mathbb{E}_t[r_{t,t+n} - r_{t,t+n}^f] = \underbrace{\mathbb{E}_t\left[M_{t,t+n}R_{t,t+n}r_{t,t+n} - r_{t,t+n}^f\right]}_{\mathcal{L}_t^{(n)}} - \underbrace{\operatorname{cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})}_{\mathcal{C}_t^{(n)}}, \quad (2)$$

where $M_{t,t+n} = M_{t,t+1}M_{t+1,t+2}\cdots M_{t+n-1,t+n}$ is the n-period stochastic discount factor (SDF). The first expectation on the right side of (2) is directly observable from option prices, as shown by Gao and Martin (2021), who label this term the LVIX.[4] We denote this term by $\mathcal{L}_t^{(n)}$, as in (2). The covariance term, $\mathcal{C}_t^{(n)}$, is an unobserved risk adjustment. The size and sign of this adjustment can be controlled by theory, as discussed in detail below.

To build intuition, we first make the simplifying assumption that $M_{t,t+n} = 1/R_{t,t+n}$. This stochastic discount factor would, for instance, arise if the representative investor is fully invested in the stock market and has log utility over terminal wealth. In this case, the covariance term in (2) is equal to zero, and the LVIX directly identifies expected excess returns at different horizons. Given the identity in (2) and the definitions in Section 1.1, we can thus calculate spot rates, forward rates, and forecast errors from the data as follows.

Proposition 1 (Log Utility Identification). *Assuming that $M_{t,t+n} = 1/R_{t,t+n}$, spots, forwards, and forecast errors are given, respectively, by:*

$$\begin{aligned}
\mu_t^{(n)} &= \mathcal{L}_t^{(n)} \\
f_t^{(n,m)} &= \mathcal{L}_t^{(n+m)} - \mathcal{L}_t^{(n)} \\
\varepsilon_{t+n}^{(m)} &= \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_t^{(n+m)} + \mathcal{L}_t^{(n)}.
\end{aligned}$$

[4] One can equivalently write the LVIX as $\mathcal{L}_t^{(n)} = (R_{t,t+n}^f)^{-1}\,\mathbb{E}_t^*\left[R_{t,t+n}r_{t,t+n}\right] - r_{t,t+n}^f$, where $\mathbb{E}_t^*$ denotes the risk-neutral expectation.

Proposition 1 follows straightforwardly from the fact that $M_{t,t+n}R_{t,t+n} = 1$ under log utility. Thus given this assumption, we can directly identify forecast errors from option prices and thereby test whether expectations are intertemporally consistent (i.e., whether $\mathbb{E}[\varepsilon_{t+n}^{(m)}] = 0$ and $\mathbb{E}[Z_t\varepsilon_{t+n}^{(m)}] = 0$ for Z_t observable as of t). And this result does not in fact require the existence of a representative agent: the LVIX-based estimates reflect the expectations of any unconstrained log investor who is content to hold the market portfolio. These expectations can, for instance, be thought of as those of Mr. Market in the heterogeneous-agent log utility model of Martin and Papadimitriou (2022).

If we go beyond log utility, our estimates of spot and forward rates are contaminated by the covariance terms in (2). When considering spot rates by themselves, it may be reasonable to assume that $\mathcal{C}_t^{(n)} = \mathrm{cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n}) \leq 0$, so that the LVIX provides a lower bound for $\mu_t^{(n)}$; this is the tack taken by Gao and Martin (2021). But when considering forward rates, $f_t^{(n,m)} = \mathscr{L}_t^{(n+m)} - \mathscr{L}_t^{(n)} - (\mathcal{C}_t^{(n+m)} - \mathcal{C}_t^{(n)})$, it is unclear whether $\mathcal{C}_t^{(n+m)} \leq \mathcal{C}_t^{(n)}$ or vice versa.

This is not, however, the end of the story. Our main innovation is to consider *forecast errors*, for which we show that the unobserved covariance terms largely cancel in expectation. We present two versions of this result. First, to continue building intuition, Proposition 2 considers identification in a log-normal world given general $M_{t,t+n}$. Proposition 3 then provides a fully general identification result, which shows that the main insights from Proposition 2 carry through.

Before presenting these results, we define our empirical proxy for forecast errors:

$$\widehat{\varepsilon}_{t+n}^{(m)} = \mathscr{L}_{t+n}^{(m)} - \mathscr{L}_t^{(n+m)} + \mathscr{L}_t^{(n)},$$

which are the forecast errors one would obtain under the log-utility assumption. To streamline notation, we also define $MR_{t,t+n} = M_{t,t+n}R_{t,t+n}$. We can now show that we can study expected forecast errors up to a single covariance term.

Proposition 2 (Log-Normal Identification). *Assume a general SDF $M_{t,t+n}$, and assume that $M_{t,t+n}$ and $R_{t,t+n}$ are jointly log-normal. Then the expected value of the forecast-error proxy satisfies*

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \mathrm{cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])\,, \tag{3}$$

where $\varepsilon_{t+n}^{(m)}$ is the true forecast error.

Proposition 2 shows that in a log-normal world, our forecast-error estimate is equal

in expectation to the true forecast error minus an unobserved covariance term related to the pricing of shocks to expected returns (or discount-rate risk). Note that when considering spot rates, the risk adjustment term $\mathcal{C}_t^{(n)} = \text{cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})$ depends on a covariance with *realized* returns $r_{t,t+n}$. By contrast, when considering forecast errors, the relevant risk adjustment depends on a covariance with *expected* returns $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$. By replacing (highly volatile) realized returns with (much less volatile) expected returns, the covariance in (3) is likely to be substantially smaller than the covariance in (2). We discuss how one might quantify this covariance in greater detail in Section 4.

The following proposition shows that the above intuition carries through to more general non-log-normal settings, in which case the expected return $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$ is replaced by a closely related LVIX-based proxy.

Proposition 3 (Generalized Identification). *For any SDF $M_{t,t+n}$ and any data-generating process for which the relevant expectations exist, the expected value of the forecast-error proxy satisfies*

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \text{cov}_t\left(MR_{t,t+n}, \widehat{\mu}_{t+n}^{(m)}\right), \tag{4}$$

where $\varepsilon_{t+n}^{(m)}$ is the true forecast error and $\widehat{\mu}_{t+n}^{(m)} = \mathscr{L}_{t+n}^{(n)} + r_{t+n,t+n+m}^f$ is a proxy for $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$.

The appearance of the LVIX-based proxy $\widehat{\mu}_{t+n}^{(m)}$ in the risk adjustment term in (4) has one benefit relative to (3): the LVIX is directly observable in the data. We can thus quantify the degree to which $\widehat{\mu}_{t+n}^{(m)}$ is less volatile than the realized return $r_{t,t+n}$ in (2). In particular, under our main specification (discussed further in Section 2 below), we find that the unconditional volatility of $\widehat{\mu}_{t+n}^{(m)}$ is one-tenth the volatility of the realized market return in our sample. As such, our empirical estimate of forecast errors is likely to be useful even for SDFs different from $M_{t,t+n} = 1/R_{t,t+n}$.

Before moving to our empirical estimation, we note in passing that expected log returns $\mathbb{E}_t[r_{t,t+n}]$ are conceptually distinct from log expected returns $\ln \mathbb{E}_t[R_{t,t+n}]$ (with the difference depending on higher moments of the return distribution). Expectations of the former are more relevant for prices, given that prices depend on geometric-average expected returns (as can be seen in a Campbell-Shiller decomposition). This motivates our focus on expected log returns, but this distinction should be kept in mind in interpreting our results.

2 Data and Implementation

We now turn to our empirical estimation of spot rates, forward rates, and forecast errors in a large global panel of index option prices.

2.1 Data

We use a large dataset of option prices largely from OptionMetrics.[5] The dataset contains data on European (in terms of exercise) put and call options for major stock market indexes around the world. In our full sample, we have a total of 20 different indexes from at least 15 different countries and three pan-European indexes, as shown in Table A1. The sample starts in 1990 for the S&P 500 and runs through 2021. For international indexes, the respective samples start substantially later. In addition to focusing separately on the S&P 500 and the Euro Stoxx 50, our main panel analysis is conducted in a constrained sample of the 10 largest, most liquid, and most dense indexes in our sample, as detailed again in Table A1. Options for these exchanges are available since 2006 at latest, and again run through 2021. In all cases, we sample monthly and apply standard filters. Details on the data, sample selection, and filters are relegated to Appendix B.

2.2 Measuring LVIX

We measure $\mathscr{L}_t^{(n)}$ following Breeden and Litzenberger (1978) and Carr and Madan (2001) (see Gao and Martin 2021). We make the simplifying assumption that the ex dividend payment at time $t+n$ is known ex ante (at time t). We denote the relevant index at time t as P_t, the time t prices for call and put options on P_{t+n} with strike K as $\text{call}_t^{(n)}(K)$ and $\text{put}_t^{(n)}(K)$, and the time t forward price for the index at $t+n$ as $F_t^{(n)}$. With these definitions,

$$\mathscr{L}_t^{(n)} = \frac{1}{P_t}\left[\int_0^{F_t^{(n)}} \frac{\text{put}_t^{(n)}(K)}{K} dK + \int_{F_t^{(n)}}^{\infty} \frac{\text{call}_t^{(n)}(K)}{K} dK\right] \tag{5}$$

which means that LVIX is a function of put and call prices written on the relevant index (as well as the current and forward price of the index).

[5] We augment our sample with data from CBOE Market Data Express to cover the period prior to 1996 in the U.S. data.

2.3 Implementation

We run our baseline analysis at the 6-month horizon ($n = 6$); when considering forward rates, our baseline also uses forward maturities of $m = 6$ months. Our choice of horizon reflects a tradeoff: longer horizons are more economically meaningful, but long-maturity options are generally less liquid and have relatively sparse density. Longer-dated options with multi-year horizons do, however, trade on a relatively liquid basis for the Euro Stoxx 50; we make use of these options when estimating the magnitude of forecast errors over the entire term structure. We also provide a robustness analysis for our main empirical results at alternative horizons.

We do not observe options for all positive strike values, as is needed in (5), and we therefore truncate the integral after extrapolating well past the range of observable contracts. We examine this assumption, along with a number of other assumptions with respect to measurement, in detail in the Appendix.

3 Empirical Results

After measuring the LVIX, we construct empirical proxies for spot rates, forward rates, and forecast errors using the expressions provided in Proposition 1. As in that proposition, these proxies are exact under log utility ($M_{t,t+n} = 1/R_{t,t+n}$), which provides a useful benchmark for interpreting the following results. We begin by considering the relationship between spot and forward rates. We then turn to forecast errors; given the results in Propositions 2–3, these forecast-error estimates are less heavily reliant on the assumption of log utility.

3.1 Spot and Forward Rates: Descriptive Statistics and Figures

We begin with an overview of our estimated spot and forward rates. Table A2 in the Appendix provides summary statistics by exchange. For a brief intuitive description of the dynamics of spots and forwards, we focus here on descriptive figures for the U.S. (S&P 500) sample. Figure 2 shows contemporaneous time-series estimates for 6-month spot rates and 6×6-month forward rates. Consistent with Figure 1 in the introduction, spot and forward rates increase significantly in crises, with forward rates increasing less than one-for-one with contemporaneous spot rates. The "contemporaneous" qualifier for spot rates is important here, as will be seen below.

Figure 3 instead compares forward rates with ex post realized spot rates. (The 6×6 forward rate is a predicted value using a shorter-horizon forward as an instrument, as discussed further below.) The gap between the realized spot rate and the forward rate represents the forecast error. Again as in Figure 1, forecast errors tend to be negative after crises, when spot rates decline from their crisis peaks. By contrast, forecast errors are frequently (though not always) positive outside of crises.

To formalize this visual analysis, we move now to a set of regression analyses.

3.2 Forward Rates as Predictors of Future Spot Rates

We first consider Mincer-Zarnowitz (1969) regressions of realized spot rates on forward rates:

$$\mu_{i,t+6}^{(6)} = \beta_0 + \beta_1 f_{i,t}^{(6,6)} + e_{i,t+6}, \tag{6}$$

where i now indexes the exchange. We consider both panel regressions and exchange-specific time series regressions, and in both cases we conduct rolling monthly regressions for $n = m = 6$ months, following Section 2.3. Standard errors are clustered by exchange and date in the panel case; in the time-series case, we use heteroskedasticity- and autocorrelation-robust Newey-West standard errors, with lags selected following Lazarus et al. (2018). The relevant joint null for forecast predictability features $\beta_0 = 0$, $\beta_1 = 1$, which should hold in our case under the joint assumptions of rational expectations and $M_{t,t+n} = 1/R_{t,t+n}$.

Table 1 presents the regression results. Column (1) shows results from the main panel, as defined in Section 2.1, without any fixed effects. The slope coefficient is around 0.65, and the intercept is statistically significant at 1.02 annualized percentage points. We can therefore reject the null of $\beta_0 = 0$, $\beta_1 = 1$ with substantial confidence for this specification. The fact that $\widehat{\beta}_1 < 1$ suggests that forward rates tend to overshoot future spot rates in the data. Thus, even though forward rates move less than one-for-one with *contemporaneous* spot rates as in Figure 2, they move *more* than one-for-one with *future* spot rates. But in spite of this overshooting of forward rates, the R^2 for this regression is around 0.2, so forward rates do predict a substantial portion (one fifth) of the variation in the equity premium ex ante.

The next columns consider different specifications and samples. Column (2) includes an exchange fixed effect; this is in fact our preferred specification for estimating slope coefficients in the main panel, as it cleanly identifies within-country (or within-exchange)

time-series predictability. The slope coefficient in this case is slightly further below 1 than in column (1), and the within-exchange R^2 is slightly below 0.2. Column (3) includes both index and date fixed effects for completeness, though these results are somewhat more challenging to interpret. Column (4) excludes the U.S. from the specification with only exchange fixed effects, while (5) and (6) consider the S&P 500 and the Euro Stoxx 50, respectively, in isolation. The results are largely similar in all cases, though with some variation in slopes and R^2 values across exchanges.

One might, however, be concerned with attenuation bias for the slope coefficients in Table 1 given possible measurement error in forward rates.[6] We now address this concern by instrumenting variation in the forward rate with another forward rate.[7] We generally use shorter-maturity forward rates as instruments, as these are likely to be better measured given that options are denser at shorter maturities.

In particular, for regression (6), we use the 2×1-month forward rate $f_{i,t}^{(2,1)}$ as an instrument for the 6×6 rate $f_{i,t}^{(6,6)}$. Table 2 presents the resultant two-stage least squares results.[8] The estimated slope coefficients increase slightly and are now quite consistent across specifications at around 0.7–0.8. The one exception is column (3), which uses date fixed effects; again, this estimate is less informative and less interpretable than the others, given that it absorbs common time variation in forward rates. In all other specifications, we continue to reject the null of $\beta_0 = 0$, $\beta_1 = 1$. In columns (1)–(2) and (4)–(5), the estimated slopes are significantly different from 1; for the Euro Stoxx 50 in column (6), β_1 is noisily estimated, but β_0 is significantly different from zero.

Taken together, we find that while forward rates have substantial predictive power over future spot rates, they tend to overshoot those spot rates. When forward rates increase by 1% relative to their within-exchange mean, this corresponds on average to an increase in future spot rates of only about 0.7%. Separate estimation of forward and spot rates requires log utility, however, so in order to soften this assumption, we now consider the behavior of forecast errors.

[6] Recall that forwards are estimated from discrete approximations of the integrals in (5), for which extrapolation past the range of observable strikes is necessary. Again see Appendix B for details.

[7] We note that this IV specification will account for measurement error idiosyncratic to a given forward rate, but not for measurement error in the overall forward curve.

[8] The first stage in these regressions (not shown) has an R^2 close to 70% when excluding all fixed effects, suggesting the 2×1 forward rate is a strong instrument for the 6×6 rate.

3.3 The Insignificance of Average Forecast Errors

We first consider unconditional averages of forecast errors, $\overline{\varepsilon}^{(6)}_{i,t+6}$, across different sets of exchanges and subsamples. These values are reported in Table 3. For our main panel, in column (1), we find average forecast errors that are not just statistically insignificant, but effectively zero: $\overline{\varepsilon}^{(6)}_{i,t+6}$ is below 20 basis points on an annualized basis. Realized spot rates have thus been very slightly (and insignificantly) higher than forward rates on average.[9] This rough magnitude applies to different exchanges, though the U.S. average in column (3) is even lower, at roughly 2 basis points.

The fact that forecast errors are close zero on average means that we cannot reject the log-utility assumption along this dimension. This test is, of course, unlikely to be the highest-powered test of the log-utility null. But the result will nonetheless be important when attempting to rationalize the time variation in forecast errors through the behavior of the SDF (in Section 4).

While forecast errors are close to zero unconditionally, this average masks substantial predictability over time, which we turn to now.

3.4 The Predictability of Forecast Errors

As shown in Figure 1 in the introduction, elevated forward rates during particular crisis periods appear to be followed by negative forecast errors (with realized spot rates below ex ante forward rates). Did these arise by random chance, or are forecast errors systematically predictable from (i) forward rates and/or (ii) forward-rate revisions? While the results of Section 3.2 are suggestive for question (i), the significance of forecast-error predictability and question (ii) remain open.

For a more formal consideration of forecast-error predictability, we begin by testing whether forward rates predict forecast errors in regressions of the following form:

$$\varepsilon^{(6)}_{i,t+6} = \beta_0 + \beta_1 \, f^{(2,1)}_{i,t} + e_{i,t+6}. \tag{7}$$

We use the 2×1 forward rate $f^{(2,1)}_{i,t}$ here instead of the 6×6 rate, as we would otherwise

[9] In Appendix B, we consider whether the small positive forecast errors might arise due to measurement noise. As discussed there, our LVIX estimate could be biased downwards and more so at long maturities. Nonetheless, in all the robustness tests we consider there (with alternative estimation procedures or specifications), we find that measurement error is unlikely to be large enough to generate even the 20 basis points estimated for average errors. Along similar lines, forecast errors in Table 3 are largest for the SX5E index, which has the best availability of options.

have the same forward rate on both sides of the regression, leading to a mechanical upward bias in the slope coefficient in the presence of measurement error.[10] All other aspects of the regressions are identical to those in Table 1.

Table 4 reports the results. In the main panel without any fixed effects (column (1)), the slope coefficient on the forward rate is -0.12 and is significant at the 5% level.[11] In our preferred panel specification including exchange fixed effects (column (2)), the slope coefficient is now significant at the 1% level. Given the common time variation in forecast-error predictability across exchanges, a time fixed effect absorbs the predictive power (column (3)). Results are similar in the ex-U.S. and U.S.-only samples (though are no longer significant for SX5E alone).

Given these results, we conclude again that forward rates significantly overshoot future spot rates, generating predictable forecast errors. We next study the predictability of forecast errors using forward-rate *changes*, using the method in Coibion and Gorodnichenko (2015). This allows us to address more specifically whether forward rates exhibit excess sensitivity to news. These regressions take the following form:

$$\varepsilon^{(3)}_{i,t+6} = \beta_0 + \beta_1 \left(f^{(6,3)}_{i,t} - f^{(9,3)}_{i,t-3} \right) + e_{i,t+6}.$$

We change the horizon relative to (7) because we do not have forward rates with final maturity beyond $n + m = 12$ months with which to compute forecast revisions. Instead, we examine the expected 3-month equity premium 6 months from t, the longest risk premium for which can conduct these regressions. This requires we examine quarterly, not monthly, forecast revisions.[12]

Table 5 reports the results, and we again find significant error predictability on the basis of forward-rate revisions, with positive forecast revisions predicting negative forecast errors. Interestingly, unlike in Table 4, the time fixed effect does not fully capture the predictive power of forecast revisions in column (3), pointing to a substantial idiosyncratic component to forecast revisions across countries. This column's slope coefficient is significant at the 1% level.

The above findings are at least consistent with the notion that the market overesti-

[10] Moreover, with the same conditioning variable, the difference between the slope in Mincer-Zarnowitz and error-predictability regressions would be one by construction.

[11] The magnitudes presented in the table are not directly comparable to those in Tables 1–2 given the use of $f^{(2,1)}_{i,t}$ as the predictor. Instead, regression (7) can be viewed as an analogue to the reduced form of the IV specification for (6) presented in Table 2.

[12] In a series of robustness checks in the Appendix, we also consider monthly revisions, but this comes with the cost of a shorter, arguably less economically interesting, risk premium.

mates the degree of persistence in the equity premium. That said, these results do not represent an unqualified success for all possible notions of overreaction or extrapolation. While the results are consistent with overextrapolation of forward-looking expected returns, they are not consistent with extrapolation of *past* returns, as forward rates tend to be too high precisely when past returns have been low. To illustrate this, Figure 4 plots forward rates and forecast errors, alongside annotations describing each significant period of market distress in the sample. In all six such episodes, forward rates *increase* in bad times (when realized returns are low), and future forecast errors are then negative. We return to this issue in Section 5.

In Appendices B–C, we consider a number of robustness checks with alternative measurement methodologies and sample cuts. Results are similar in all cases (see Tables A3–A7). We also expand the sample to all 20 available exchanges and find somewhat stronger error predictability than in the main sample (Table A4).

3.5 How Significant Are Forecast Errors for Prices?

In light of the foregoing results, forward rates are more volatile than the predictable component of future spot rates, generating predictable forecast errors. While our theory suggests that the risk-premium component of forecast errors is likely to be qualitatively small, the subsequent sections provide a more thorough quantitative analysis of the potential role for rational risk premia versus expectation errors. Before taking any stance on the source of our estimated forecast errors, however, we return to the issue raised at the outset of the paper: how significant are predictable forecast errors for overall stock-price movements?[13]

Answering this question requires quantifying the discounted sum of predicted forecast errors at all horizons; we conduct an illustrative quantification here. In particular, starting from a Campbell-Shiller (1988) decomposition for the log price-dividend ratio

[13]One could ask other versions of this question; for example, how much price variation is attributable to expected equity premia? We focus on the predictable forecast-error component of forward rates because (i) our paper's main innovation is in isolating these forecast errors specifically, and (ii) these forecast errors represent the excessively volatile component of expected equity premia.

$p_t - d_t$ and using the definition of forward rates as expected equity premia:

$$\begin{aligned} p_t - d_t &= k + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[\Delta d_{t+j+1}] \; - \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[r_{t+j,t+j+1}] \\ &= k + \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[\Delta d_{t+j+1}]}_{CF_t} \; - \underbrace{\sum_{j=0}^{\infty} \rho^j \, f_t^{(j,1)}}_{\mathcal{F}_t} \; - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[r^f_{t+j,t+j+1}]}_{RF_t}, \end{aligned} \tag{8}$$

where k is a constant and $0 < \rho < 1$. We further split the forward-rate term $\mathcal{F}_t$ into

$$\underbrace{\sum_{j=0}^{\infty} \rho^j \, f_t^{(j,1)}}_{\mathcal{F}_t} \; = \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[\mu^{(1)}_{t+j}]}_{\mathcal{M}_t} \; - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t[\varepsilon^{(1)}_{t+j}]}_{\mathcal{E}_t}. \tag{9}$$

Our interest is in the term $\mathcal{E}_t$, which quantifies the component of the log price-dividend ratio attributable to all future predicted forecast errors. The length of one period in (8)–(9) is arbitrary, so we set it to 6 months to correspond to our baseline horizon.

In our main sample, we estimate forecast errors for horizons only up to one year. To obtain estimates for longer horizons, we re-estimate spot and forward rates out to an 8-year horizon for the Euro Stoxx 50, which is the only exchange in our sample with index options available at such maturities.[14] For each horizon, we predict forecast errors as in (7) by regressing realized forecast errors on shorter-horizon forward rates. We then estimate a decay function $\phi^{(n,m)}$ for forecast errors, such that $\mathbb{E}_t[\varepsilon^{(m)}_{t+n}] = \phi^{(n,m)} \, \mathbb{E}_t[\varepsilon^{(1)}_{t+1}]$. Details are in Appendix C. As reported in Table A8, the decay parameters are very close to 1 at all horizons. We therefore use the assumption that predictable forecast errors have a flat term structure, $\mathbb{E}_t[\varepsilon^{(1)}_{t+j+1}] = \mathbb{E}_t[\varepsilon^{(1)}_{t+j}]$ for all j in (9), to obtain a back-of-the-envelope quantification of $\mathcal{E}_t$ for the U.S. sample:

$$\mathcal{E}_t = \frac{\rho}{1-\rho} \, \mathbb{E}_t[\varepsilon^{(1)}_{t+1}], \tag{10}$$

where again one period is 6 months. We then compare variation in $\mathcal{E}_t$ to variation in $p_t - d_t$. To account for share repurchases and ensure that $p_t - d_t$ is stationary, we use the repurchase-adjusted price-dividend series provided by Nagel and Xu (2022a).[15]

[14] This is similar to the case for the dividend futures market (Binsbergen and Koijen 2017).

[15] We estimate ρ for the 6-month (half-yearly) horizon in (10) as $\rho = (1 + \exp(\overline{d-p}))^{-1/2}$, and we use predicted forecast errors $\mathbb{E}_t[\varepsilon^{(1)}_{t+1}]$ expressed on a non-annualized basis.

Figure 5 presents the results, with both discounted forecast errors $\mathcal{E}_t$ (red) and the repurchase-adjusted price-dividend ratio $p_t - d_t$ (blue) shown as log differences from their full-sample means. The two series do not always comove positively. During the Russian debt crisis in the late 1990s, for example, forward rates were quite elevated (leading to negative $\mathcal{E}_t$), but the stock market's dot-com boom largely continued apace (leading to high $p_t - d_t$). But during large stock-market declines (low $p_t - d_t$), predictable forecast errors generally play a significant role in the decomposition in (8)–(9). During the depths of the 2008 financial crisis, for example, when the price-dividend ratio reached more than 70 log points (about 50 percentage points) below its mean, discounted forecast errors accounted for more than half of this decline. A meaningful component of the stock-price collapse therefore consisted of elevated expected future equity premia, which, in a predictable manner, declined faster than suggested by the forward curve. Overall, the price-dividend ratio moves roughly two-for-one with discounted forecast errors.[16] While the $\mathcal{E}_t$ estimates should be viewed as rough approximate values, this exercise illustrates a meaningful potential role for forecast errors in future expected equity premia in explaining stock-price variation.

4 Rationalizing Forecast Errors

We now consider potential sources for the forecast errors documented in the previous section. We begin by asking what would be required in order for forecast errors to be rationalized by the behavior of the stochastic discount factor alone. When going beyond log utility, our estimates of spot and forward rates partly reflect risk premium adjustments. While most of these cancel out in forecast errors, a risk premium term remains: from Propositions 2–3, expected forecast errors are

$$\mathbb{E}_t[\widehat{\varepsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\varepsilon^{(m)}_{t+n}] - \varsigma_t,$$

$$\text{where} \quad \varsigma_t = \begin{cases} \operatorname{cov}_t\big(MR_{t,t+n}, \mathbb{E}_{t+n}\, r_{t+n+m}\big) & (\textit{log-normal case}), \\ \operatorname{cov}_t\big(MR_{t,t+n}, \widehat{\mu}^{(m)}_{t+n}\big) & (\textit{general case}). \end{cases}$$

To simplify analysis, we maintain focus on the $n = m = 1$ case (where one period is 6 months) and suppress the dependence of ς_t on n and m.

As a starting point for the potential role of risk premia, Figure 6 plots the predicted

[16] The slope coefficient from a regression of $p_t - d_t$ on $\mathcal{E}_t$ is 0.50.

values of the forecast errors from the regressions in Table 4. These are empirical estimates of $\mathbb{E}_t[\widehat{\varepsilon}^{(1)}_{t+1}]$, and $-\varsigma_t$ must take on these values in order to rationalize our results through risk premia alone (so that true forecast errors satisfy $\mathbb{E}_t[\varepsilon^{(1)}_{t+1}] = 0$). The main challenge is that the covariance term $-\varsigma_t$ must flip sign over time: it must be positive in good times and significantly negative in bad times. We show below that for this to happen, the price of discount rate risk must be highly volatile: it must be close to zero during good times and high during bad times.[17]

4.1 Examining the Risk Premium Term: Decompositions

To better understand the properties required of ς_t in order to match Figure 6, it is useful to formalize its relation to a discount-rate risk premium. As in Proposition 2, assume for now that the SDF and returns are jointly log-normal. Then by Stein's lemma and the fact that $\mathbb{E}_t[MR_{t+1}] = 1$, we have that

$$\text{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}\, r_{t+2}) = \text{cov}_t(m_{t+1} + r_{t+1}, \mathbb{E}_{t+1}\, r_{t+2}). \tag{11}$$

For ς_t to flip signs, the covariance in (11) must thus flip signs. To see how this covariance is closely tied to discount-rate risk, define $P_{\mathbb{E},t}$ as the price of the claim that pays $-\,\mathbb{E}_{t+1}\, r_{t+2}$ next period, and define its ex ante Sharpe ratio (in log-return terms) as $SR_{\mathbb{E},t}$.[18] This Sharpe ratio captures the price of risk for exposure to increases in expected stock returns (or discount rates).

Rewriting (11), the sign of ς_t is given by

$$\text{Sign}\,(\varsigma_t) = \text{Sign}\Big(SR_{\mathbb{E},t} + \rho_t(r, \mathbb{E}_{t+1}\, r)\sigma_t(r_{t+1})\Big),$$

where $\rho_t(r, \mathbb{E}_{t+1}\, r) = \text{Corr}_t(r_{t+1}, \mathbb{E}_{t+1}\, r_{t+2})$. This correlation is likely negative, as low realized returns are associated with higher expected returns going forward; for simplicity, start by assuming that $\rho_t(r, \mathbb{E}_{t+1}\, r) = -1\ \forall\, t$.[19] In this case, the above expression shows that $SR_{\mathbb{E},t}$ must be higher than $\sigma_t(r)$ for ς_t to be positive, and lower than $\sigma_t(r)$ for ς_t to

[17] As we will show below, it is possible for standard models to generate a ς_t that is sufficiently volatile, but if ς_t does not change sign, the average of $\widehat{\varepsilon}$ will be far from zero, which is inconsistent with the average of $\widehat{\varepsilon}$ being close to zero in the data.

[18] That is, $SR_{\mathbb{E},t} = (\mathbb{E}_t[r_{\mathbb{E},t+1}] - r^f_{t,t+1} + \sigma^2_{\mathbb{E},t}/2)/\sigma_{\mathbb{E},t}$, where $r_{\mathbb{E},t+1}$ is the claim's log return and $\sigma_{\mathbb{E},t}$ is its standard deviation.

[19] This is an extreme case in which all movements in realized returns reflect variation in discount rates (Campbell and Ammer 1993).

be negative. Since ς_t must be positive in bad times and negative in good times, $SR_{\mathbb{E},t}$ must vary more than $\sigma_t(r)$ between good and bad times.

As a benchmark, Figure 7 plots the time variation in $\sigma_t(r)$ at the 6-month horizon from the standpoint of an investor with log utility.[20] The figure shows that volatility varies from close to 15% in good times to more than 50% in bad times. As such, the price of discount-rate risk must vary significantly and countercyclically to generate the time variation in ς_t observed in the data.

Moreover, in the case that $\rho_t(r, \mathbb{E}_{t+1} r) = -1$, then $-\varsigma_t$ is a scaled version of $\text{cov}_t(MR_{t,t+1}, r_{t+1})$. Under the *modified negative correlation condition* (mNCC) used throughout Gao and Martin (2021), this covariance is always negative, so ς_t cannot flip sign. This illustrates that the degree of countercyclical variation in discount-rate risk prices required under $\rho_t(r, \mathbb{E}_{t+1} r) = -1$ is quite restrictive, and it is ruled out by a range of standard models considered by Gao and Martin (2021).

In practice, however, we expect $\rho_t(r, \mathbb{E}_{t+1} r)$ to be higher than -1, as not all changes in stock returns are driven by discount rates. A $\rho_t(r, \mathbb{E}_{t+1} r) > -1$ implies a less volatile price of discount-rate risk than discussed above. However, it also implies that the price of discount-rate risk must be even lower during good times than above; for example, in the opposing extreme in which $\rho_t(r, \mathbb{E}_{t+1} r) = 0$ (i.e., with i.i.d. returns), the Sharpe ratio on the discount-rate claim must be *negative* in good times in order for ς_t to flip sign.

The discussion above can also be generalized beyond the log-normal case. For example, assume that the SDF can be written as

$$M_{t,t+1} = \beta \frac{V_W(W_{t+1}, z_{t+1})}{V_W(W_t, z_t)},$$

where V_W is an unconstrained investor's marginal utility of wealth, z_t is a vector of state variables that includes $-\widehat{\mu}_{t+1}^{(1)}$, and V_W is weakly decreasing in each entry of z_t. If this agent is fully invested in the market, has relative risk aversion $-WV_{WW}/V_W$ of at least 1, and R_{t+1} and the elements of z_{t+1} are *associated* random variables,[21] then Result 4 of Gao and Martin (2021) can be applied to obtain that ς_t again cannot change sign under the benchmark $\rho_t(r, \widehat{\mu}) = -1$.

[20] This is likely to be a conservative benchmark for the degree of variation in conditional volatility if investors are more risk-averse than implied by log utility.

[21] Formally, the elements of the vector $X_{t+1} = (R_{t+1}, z'_{t+1})'$ are associated random variables if $\text{cov}_t(f(X_{t+1}, g(X_{t+1})) \geq 0$ for all nondecreasing functions f and g for which this covariance exists.

4.2 Further Intuition Under Power Utility

The above emphasizes that the price of discount-rate risk must vary substantially over time to rationalize our results. To better understand this result, we next reconduct our analysis under various utility functions generating a constant price of risk, and show how such functions will not be able to explain the data. This analysis will help provide intuition on the role of risk aversion in our results.

We calculate spot and forward rates using CRRA utility with γ ranging from 0.5 to 3 (see Appendix B.4 for details). As shown in Table 6, none of the power utility functions can rationalize the results. The functions with $\gamma > 1$ explain the time variation in the forecast errors, but they cannot explain the average being close to zero. To understand the intuition, consider the results derived above for log-normally distributed variables. When $\gamma > 1$, the covariance term ς_t correctly increases in bad times. However, the covariance term is positive because $\text{cov}_t(m_{t+1} + r_{t+1}, \mathbb{E}_{t+1}\, r_{t+2}) > 0$ at all times, which leads to to large average forecast errors. The opposite mechanics are at play for $\gamma < 1$, for which $\text{cov}_t(m_{t+1} + r_{t+1}, \mathbb{E}_{t+1}\, r_{t+2}) < 0$ for all t.

Figure 8 illustrates this challenge in getting both the average and the time variation in the covariance term correct. When γ is low, the average error is close to zero, but implied forecast errors are highly predictable. Conversely, as γ increases, the implied errors become less predictable, but the average error increases substantially.

5 A Model of Expectation Errors

We now examine whether our empirical findings could plausibly arise from a combination of log utility and expectation errors. To do so, we introduce a simple model in which expectations formation deviates from full rationality, building on the framework proposed by Bordalo, Gennaioli, and Shleifer (2018).[22] We then calibrate the model and compare it to the data under the assumption that the representative investor has log utility over the market return.

[22]The model is also qualitatively similar to the natural expectations framework proposed by Fuster, Laibson, and Mendel (2010) and adapted by Giglio and Kelly (2018), as well as the overreaction and base-rate neglect versions of the Augenblick and Rabin (2021) model.

5.1 Model Setup

We assume that the 3-month spot rate follows an AR(3) process under the objective measure:

$$\mu_t^{(3)} = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \phi_1 \, \mu_{t-1}^{(3)} + \phi_2 \, \mu_{t-2}^{(3)} + \phi_3 \, \mu_{t-3}^{(3)} + e_t,$$

where $\bar{\mu} = \mathbb{E}[\mu_t^{(3)}]$ and $e_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_e^2)$.[23]

If expectations were rational, investors would use the objective AR(3) dynamics to make iterated forecasts of future 3-month spot rates:

$$\mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \sum_{j=1}^{3} \phi_j \, \mathbb{E}_t\left[\mu_{t+n-j}^{(3)}\right] \text{ for } n > 0. \tag{12}$$

In such a rational world, these objective forecasts of future 3-month spot rates would define the current term structure of quarterly spot rates recursively using (1):

$$\mu_t^{(n)} = \begin{cases} \mu_t^{(3)} & \text{for } n = 3 \\ \mu_t^{(n-3)} + \mathbb{E}_t\left[\mu_{t+n-3}^{(3)}\right] & \text{for } n \in \{6, 9, 12\}. \end{cases} \tag{13}$$

Rather than assuming rationality as in (12), however, we assume that investors potentially overreact to objective news about $\mu_t^{(n)}$ in forming their subjective forecasts. We use $\mathbb{E}_t^\theta[\cdot]$ to refer to these investors' subjective expectations, as the parameter θ indexes the degree of excess sensitivity to spot-rate news as follows:

$$\mathbb{E}_t^\theta\left[\mu_{t+n}^{(3)}\right] = \mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] + \theta \underbrace{\left(\mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] - \mathbb{E}_{t-3}\left[\mu_{t+n}^{(3)}\right]\right)}_{\text{spot-rate news}}. \tag{14}$$

In the language of Bordalo et al. (2019), θ is the "diagnosticity" parameter. This representation nests rationality when $\theta = 0$. When $\theta > 0$, investors are excessively sensitive to news.[24] After bad news, for example, spot rates increase, the news term is

[23] By focusing on the ex ante equity premium as the stochastic process of interest, our model departs somewhat from the diagnostic model considered by, e.g., Bordalo et al. (2019), as discussed in Section 5.3. We also generalize slightly by assuming an AR(3) process.

[24] While overreaction is defined relative to objective news (which investors do not directly observe), the news term in (14) is proportional to the objective innovation e_t (which they do observe). Since news is defined over a 3-month period in the objective dynamics, we assume that the relevant lagged expectations in (14) are 3-month lagged expectations. In principle, this could also be estimated from the data as in Bordalo et al. (2019).

positive, and investors' subjective expectations overreact by θ times news.

The actual term structure of quarterly spot rates $\mu_t^{(n)}$ is defined by (13), but with the objective expectation $\mathbb{E}_t[\mu_{t+n}^{(3)}]$ replaced by the subjective expectation $\mathbb{E}_t^{\theta}[\mu_{t+n}^{(3)}]$. As a consequence, long-horizon spot rates — and therefore forward rates — increase after bad news even more than they would under rationality.

5.2 Calibration and Results

To understand whether the model can match our empirical findings, we turn to Monte Carlo simulations. We first estimate the objective dynamics of the 3-month spot rate by country under the assumption of log utility. Table A9 reports these parameters. We then simulate 10,000 artificial samples of length equivalent to that in the data. In each artificial dataset, we run the same regressions as conducted in Section 3 to evaluate how the regression slopes and average errors vary with deviations from rational expectations.

While we consider a range of values for the key parameter θ, one particular focus is the value estimated by Bordalo, Gennaioli, and Shleifer (2018) and Bordalo et al. (2019), $\theta = 0.91$. They estimate this value in different settings than the one considered here, allowing us to test whether our empirical results are consistent with a fully externally calibrated θ. Given that this value of θ is close to 1, the magnitude of forecast errors is roughly comparable to news, consistent with our slope estimates in Coibion-Gorodnichenko regressions.

Figure 9 plots regression slopes and average errors by the degree of overreaction. The model is qualitatively consistent with the data. As θ increases, the Mincer-Zarnowitz slope β_1 decreases toward zero and the error-predictability and Coibion-Gorodnichenko slopes become more negative. Given (14), subjective expectations are on average unbiased, so the model cannot produce any non-zero average error. There is thus no relationship between the average error and θ. At the calibrated value $\theta = 0.91$, the model and data are roughly in line, as the empirical estimates are close to the model-implied confidence intervals in all cases; slightly lower values of θ would align the two even more closely. This simple model of expectation errors thus appears capable of explaining the excess sensitivity of forward rates we document empirically.

However, while the model is able to match overreaction reasonably well despite the stylized setup and few parameters, the model does seem to fall short of matching the data on certain additional dimensions. In particular, it appears to underestimate the degree of rational variation in forward rates observed in the data: as seen in Figure A7,

the model's regression R^2 values are somewhat below those observed in the data on average. For example, the Mincer-Zarnowitz R^2 is 20% in the data, but only 10% at the calibrated $\theta = 0.91$.[25] Nonetheless, the model still replicates the same qualitative patterns in R^2 values as one would expect.

5.3 Interpretation and Relation to Past Work

While the above model of expectation errors is capable of matching our empirical findings, the cyclicality of expectation errors distinguishes our setting from certain frameworks featuring return extrapolation, as discussed in Section 3.4. In the model, forward rates overreact to news about spot rates: when the objective equity premium is high, forward rates increase, and forecast errors are predictably negative. This echoes our empirical findings. Those findings would not be matched by specifying a model of overreaction of forward rates to realized returns: the estimated equity premium is high in bad times, when realized returns are *low*.

If the cyclicality of our model's expectation errors were to be flipped — if forward rates were too low in bad times, generating positive errors — then these errors would not be capable of explaining any of the variation in the price-dividend ratio as considered in Figure 5. Forecast errors instead would exert a *dampening* force on stock prices, making it more difficult to explain their observed volatility. We discuss and formalize this idea in more detail in Appendix D, which introduces a "trilemma" for expectation errors: it is difficult to simultaneously make sense of (i) volatile expectation errors, (ii) countercyclical expectation errors (e.g., extrapolation of past returns), and (iii) a volatile price-dividend ratio.[26] Our framework discards (ii), leaving (i) and (iii) on the table.

Our results also relate to past work on the equity term structure. This literature studies how expected returns vary across dividend claims with different maturity; this is a conceptually different focus than our analysis, as we focus on expected returns on the market as a whole (rather than individual dividends) and consider how these expectations vary by horizon. That said, our results relate to (and are in line with) facts about the equity term structure. Our finding of small, positive forecast errors implies lower-than-expected realized returns on long-maturity claims like the market

[25] Lower values of θ again improve this aspect of the model's match to the data, though this is not the case for the forecast-error predictability R^2 values.

[26] This discussion builds on Campbell (2017), who introduces related trilemmas for present value and portfolio choice.

portfolio, potentially leading to a slightly negative observed term premium.[27] Moreover, Gormsen (2021) finds that this premium is positive in good times; we similarly find that realized spot rates are lower than expected ex ante in bad times, which would imply an unexpectedly high ex post realized return on the market, potentially generating a positive observed term premium.[28] As such, the simple model of expectation errors that we study can potentially also explain many of the dynamics of the equity term structure.

6 Conclusion

We introduce a new methodology to test whether the market understands time variation in the equity risk premium. We find evidence consistent with the notion that it does, at least to an extent. Forward rates get the direction of future spot rates right, and they explain as much of 20% of the variation in the equity premium at the 6-month horizon. This degree of spot-rate predictability exceeds the value typically observed for the fixed-income term structure, where forward rates contain less predictive power over future spot rates.[29] Nonetheless, forward rates are less-than-perfect forecasters of future spot rates: they appear excessively volatile relative to the predictable component of future spot rates, generating predictable forecast errors. In times of crisis, for example, the equity premium mean reverts more quickly than suggested by the forward curve.

We offer guidance on the elements needed to rationalize these findings through the behavior of the stochastic discount factor alone, showing that such a stochastic discount factor must feature a highly volatile and countercyclical price of discount-rate risk. The specifics of such a possible model are left to future work. We do, however, provide a specific model illustrating the viability of expectation errors, the leading alternative explanation for our results. Both the model-based and empirical variation in forward rates suggest a meaningful role for forecast errors in accounting for overall stock-price movements.

[27] See, e.g., Binsbergen and Koijen (2017) for results from dividend futures, Lazarus (2022) for results from short-term options, and Gormsen and Lazarus (2022) for results based on the equity cross-section. Separately, Callen and Lyle (2020) study firm-level equity term structures with options.

[28] Gormsen (2021) also finds that a highly volatile and countercyclical price of discount rate risk can in principle rationalize time variation in the equity term structure. In addition, our cyclicality results echo those of Giglio and Kelly (2018) for other term structures.

[29] While our tests have parallels to tests of the fixed-income expectations hypothesis, our framework by design allows for less room for discount-rate variation to explain our results.

Tables and Figures

Table 1
Mincer-Zarnowitz Regressions

This table reports Mincer-Zarnowitz regressions of future realized spot rates on current forward rates:

$$\mu_{i,t+6}^{(6)} = \beta_0 + \beta_1 f_{i,t}^{(6,6)} + e_{i,t+6}$$

The realized spot rate

$$\mu_{i,t+6}^{(6)} = \mathbb{E}_{t+6}\left[r_{i,t+6}^{(6)}\right]$$

is the future expectation of the 6-month equity premium. The forward rate

$$f_{i,t}^{(6,6)} = \mathbb{E}_t\left[r_{i,t+6}^{(6)}\right]$$

is the current expectation of the same risk premium. The units are annualized percentage points. Panel regressions, in columns (1)–(4), report standard errors clustered by exchange and date. Time-series regressions, in columns (5)–(6), report Newey-West standard errors with lags selected following Lazarus et al. (2018) and fixed-b p-values. The sample is the longest available for each exchange in the main sample.

	(1) Main	(2) Main	(3) Main	(4) Main excl U.S.	(5) U.S. Only	(6) SX5E Only
$f_t^{(6,6)}$	0.64***	0.56***	0.42***	0.55***	0.67***	0.65***
	(0.049)	(0.055)	(0.096)	(0.056)	(0.096)	(0.12)
Intercept	1.04***				0.74***	1.18**
	(0.17)				(0.28)	(0.48)
p-value: $\beta_1 = 1$	0.0000	0.0000	0.0002	0.0000	0.0025	0.0090
Observations	2227	2227	2080	1849	378	231
Fixed Effects	None	Ex	Ex/Date	Ex	None	None
Standard Errors	Cluster	Cluster	Cluster	Cluster	Newey-West	Newey-West
Cluster	Ex/Date	Ex/Date	Ex/Date	Ex/Date	-	-
Adjusted R^2	0.20	0.22	0.88	0.21	0.22	0.14
Within R^2	-	0.15	0.14	0.14	-	-

Table 2
Instrumented Mincer-Zarnowitz Regressions

This table reports instrumented Mincer-Zarnowitz regressions of future realized spot rates on current forward rates:

$$\mu^{(6)}_{i,t+6} = \beta_0 + \beta_1 f^{(6,6)}_{i,t} + e_{i,t+6}$$

The realized spot rate is the future expectation of the 6-month equity premium and the forward rate is the current expectation of the same risk premium. The instrument is the current expectation of the 1-month equity premium in 2 months:

$$f^{(2,1)}_{i,t} = \mathbb{E}_t \left[r^{(1)}_{i,t+2} \right]$$

The units are annualized percentage points. Panel regressions, in columns (1)–(4), report standard errors clustered by exchange and date. Time-series regressions, in columns (5)–(6), report Newey-West standard errors with lags selected following Lazarus et al. (2018) and fixed-b p-values. The sample is the longest available for each exchange in the main sample.

	(1) Main	(2) Main	(3) Main	(4) Main excl U.S.	(5) U.S. Only	(6) SX5E Only
$f^{(6,6)}_t$	0.77***	0.70***	0.92***	0.69***	0.73***	0.77***
	(0.071)	(0.074)	(0.20)	(0.078)	(0.062)	(0.11)
Intercept	0.73***				0.59***	0.86***
	(0.19)				(0.13)	(0.29)
p-value: $\beta_1 = 1$	0.0012	0.0028	0.6858	0.0043	0.0177	0.1677
Observations	2227	2227	2080	1849	378	231
Fixed Effects	None	Ex	Ex/Date	Ex	None	None
Standard Errors	Cluster	Cluster	Cluster	Cluster	Newey-West	Newey-West
Cluster	Ex/Date	Ex/Date	Ex/Date	Ex/Date	-	-
Adjusted R^2	0.19	-	-	-	0.22	0.14
Within R^2	-	0.14	-0.05	0.13	-	-

Table 3
Average Forecast Errors

This table reports average forecast errors. The forecast error is the difference between the future realized spot rate and the current forward rate:

$$\varepsilon_{i,t+6}^{(6)} = \mu_{i,t+6}^{(6)} - f_{i,t}^{(6,6)}$$

The realized spot rate is the future expectation of the 6-month equity premium and the forward rate is the current expectation of the same risk premium. The units are annualized percentage points. Panel regressions, in columns (1)–(2), report standard errors clustered by exchange and date. Time-series regressions, in columns (3)–(5), report Newey-West standard errors with lags selected following Lazarus et al. (2018). The sample is the longest available for each exchange in the main sample.

	(1) Main	(2) Main excl U.S.	(3) U.S. Only	(4) SX5E Only	(5) U.S. Post-2006
Intercept	0.17	0.20	0.021	0.24	0.029
	(0.11)	(0.11)	(0.15)	(0.25)	(0.26)
Observations	2227	1849	378	231	174
Standard Errors	Cluster	Cluster	Newey-West	Newey-West	Newey-West
Cluster	Ex/Date	Ex/Date	-	-	-

Table 4
Predictability of Forecast Errors

This table reports predictability regressions of future realized forecast errors on current forward rates:

$$\varepsilon^{(6)}_{i,t+6} = \beta_0 + \beta_1 f^{(2,1)}_{i,t} + e_{i,t+6}$$

The realized spot rate is the future expectation of the 6-month equity premium, the forward rate is the current expectation of the same risk premium, and the forecast error is the realized spot rate minus the forward rate. The predictor is the 2×1-month forward rate, the current expectation of the 1-month equity premium in 2 months. The units are annualized percentage points. Panel regressions, in columns (1)–(4), report standard errors clustered by exchange and date. Time-series regressions, in columns (5)–(6), report Newey-West standard errors with lags selected following Lazarus et al. (2018). The sample is the longest available for each exchange in the main sample.

	(1) Main	(2) Main	(3) Main	(4) Main excl U.S.	(5) U.S. Only	(6) SX5E Only
$f^{(2,1)}_t$	-0.13**	-0.16***	-0.028	-0.16**	-0.17**	-0.12
	(0.047)	(0.047)	(0.076)	(0.049)	(0.066)	(0.083)
Intercept	0.51***				0.39*	0.59
	(0.15)				(0.23)	(0.39)
Observations	2227	2227	2080	1849	378	231
Fixed Effects	None	Ex	Ex/Date	Ex	None	None
Standard Errors	Cluster	Cluster	Cluster	Cluster	Newey-West	Newey-West
Cluster	Ex/Date	Ex/Date	Ex/Date	Ex/Date	-	-
Adjusted R^2	0.02	0.03	0.82	0.03	0.03	0.01
Within R^2	-	0.03	0.00	0.03	-	-

Table 5
Coibion-Gorodnichenko Regressions

This table reports Coibion-Gorodnichenko regressions of future realized forecast errors on current 3-month forecast revisions:

$$\varepsilon^{(3)}_{i,t+6} = \beta_0 + \beta_1 \left(f^{(6,3)}_{i,t} - f^{(9,3)}_{i,t-3} \right) + e_{i,t+6}$$

The realized spot rate is the future expectation of the 3-month equity premium, the forward rate is the current expectation of the same risk premium, and the forecast error is the realized spot rate minus the forward rate. The units are annualized percentage points. Panel regressions, in columns (1)–(4), report standard errors clustered by exchange and date. Time-series regressions, in columns (5)–(6), report Newey-West standard errors with lags selected following Lazarus et al. (2018). The sample is the longest available for each exchange in the main sample.

	(1) Main	(2) Main	(3) Main	(4) Main excl U.S.	(5) U.S. Only	(6) SX5E Only
$f^{(6,3)}_t - f^{(9,3)}_{t-3}$	-0.24*	-0.24*	-0.30***	-0.24*	-0.29**	-0.12
	(0.12)	(0.12)	(0.087)	(0.12)	(0.12)	(0.19)
Intercept	0.25				0.0036	0.26
	(0.14)				(0.19)	(0.27)
Observations	2186	2186	2039	1811	375	228
Fixed Effects	None	Ex	Ex/Date	Ex	None	None
Standard Errors	Cluster	Cluster	Cluster	Cluster	Newey-West	Newey-West
Cluster	Ex/Date	Ex/Date	Ex/Date	Ex/Date	-	-
Adjusted R^2	0.01	0.02	0.84	0.02	0.01	-0.00
Within R^2	-	0.01	0.06	0.01	-	-

Table 6
Power Utility Regressions

This table reports regression estimates from the standpoint of an unconstrained power utility investor fully invested in the market. See Appendix B.4 for more details. Panel A reports estimates from panel regressions in the main sample. The sample is the longest available for each exchange. Panel B reports estimates from time-series regressions in the United States. The sample is from January 1990 to June 2021. Mincer-Zarnowitz regressions test $H_0 : \beta_1 = 1$, as in Table 1. The average forecast error tests $H_0 : \bar{\varepsilon}_t = 0$, as in Table 3. Error-predictability regressions test $H_0 : \beta_1 = 0$, as in Table 4. Coibion-Gorodnichenko regressions test $H_0 : \beta_1 = 0$, as in Table 5. The risk premium is the 6-month spot rate in 6 months for Mincer-Zarnowitz, average error, and error-predictability regressions. The risk premium is the 3-month spot rate in 6 months for Coibion-Gorodnichenko regressions. The units are annualized percentage points. All regressions include exchange fixed effects and report a within R^2. Panel regressions, in the main sample, report standard errors clustered by exchange and date. Time-series regressions, in the United States, report Newey-West standard errors with lags selected following Lazarus et al. (2018) and fixed-b p-values.

	$\mu^{(6)}_{t+6}$												$\mu^{(3)}_{t+6}$				
	Mincer-Zarnowitz				Average Error			Error Predictability					Coibion-Gorodnichenko				
	β_1	$se(\beta_1)$	p-val	R^2	$\bar{\varepsilon}_t$	$se(\bar{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	N	β_1	$se(\beta_1)$	p-val	R^2	N
Panel A. Main Sample																	
$\gamma = 0.50$	0.18	0.041	***	0.05	-0.047	0.010	***	-0.40	0.10	***	0.11	2094	-0.30	0.11	**	0.08	1999
$\gamma = 0.75$	0.51	0.059	***	0.14	0.046	0.057		-0.19	0.045	***	0.04	2202	-0.29	0.11	**	0.02	2149
$\gamma \to 1.00$	0.57	0.054	***	0.15	0.17	0.11		-0.17	0.046	***	0.03	2195	-0.26	0.12	*	0.01	2140
$\gamma = 1.25$	0.60	0.056	***	0.16	0.36	0.15	**	-0.14	0.049	**	0.02	2181	-0.23	0.13		0.01	2122
$\gamma = 1.50$	0.62	0.061	***	0.16	0.58	0.19	**	-0.10	0.054	*	0.01	2164	-0.20	0.13		0.01	2097
$\gamma = 2.00$	0.64	0.075	***	0.16	1.12	0.28	***	-0.054	0.059		0.00	2114	-0.16	0.16		0.00	2016
Panel B. United States																	
$\gamma = 0.50$	0.31	0.059	***	0.17	-0.041	0.015	***	-0.63	0.12	***	0.28	378	-0.26	0.11	**	0.03	375
$\gamma = 0.75$	0.61	0.094	***	0.21	-0.022	0.083		-0.20	0.069	***	0.05	378	-0.30	0.12	***	0.01	375
$\gamma \to 1.00$	0.67	0.096	***	0.22	0.021	0.15		-0.17	0.066	**	0.03	378	-0.29	0.12	**	0.01	375
$\gamma = 1.25$	0.72	0.098	**	0.23	0.12	0.20		-0.13	0.064	**	0.02	378	-0.27	0.13	**	0.01	375
$\gamma = 1.50$	0.76	0.100	**	0.24	0.25	0.26		-0.099	0.062		0.01	378	-0.24	0.14	*	0.00	375
$\gamma = 2.00$	0.85	0.10		0.26	0.57	0.35		-0.048	0.059		0.00	378	-0.18	0.15		0.00	375
$\gamma = 2.50$	0.91	0.10		0.27	0.94	0.44	**	-0.0087	0.057		-0.00	378	-0.12	0.17		-0.00	375
$\gamma = 3.00$	0.97	0.10		0.28	1.32	0.52	**	0.022	0.056		-0.00	378	-0.075	0.18		-0.00	375

Figure 2
Current Spot Rates and Forward Rates

This figure plots the current 6-month spot rate $\mu_t^{(6)}$ (blue) and the 6×6-month forward rate $f_t^{(6,6)}$ (red) in the United States. Gray bands are NBER recessions. The sample is from January 1990 to June 2021.

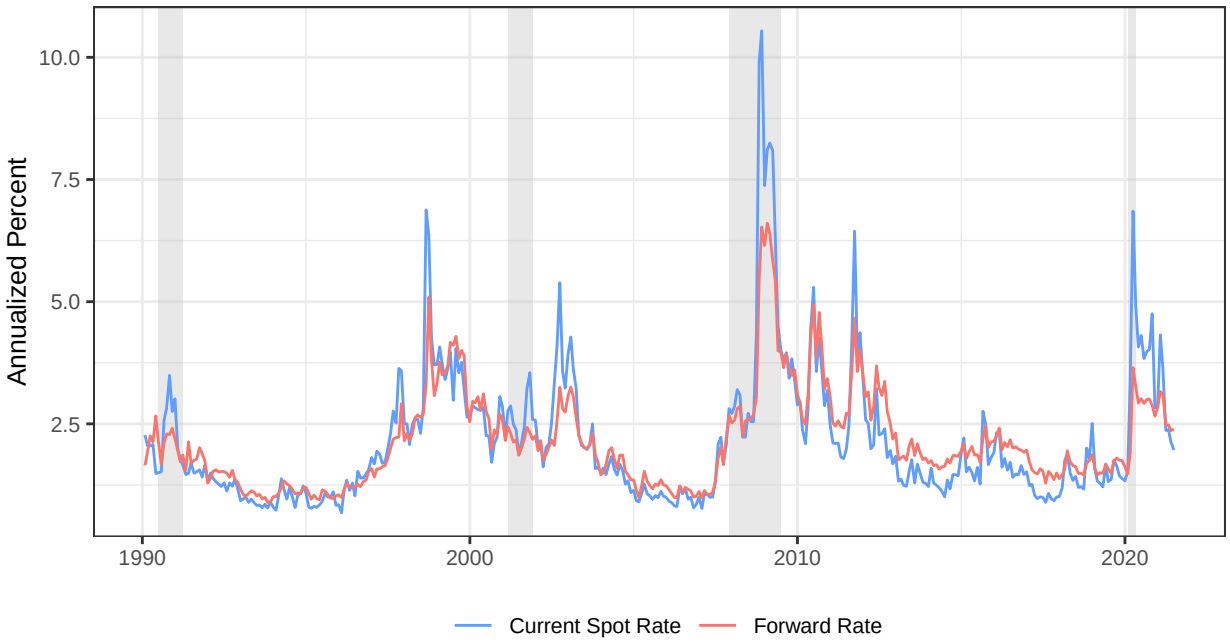


Figure 3
Forward Rates and Realized Spot Rates

This figure plots the instrumented 6×6-month forward rate $f_t^{(6,6)}$ (red) and the corresponding realized spot 6-month spot rate $\mu_{t+6}^{(6)}$ (blue) in the United States. The instrument is the 2×1-month forward rate: see Table 2 for more details. Gray bands are NBER recessions. The sample is from January 1990 to June 2021.

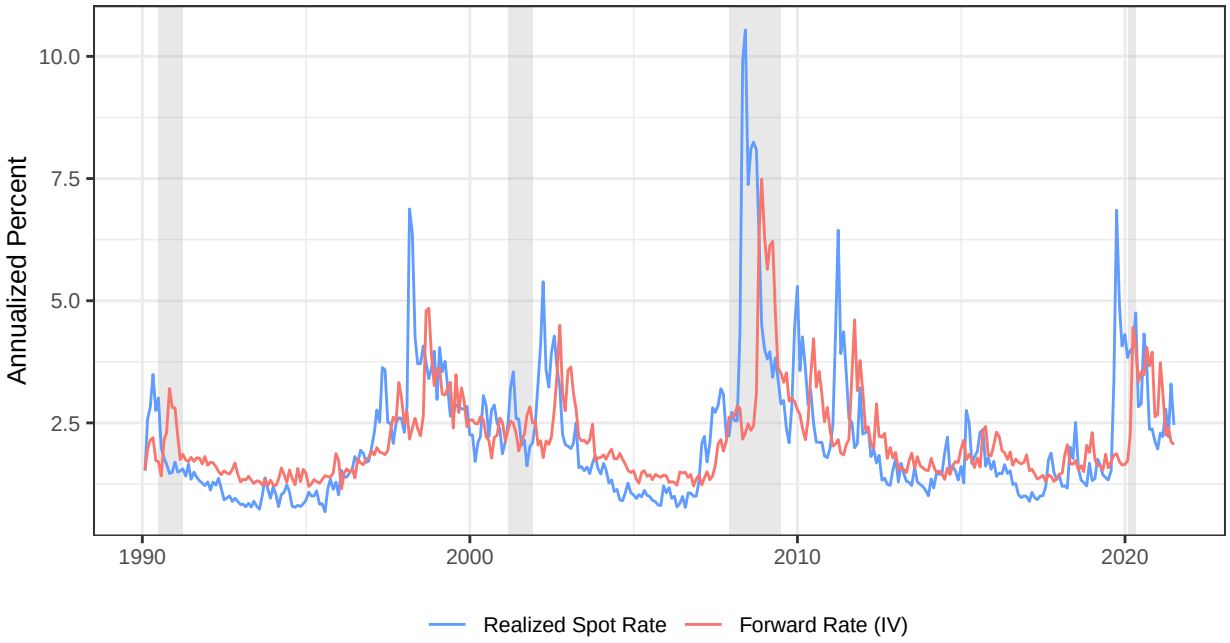


Figure 4
Forward Rates and Realized Forecast Errors

This figure plots the current 6×6-month forward rate $f_t^{(6,6)}$ (blue) and the corresponding realized forecast error $\varepsilon_{t+6}^{(6)}$ (gray) in the United States. The sample is from January 1990 to June 2021.

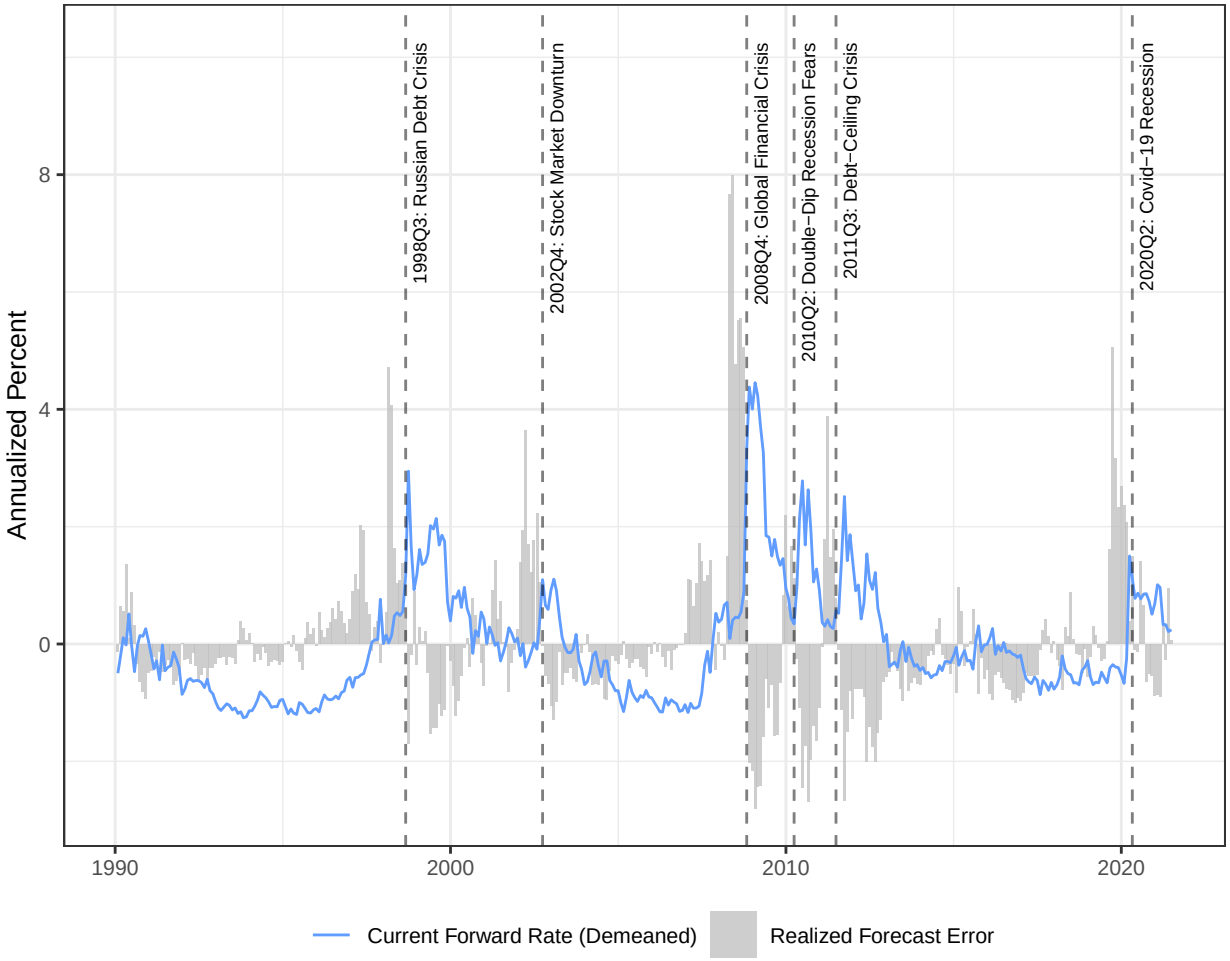


Figure 5
Discounted Forecast Errors and the Price-Dividend Ratio

This figure plots discounted predicted forecast errors over all future horizons $\mathcal{E}_t$ (red) and the log repurchase-adjusted price-dividend ratio $p_t - d_t$ (blue) in the United States. Discounted forecast errors $\mathcal{E}_t$ are calculated as in (10) using the 6×6 predicted forecast error (non-annualized), with ρ calculated using the full-sample average price-dividend ratio. The monthly repurchase-adjusted $p_t - d_t$ for the CRSP value-weighted stock index is obtained from Nagel and Xu (2022a) via Zhengyang Xu's website. Both series are plotted as log differences from their full-sample means. Gray bands are NBER recessions. The sample is from January 1990 to June 2021.

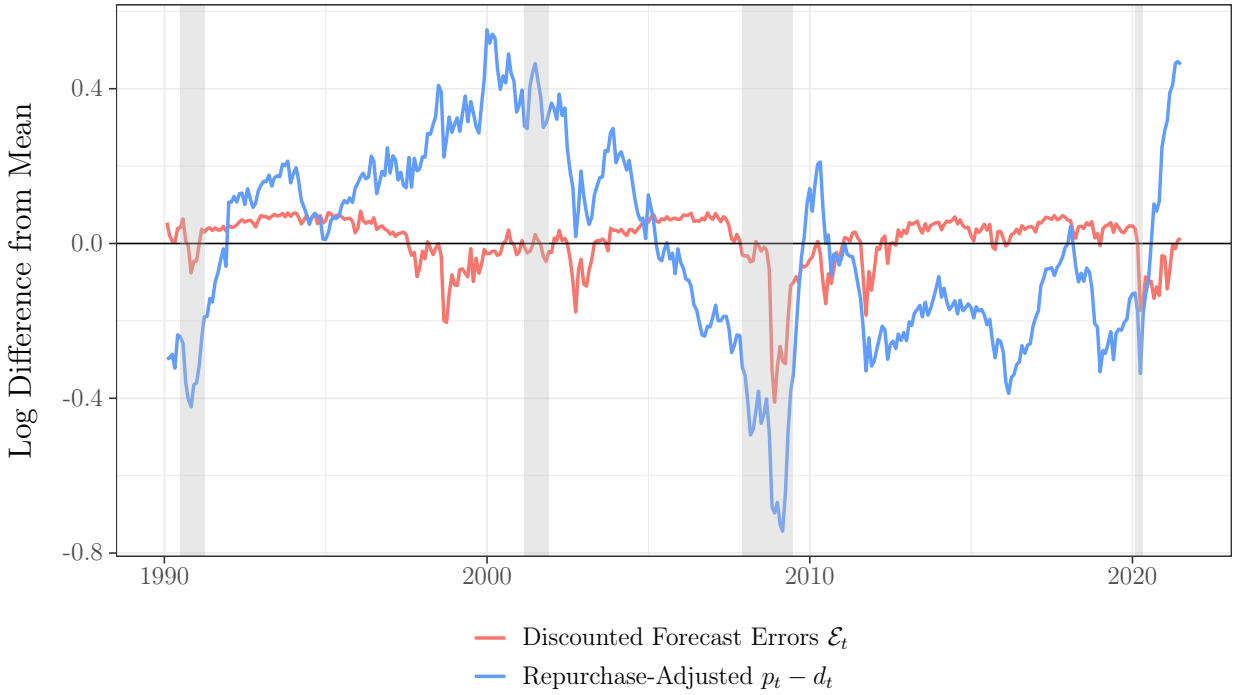


Figure 6
Forward Rates and Predicted Forecast Errors

This figure plots the instrumented 6×6-month forward rate $f_t^{(6,6)}$ (red) and the corresponding predicted forecast error $\varepsilon_{t+6}^{(6)}$ (blue) in the United States. The instrument is the 2×1-month forward rate: see Table 2 for more details. The predicted forecast error is from a time-series regression of future realized forecast errors on current forward rates: see Table 4 for more details. Gray bands are NBER recessions. The sample is from January 1990 to June 2021.

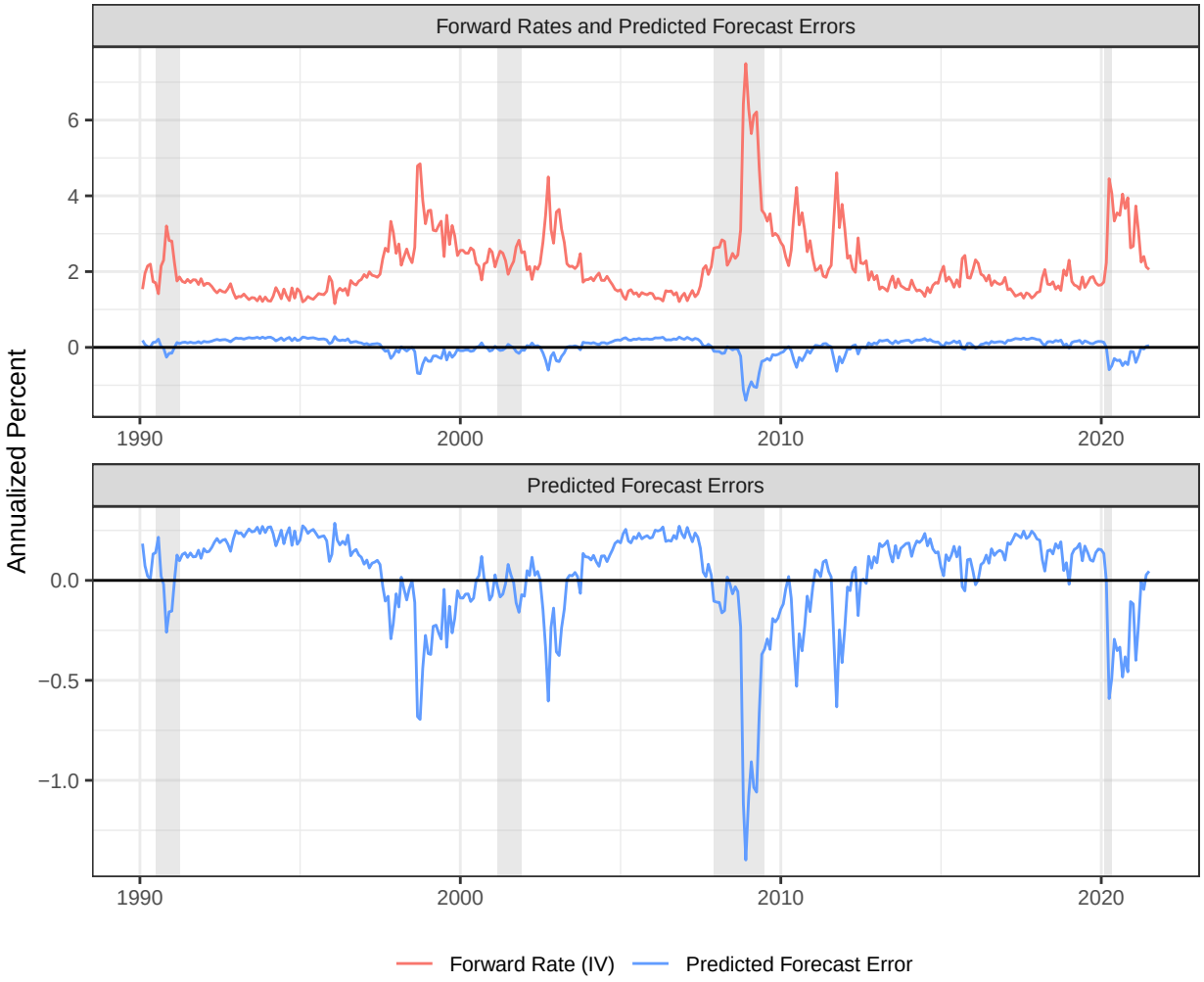


Figure 7
Conditional Volatility of the Market Return

This figure plots the conditional volatility of the 6-month market return $\sigma_t\left(\ln R_{t,t+6}\right)$ from the standpoint of an unconstrained log utility investor fully invested in the market. See Appendix B.4 for more details. Gray bands are NBER recessions. The sample is from January 1990 to June 2021 in the United States.

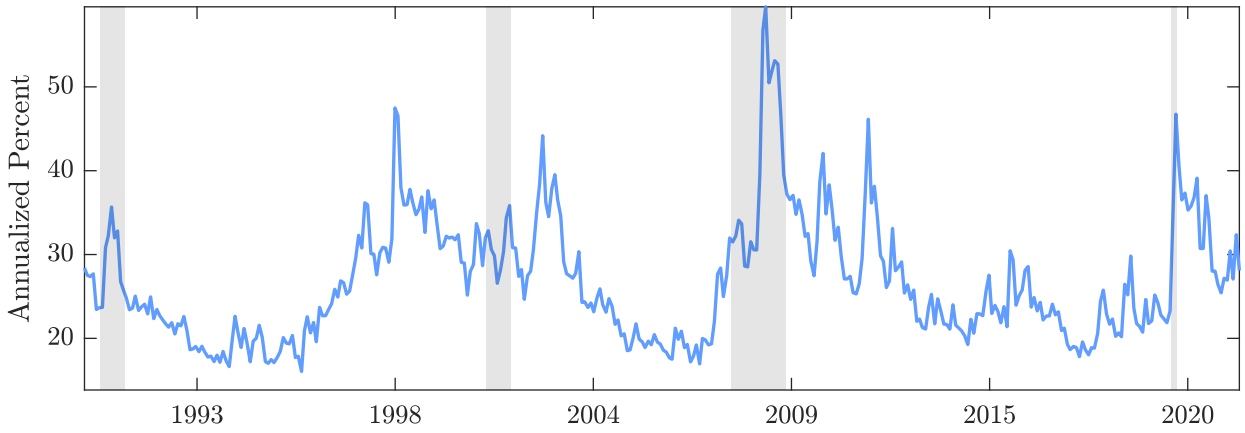


Figure 8
Power Utility: Regression Slopes and Average Forecast Errors

This figure reports regression slopes and average forecast errors from the standpoint of an unconstrained power utility investor fully invested in the market. See Panel A of Table 6 for more details. The sample is the longest available for each exchange in the main sample.

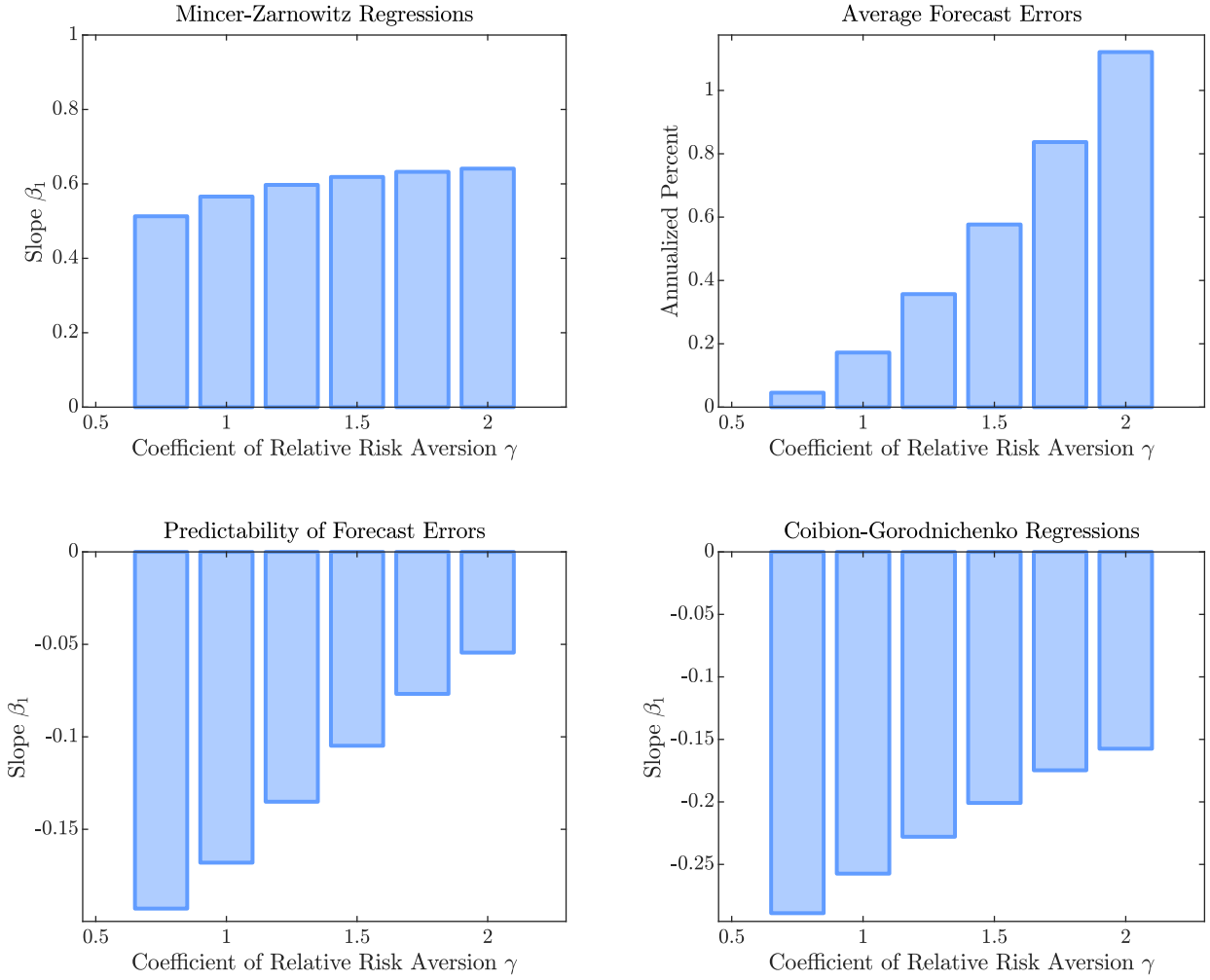
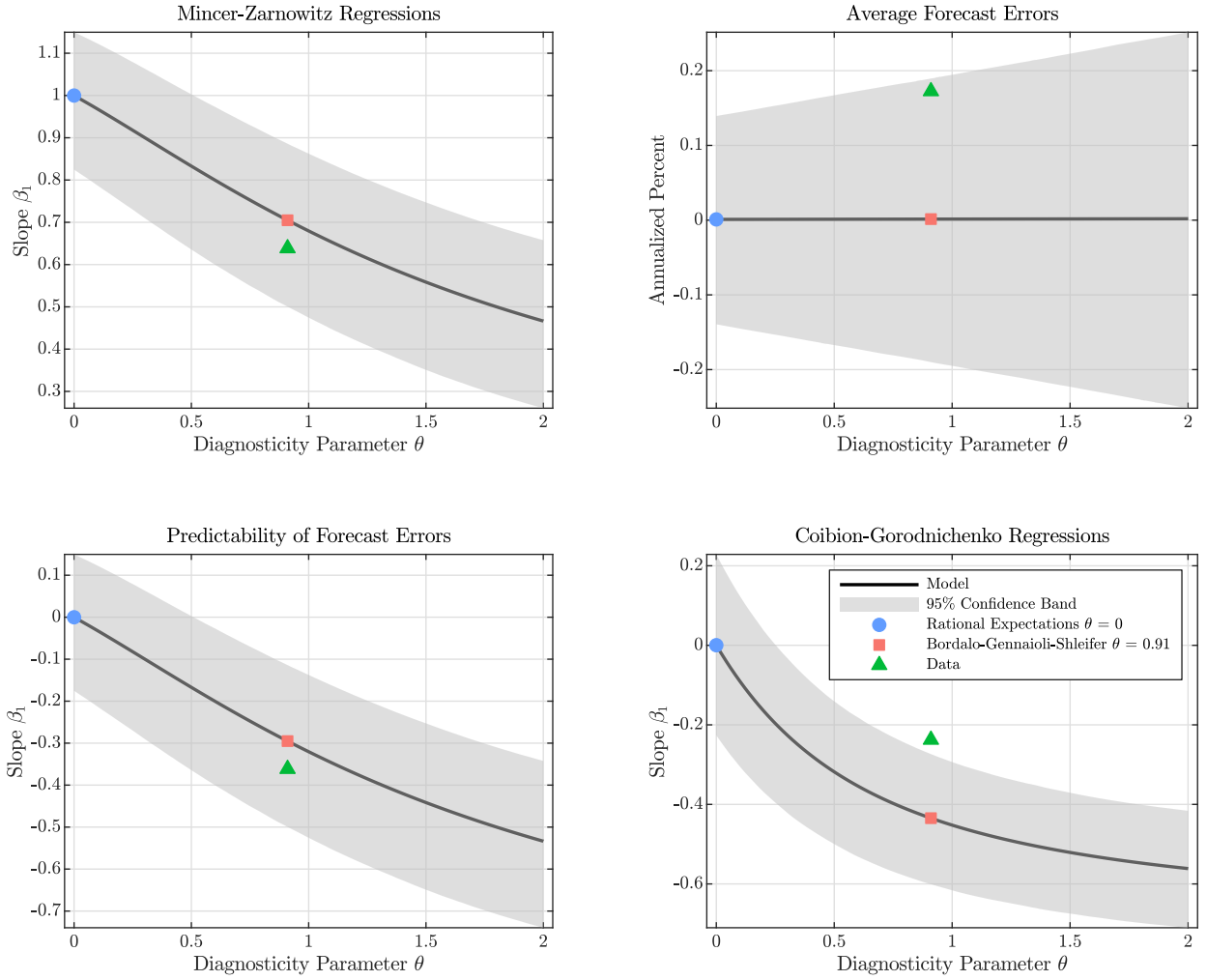


Figure 9
Model Calibration: Regression Slopes and Average Forecast Errors

This figure reports regression slopes and average forecast errors in the calibrated model of expectation errors. The model is calibrated from the standpoint of an unconstrained log utility investor fully invested in the market. Table A9 reports the objective parameters. The solid lines are model-implied population moments in a single long sample. The shaded regions are model-implied 95% confidence bands in 10,000 short samples. The blue circles are model-implied moments under rational expectations with $\theta = 0$. The red squares are model-implied moments under diagnostic expectations with $\theta = 0.91$ from Bordalo, Gennaioli, and Shleifer (2018). The green triangles are moments in the data. The sample is the longest available for each exchange in the main sample.



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For Online Publication

Online Appendix for:

Does the Market Understand Time Variation in the Equity Premium?*

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December 2022

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A Proofs

Proof of Proposition 1. Given that $M_{t,t+n}R_{t,t+n} = 1$ by assumption, $\mathcal{C}_t^{(n)} = 0$ in (2). The stated results then follow immediately. □

Proof of Proposition 2. First consider $n = m = 1$, and write

$$\begin{aligned}\varepsilon_{t+1}^{(1)} = \mu_{t+1}^{(1)} - f_t^{(1)} = \mathscr{L}_{t+1}^{(1)} - \mathscr{L}_t^{(2)} + \mathscr{L}_t^{(1)} + \operatorname{cov}_t(MR_{t,t+2}, r_{t,t+2}) \\ - \operatorname{cov}_t(MR_{t,t+1}, r_{t,t+1}) - \operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2}).\end{aligned} \tag{A1}$$

Consider the first covariance term. Given the joint log-normality of the SDF and returns (and the normality of $r_{t,t+n}$), Stein's lemma gives that

$$\begin{aligned}\operatorname{cov}_t(MR_{t,t+2}, r_{t,t+2}) &= \operatorname{cov}_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+2}) \\ &= \operatorname{cov}_t(mr_{t,t+1} + mr_{t+1,t+2}, r_{t,t+2})\,\mathbb{E}_t[MR_{t,t+2}] \\ &= \operatorname{cov}_t(mr_{t,t+1} + mr_{t+1,t+2}, r_{t,t+1} + r_{t+1,t+2})\,,\end{aligned}$$

where $mr_{t,t+n} = \ln(MR_{t,t+n})$, and where the last line uses that $\mathbb{E}_t[MR_{t,t+2}] = 1$. Having separated the two MR terms, apply Stein's lemma again to obtain

$$\begin{aligned}\operatorname{cov}_t(MR_{t,t+2}, r_{t,t+2}) &= \operatorname{cov}_t(mr_{t,t+2}, r_{t,t+1}) + \operatorname{cov}_t(mr_{t,t+1}, r_{t+1,t+2}) + \operatorname{cov}_t(mr_{t+1,t+2}, r_{t,t+2}) \\ &= \operatorname{cov}_t(MR_{t,t+2}, r_{t,t+1}) + \operatorname{cov}_t(MR_{t,t+1}, r_{t+1,t+2}) \\ &\quad + \operatorname{cov}_t(MR_{t+1,t+2}, r_{t+1,t+2})\,.\end{aligned} \tag{A2}$$

For the first two terms in (A2), by the law of total covariance and using that $\mathbb{E}_{t+1}[MR_{t+1,t+2}] = 1$,

$$\begin{aligned}\operatorname{cov}_t(MR_{t,t+2}, r_{t,t+1}) &= \mathbb{E}_t\big[MR_{t,t+1}r_{t,t+1}\operatorname{cov}_{t+1}(MR_{t+1,t+2}, 1)\big] \\ &\quad + \operatorname{cov}_t(MR_{t,t+1}\,\mathbb{E}_{t+1}[MR_{t+1,t+2}], r_{t,t+1}) \\ &= \operatorname{cov}_t(MR_{t,t+1}, r_{t,t+1})\,,\end{aligned} \tag{A3}$$

$$\begin{aligned}\operatorname{cov}_t(MR_{t,t+1}, r_{t+1,t+2}) &= \mathbb{E}_t[MR_{t,t+1}\operatorname{cov}_{t+1}(1, r_{t+1,t+2})] + \operatorname{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]) \\ &= \operatorname{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}])\,.\end{aligned} \tag{A4}$$

Turning now to the last term in (A1), the law of total covariance can similarly be applied to obtain that as of time t,

$$\mathbb{E}_t[\operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})] = \operatorname{cov}_t(MR_{t+1,t+2}, r_{t+1,t+2}). \tag{A5}$$

Taking expectations in (A1), substituting in results (A2)–(A5), and applying the definition of $\widehat{\varepsilon}_{t+1}^{(1)}$, we obtain:

$$\mathbb{E}_t\left[\varepsilon_{t+1}^{(1)}\right] = \mathbb{E}_t\left[\widehat{\varepsilon}_{t+1}^{(1)}\right] + \operatorname{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}])\,. \tag{A6}$$

Rearranging to solve for $\mathbb{E}_t[\widehat{\varepsilon}_{t+1}^{(1)}]$ yields the stated result for the $n = m = 1$ case. While this case is convenient for straightforward derivations, note that all the above steps apply when using $t + n$ in place of $t + 1$ and using $t + n + m$ in place of $t + 2$, so the stated result holds for general n, m. □

Proof of Proposition 3. Starting again with (A1) and expanding the first covariance term,

$$\operatorname{cov}_t(MR_{t,t+2}, r_{t,t+2}) = \operatorname{cov}_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+1}) + \operatorname{cov}_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t+1,t+2}). \quad \text{(A7)}$$

We consider each of the two terms on the right side of (A7) in turn, and in both cases apply the law of total covariance. For the first term, as in (A3),

$$\operatorname{cov}_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+1}) = \operatorname{cov}_t(MR_{t,t+1}, r_{t,t+1})\,. \quad \text{(A8)}$$

For the second term,

$$\begin{aligned}\operatorname{cov}_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+2}) = \mathbb{E}_t\big[MR_{t,t+1}\operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})\big] \\ + \operatorname{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}])\,. \quad \text{(A9)}\end{aligned}$$

Using (A8) and (A9) in (A1), applying the definition of $\widehat{\varepsilon}^{(1)}_{t+1}$, and taking expectations,

$$\begin{aligned}\mathbb{E}_t\left[\varepsilon^{(1)}_{t+1}\right] &= \mathbb{E}_t\left[\widehat{\varepsilon}^{(1)}_{t+1}\right] + \mathbb{E}_t[(MR_{t,t+1}-1)\operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})] \\ &\quad + \operatorname{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]) \\ &= \mathbb{E}_t\left[\widehat{\varepsilon}^{(1)}_{t+1}\right] + \operatorname{cov}_t(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}] + \operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2}))\,. \quad \text{(A10)}\end{aligned}$$

Note from (2) that $\mathscr{L}^{(1)}_{t+1} + r^f_{t+1,t+2} = \mathbb{E}_{t+1}[r_{t+1,t+2}] + \operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})$. Using this in (A10) along with the definition of $\widehat{\mu}^{(1)}_{t+1}$,

$$\mathbb{E}_t\left[\widehat{\varepsilon}^{(1)}_{t+1}\right] = \mathbb{E}_t\left[\varepsilon^{(1)}_{t+1}\right] - \operatorname{cov}_t\left(MR_{t,t+1}, \widehat{\mu}^{(1)}_{t+1}\right).$$

The above steps again apply when using $t+n$ in place of $t+1$ and using $t+n+m$ in place of $t+2$, completing the proof. □

Proof of Lemma A1. To compute the risk-neutral expectation of $H[P_T] = R^\alpha (\ln R)^\beta$, we apply standard spanning theorems (Bakshi and Madan 2000, Carr and Madan 2001). We have

$$\begin{aligned}\frac{1}{R_f}\mathbb{E}^*_t\left[R^\alpha(\ln R)^\beta\right] &= \left(H\left[\bar{P}\right] - \bar{P}H_P\left[\bar{P}\right]\right)\frac{1}{R_f} + H_P\left[\bar{P}\right]P_t \\ &\quad + \int_0^{\bar{P}} H_{PP}[K]\,\text{put}^{(n)}_t(K)dK + \int_{\bar{P}}^{\infty} H_{PP}[K]\,\text{call}^{(n)}_t(K)dK.\end{aligned}$$

The result follows by setting $\bar{P} = F^{(n)}_t$ and simplifying. □

Proof of Proposition A1. (A13) is immediate from Martin (2017) Result 8. (A14) and (A15) follow from Lemma A1 by setting the appropriate α and β and simplifying. □

B Measurement Details

B.1 Data

United States Data. For the 1996 to 2021 period, we obtain end-of-day option prices, index prices, projected dividend yields, and risk-free rates from OptionMetrics. To maximize the sample size, we

use options with both AM and PM settlement. We use the bid/ask midpoint as the option price in the main analysis. We linearly interpolate the risk-free rate curve to match option maturities. If either the dividend yield or risk-free rate is missing, we use the last non-missing observation.

For the 1990 to 1995 period, we obtain intraday option quotes from CBOE Market Data Express, as in Kelly, Pástor, and Veronesi (2016) and Culp, Nozawa, and Veronesi (2018). We obtain end-of-day index prices/returns from CRSP and estimate dividend yields from lagged one-year cum/ex-dividend index returns. We obtain Treasury bill rates and constant maturity Treasury yields from FRED to construct risk-free rates, as in Culp, Nozawa, and Veronesi.

Unlike OptionMetrics, CBOE provides intraday quotes. To construct end-of-day prices, we first apply filters to the intraday data and then use the last available quote. We drop quotes with the missing codes of 998 or 999. We drop quotes with negative bid-ask spreads. We correct erroneously recorded quotes – quotes with strike price less than 100 – by multiplying the strike/option price by 10. We drop end-of-day quotes that increase and then decrease fourfold (or vice versa), following similar filters in Andersen, Bondarenko, and Gonzalez-Perez (2015) and Duarte, Jones, and Wang (2022). We interpret these large reversals as probable data errors. To validate these filters, we compare data from CBOE and OptionMetrics in 1996. We match approximately 99.3% of option prices in OptionMetrics, suggesting these filters are not unreasonable.

We apply standard filters to the end-of-day data (Constantinides, Jackwerth, and Savov 2013). (1) We drop options with special settlement. (2) To eliminate duplicate quotes, we select the quote with highest open interest. (3) We drop options with fewer than seven days-to-maturity. (4) We drop options with price less than 0.01. (5) We drop options with zero bid prices or negative bid-ask spreads. (6) We drop options that violate static no-arbitrage bounds:

$$\text{put}_t^{(n)}(K) \leq Ke^{-r\tau} \qquad \text{call}_t^{(n)}(K) \leq P_t.$$

(7) We drop options for which the Black-Scholes implied volatility computation does not converge and options with implied volatility less than 5% or greater than 100%.

International Data. We again obtain end-of-day option prices, index prices, projected dividend yields, and risk-free rates from OptionMetrics. Unlike the United States, most option prices are either end-of-day settlement prices or last traded prices. Only a small fraction are from either bid/ask prices. The index price is time synchronized to the option price. If the index price is missing, we obtain the end-of-day price from Compustat Global. Risk-free rates are from currency-matched LIBOR curves. Dividend yields are from put-call parity and so are maturity-specific. As before with risk-free rates, we linearly interpolate the dividend yield curve to match option maturities. We apply the same filters to the end-of-day data as with the United States, except for filters that require bid/ask prices.

Table A1 describes the international sample. The European sample begins in January 2002 and ends in September 2021. The Asian sample begins in January 2004 and ends in April 2021. Our international sample closely follows Kelly, Pástor, and Veronesi (2016) and Dew-Becker and Giglio (2021), but we also use pan-European Stoxx indexes. These indexes represent a substantive addition to the sample. At long maturities, the Euro Stoxx 50 is arguably the most liquid options market in the world, as is the case with the dividend futures market (Binsbergen and Koijen 2017, Binsbergen et al. 2013).

Main Sample. As the international data is not equally robust across exchanges, we select the most reliable exchanges for the main analysis. We select the main sample by elimination. We drop Netherlands and Japan because they do not have sufficiently dense options, as seen in Panel A of

Figure A1. We drop Finland, the Stoxx Europe 50, and the Stoxx Europe 600 because they do not have reliable open interest data, as seen in Panel B of Figure A1. We drop Belgium, Korea, and Taiwan because they do not consistently have long-maturity options, as seen in Panel D of Figure A2. We drop China and Sweden because they do not have sufficiently deep out-of-the-money options, as seen in Figure A2. This leaves the 10 exchanges in the last column of Table A1 for the main sample. As a robustness check, we examine the full sample of 20 exchanges in Table A4 and Table A5.

B.2 Baseline Measures

Methodology. On each date and separately for puts/calls,

1. We convert option prices to implied volatilities via Black-Scholes. Here we follow an extensive literature on option-implied risk-neutral densities that finds interpolation more conducive in the space of implied volatilities, not option prices (Figlewski 2010, Malz 2014).

2. We fit a Delaunay triangulation to implied volatilities. The grid consists of strike prices between $\underline{K} = 0.10 \times P_t$ and $\overline{K} = 2.00 \times P_t$ with $\Delta K = 0.001 \times P_t$ and maturities $\tau = 30$, 60, 91, 122, 152, 182, 273, 365 days. The triangulation extrapolates as necessary with the nearest implied volatility in moneyness and time-to-maturity space.

3. We convert the triangulation of implied volatilities back to option prices via Black-Scholes. We then use the implied triangulation of option prices to evaluate the LVIX integral in (5) via Gaussian quadrature.

4. With the LVIX in hand, we can immediately compute spot rates, forward rates, and forecast errors under log utility via Proposition 1, as shown in Figure A3. Figure A4 plots contemporaneous 6-month spot rates and 6×6-month forward rates in the full sample, analogous to Figure 2 in the United States.

5. We occasionally find negative forward rates. Gao and Martin (2021) argue that negative forward rates are unlikely theoretically and likely represent data errors. We follow Gao and Martin and drop such observations, but our results are not quantitatively sensitive to this choice.

Discussion. Three empirical challenges in the computation of option-implied moments – discretization, truncation, and interpolation bias – motivate our baseline methodology (Carr and Wu 2009, Jiang and Tian 2007). We discuss each in turn. First, discretization bias arises because (5) requires numerical integration. To minimize this bias, we integrate on a fine grid of interpolated option prices in step 3. Second, truncation bias arises because (5) requires integration over an infinite range of strike prices in theory. In practice, we truncate the integral. To minimize this bias, we extrapolate and integrate over strike prices well beyond the range of observable option prices in step 2. Finally, interpolation bias arises because (5) usually requires options with unavailable maturities. To address this bias, we interpolate the option surface at target maturities in step 2.

B.3 Alternative Measures

Table A3 reports robustness checks where we use alternative choices to measure spot rates, forward rates, and forecast errors. As we discuss below, the main results are largely robust to these choices.

Integration Bounds. Panel A repeats the analysis with alternative integration bounds. The first four rows consider static bounds without extrapolation. As an example, the first row evaluates the integral in (5) between strike prices $\underline{K} = 0.65 \times P_t$ and $\overline{K} = 1.35 \times P_t$ at each maturity. The fifth row uses observable option prices between strike prices $\underline{K} = 0.10 \times P_t$ and $\overline{K} = 2.00 \times P_t$, again without extrapolation. The bounds in the first five rows naturally vary both by time and maturity with the availability of option prices. The sixth row considers static bounds with extrapolation, following a similar robustness check in Gormsen and Jensen (2022):

$$\left[\underline{K}^{(n)}, \overline{K}^{(n)}\right] = \begin{cases} [0.75, 1, 25] \times P_t & n \in \{1, 2\} \\ [0.55, 1.45] \times P_t & n \in \{3, 4, 5\} \\ [0.35, 1.65] \times P_t & n \in \{6, 9\} \\ [0.20, 1.80] \times P_t & n \in \{12\}. \end{cases}$$

These bounds vary by maturity, but not by time. The seventh row considers dynamic bounds with extrapolation, again following a similar robustness check in Gormsen and Jensen:

$$\underline{K}^{(n)} = \max\left\{0.10, 1.00 - 5\sigma_t^{(n)}\sqrt{\tau}\right\} \times P_t \qquad \overline{K}^{(n)} = \min\left\{2.00, 1.00 + 5\sigma_t^{(n)}\sqrt{\tau}\right\} \times P_t,$$

where $\sigma_t^{(n)}$, the price of the volatility contract in Bakshi, Kapadia, and Madan (2003), proxies for the risk-neutral volatility of the market return:

$$\begin{aligned} \left(\sigma_t^{(n)}\sqrt{\tau}\right)^2 &= \frac{1}{R_{t,t+n}^f}\mathbb{E}_t^*\left[\left(\ln R_{t,t+n}\right)^2\right] \\ &= \int_0^{F_t^{(n)}} \frac{2\left(1 + \ln\left[\frac{P_t}{K}\right]\right)}{K^2}\,\text{put}_t^{(n)}(K)dK + \int_{F_t^{(n)}}^{\infty} \frac{2\left(1 + \ln\left[\frac{P_t}{K}\right]\right)}{K^2}\,\text{call}_t^{(n)}(K)dK. \end{aligned} \tag{A11}$$

These bounds vary by both time and maturity with volatility. The eighth row considers the baseline integration bounds, as discussed in the main text and Appendix B.2.

In sum, this exercise illustrates the significant effect truncation/extrapolation have on the regression estimates. With shallow bounds, forecast errors are relatively large on average but less predictable. With deep bounds, forecast errors are relatively small on average but more predictable.

Liquidity Filters. Panel B repeats the analysis with alternative liquidity filters. The first row considers an outlier filter, following similar filters in Constantinides, Jackwerth, and Savov (2013) and Beason and Schreindorfer (2022). On each date and separately for puts/calls, we first fit a quadratic function to implied volatilities in terms of moneyness K/P and time-to-maturity. To minimize the effect of deep out-of-the-money, short/long-maturity options, we only use options with maturity $14 \leq \tau \leq 365$ days and moneyness $0.65 \leq K/P \leq 1.35$. We then drop influential observations via Cook's Distance. The second row considers an open interest filter. We drop options with zero open interest. We do not have open interest data before 1996. The third row combines the outlier and open interest filters. In all, this exercise is consistent with the baseline results, suggesting that option illiquidity does not explain our findings.

Volatility Surface. The first row in Panel C repeats the analysis with the interpolated volatility surface from OptionMetrics. OptionMetrics provides interpolated Black-Scholes implied volatilities on a constant moneyness/maturity grid. The literature often uses this surface for options with American exercise because OptionMetrics reports an equivalent, European exercise, implied volatility

(Kelly, Lustig, and Van Nieuwerburgh 2016, Martin and Wagner 2019). We instead simply use it as a robustness check on our own Delaunay triangulation of the volatility surface. In short, this exercise is consistent with the baseline results, although the average forecast error is somewhat smaller.

SVI Surface. The second row in Panel C repeats the analysis with the stochastic volatility inspired (SVI) surface from Jim Gatheral at Merrill Lynch (Gatheral 2011, Gatheral and Jacquier 2011, 2014). Our implementation of the SVI surface closely follows Berger, Dew-Becker, and Giglio (2020) and Beason and Schreindorfer (2022). We parameterize squared Black-Scholes implied volatilities with the function

$$\sigma_{BS}^2(t,\kappa,\tau) = a + b\left(\rho(\kappa - m) + \sqrt{(\kappa - m)^2 + \sigma^2}\right), \tag{A12}$$

where κ is standardized forward moneyness

$$\kappa = \frac{\ln K - \ln F_t^{(n)}}{\sigma_t^{(n)}\sqrt{\tau}},$$

$\sigma_t^{(n)}$ proxies for the risk-neutral volatility of the market return as in (A11), and each parameter is a linear function of time-to-maturity (e.g., $a = a_0 + a_1\tau$). On each date, we estimate parameters $\theta = (a_0, a_1, b_0, b_1, \rho_0, \rho_1, m_0, m_1, \sigma_0, \sigma_1)$ that minimize the implied volatility RMSE between the surface (A12) and the data, subject to standard no-arbitrage constraints: option prices are nonnegative and monotonic/convex in K (Aït-Sahalia and Duarte 2003). We check these constraints on a grid with moneyness between $-20 \leq \kappa \leq 0.50$ for puts, between $-0.50 \leq \kappa \leq 10$ for calls, and maturities $\tau = 30$, 60, 91, 122, 152, 182, 273, 365 days. We estimate the surface with outlier-filtered, as discussed in Appendix B.3, out-of-the-money puts/calls: puts with $\kappa \leq 0$ and calls with $\kappa \geq 0$. We estimate the surface separately for puts/calls and separately for short/long-maturity options: $14 \leq \tau \leq 122$ days and $122 < \tau \leq 365$ days, respectively.

Bid/Ask Prices. Panel D repeats the analysis with bid/ask prices, following similar robustness checks in Martin (2017) and Gao and Martin (2021). We only have bid/ask prices in the United States. The first row reports the baseline results with the bid-ask midpoint. The second row repeats the analysis with bid prices, the third ask prices. In sum, this exercise is consistent with the baseline results, although the Coibion-Gorodnichenko regression slope is somewhat smaller with ask prices.

B.4 Power Utility Measures

This section derives the power utility analogue to the LVIX. To do so, we apply results from Martin (2017) and Gao and Martin (2021). We omit time subscripts throughout to minimize clutter.

Lemma A1 (Spanning $\mathbf{R}^\alpha(\ln \mathbf{R})^\beta$). *For any α and β,*

$$\frac{1}{R_f}\mathbb{E}_t^*\left[R^\alpha(\ln R)^\beta\right] = R_f^\alpha(\ln R_f)^\beta + \int_0^{F_t^{(n)}} \omega(\alpha,\beta)\,\text{put}_t^{(n)}(K)dK + \int_{F_t^{(n)}}^{\infty} \omega(\alpha,\beta)\,\text{call}_t^{(n)}(K)dK,$$

where

$$\omega(\alpha,\beta) = -\frac{\alpha(1-\alpha)m^\beta + \beta(1-2\alpha)m^{\beta-1} + \beta(1-\beta)m^{\beta-2}}{P_t^2}\left(\frac{K}{P_t}\right)^{\alpha-2}$$

and $m = \ln K - \ln P_t$.

As is well-known, under certain regularity conditions, we can compute the price of any function of the index price via a replicating portfolio of bonds, stocks, and options. We simply apply this result to the function $R^{\alpha}\left(\ln R\right)^{\beta}$, which is useful for expectations under power utility below.

Proposition A1 (Expected Equity Premium with Power Utility). *From the standpoint of an unconstrained power utility investor fully invested in the market,*

$$\mathbb{E}_t\left[\ln R\right] - \ln R_f = \frac{\mathbb{E}_t^*\left[R^{\gamma} \ln R\right]}{\mathbb{E}_t^*\left[R^{\gamma}\right]} - \ln R_f, \tag{A13}$$

where

$$\frac{1}{R_f}\,\mathbb{E}_t^*\left[R^{\gamma} \ln R\right] = R_f^{\gamma} \ln R_f + \int_0^{F_t^{(n)}} \omega\left(\gamma, 1\right) \text{put}_t^{(n)}(K) dK + \int_{F_t^{(n)}}^{\infty} \omega\left(\gamma, 1\right) \text{call}_t^{(n)}(K) dK \tag{A14}$$

and

$$\frac{1}{R_f}\,\mathbb{E}_t^*\left[R^{\gamma}\right] = R_f^{\gamma} + \int_0^{F_t^{(n)}} \omega\left(\gamma, 0\right) \text{put}_t^{(n)}(K) dK + \int_{F_t^{(n)}}^{\infty} \omega\left(\gamma, 0\right) \text{call}_t^{(n)}(K) dK \tag{A15}$$

and γ is the investor's risk aversion.

The LVIX is a special case of (A13) with $\gamma = 1$, and so the mechanics under power utility are similar, if only messier, to that under log utility. However, there is one caveat: as risk aversion γ increases, the weights $\omega\left(\gamma, 0\right)$ and $\omega\left(\gamma, 1\right)$ on deep out-of-the-money call options become untenably large. Unfortunately, these options are largely unobservable. As such, we can only realistically measure expectations for a $\gamma \leq 3$ investor in practice.

Armed with the expected equity premium from the standpoint of a power utility investor, we can compute spot rates, forward rates, and forecast errors in the usual way. Table 6 reports results analogous to those under log utility for $0.5 \leq \gamma \leq 3$. Figure 7 plots the conditional volatility of the market return from the standpoint of a power utility investor. The mechanics are again a direct application of Martin (2017) and standard spanning theorems, as with the expected equity premium above.

B.5 Measurement Error

Spot/Forward Rates. To better understand the role of measurement error, Figure A5 examines spot/forward rates in simulations. We first compute option prices from a parametric model. Since we know the true data generating process, we then quantify how varying integration bounds affects the integral relative to the true value. The thought experiment follows a similar exercise in Jiang and Tian (2007) for the VIX.

We first truncate the integral (5) without extrapolation, as in Table A3. In Panel A, we consider a Black-Scholes model. We find a large truncation bias in bad times. In bad times, volatility is high, deep out-of-the-money options are expensive, and so the bias is large. In contrast, in good times, volatility is low, deep out-of-the-money options are cheap, and so the bias is small. The bias is especially large for 12-month spot rates and forward rates because longer-maturity option prices have more time value. In Panel B, we consider a stochastic volatility model with jumps (SVJ). We again find an uncomfortably large truncation bias. Relative to Black-Scholes, the bias is larger when volatility is low, but smaller when volatility is high because volatility mean-reverts in SVJ.

We next truncate the integral after extrapolating beyond the range of observable strikes, as in the baseline analysis. We continue with a SVJ model in Panel C. Relative to Panel B, we find that

extrapolation reduces truncation bias across the board.

We emphasize, however, that this exercise only motivates extrapolation in our baseline integration scheme and our use of shorter-maturity forward rate as instruments/predictors, as in Table 2 and Table 4. We make no claim that measurement error is unconditionally small. By construction, these simulations address only truncation bias. There is no scope for either discretization or interpolation bias, as we simulate option prices on a counterfactually dense grid. We think these biases may be non-trivial at times and especially so when options are less dense.

Coibion-Gorodnichenko Regressions. In Coibion-Gorodnichenko regressions, we use forecast revisions to predict forecast errors, as in Table 5. Since forecast revisions/errors involve the same forward rate, measurement error may produce spurious evidence of predictability. To better understand the role of measurement error, we again turn to simulations. We quantify how much correlated measurement error would be necessary to produce the Coibion-Gorodnichenko regression slopes in the data.

We assume we observe forecast revisions and forecast errors with noise: $\widetilde{x} = x\sigma_x + v\sigma_v$ and $\widetilde{y} = y\sigma_y - v\sigma_v$, respectively, with $\sigma_{xy} = \sigma_{xv} = \sigma_{yv} = 0$ and $x, y, v \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1)$. We vary σ_v^2 exogenously. In each simulation draw, we set the variance of the truth (σ_x^2 and σ_y^2) such that the observed variance ($\sigma_{\widetilde{x}}^2$ and $\sigma_{\widetilde{y}}^2$) equals that in the data. As σ_v^2 varies, these weights ensure all variation in slopes comes from variation in noise and none from variation in observed variances. Any evidence of predictability – any non-zero slope – is spurious because $\sigma_{xy} = 0$.

Figure A6 reports the results from this simulations. To produce the Coibion-Gorodnichenko regression slopes in the data, we require σ_v be about 40 basis points or more than one-quarter the volatility of forecast errors in the data. This, at least to us, seems implausibly large. We conclude that correlated measurement error cannot fully explain forecast-error predictability in Coibion-Gorodnichenko regressions, although we cannot fully rule out some bias due to measurement error.

C Additional Empirical Results and Robustness Checks

Full Sample. Table A4 considers how our results extend to the full sample with all 20 available exchanges, as opposed to the 10 developed-market exchanges used in our main panel. The table presents results from a concise set of key regressions from Tables 1–5 in the main text. The main results hold here. The Mincer-Zarnowitz regression coefficients are similar to (in fact slightly below) the results in the main sample; the average forecast error is nearly identical to that in the main sample; and the error-predictability and Coibion-Gorodnichenko results are slightly stronger in the extended sample than in the main sample.

Additional Robustness Checks. Table A5 examines additional robustness checks. Panel A reports the baseline results. Panel B winsorizes spot rates, forward rates, forecast errors, and forecast revisions at the 2.5% level by exchange. Panel C is the trimming analogue to Panel B. Panel D repeats the analysis in balanced panels. Panel E repeats the analysis in subsamples. This exercise is generally consistent with the baseline results, although the Coibion-Gorodnichenko regression slopes are sensitive to winsorization/trimming and the forecast errors are somewhat less predictable in the later subsample.

Alternative Horizons. Table A6 examines alternative horizons for Mincer-Zarnowitz, average error, and error-predictability regressions. The baseline analysis in Section 3 considers the 6-month

spot rate in 6 months ($n = m = 6$ in Panel E). This exercise illustrates the effect of the horizon on the regression estimates. Holding $n + m$ fixed, forecast errors are relatively small on average and less predictable with small n; forecast errors are relatively large on average and more predictable with large n.

Table A7 examines monthly forecast revisions for Coibion-Gorodnichenko regressions. The baseline analysis in Table 5 considers quarterly forecast revisions. The Coibion-Gorodnichenko regression slopes are generally similar between horizons, although the slope in the United States is substantially smaller with monthly revisions.

Long-Horizon Predicted Forecast Errors. For the forecast-error quantification in Section 3.5, we re-estimate spot rates, forward rates, and forecast errors at longer horizons (up to $m + n = 8$ years) for the Euro Stoxx 50. The sample runs from September 2005 through September 2014 (beyond which we cannot yet observe realized forecast errors). The combinations of m and n (in months) can be seen in Table A8. For each such combination, we predict forecast errors as in (7) using a regression of realized forecast errors on shorter-horizon forward rates; we use the $n - 12 \times 12$ forward rate (with horizons again now in months) for $n \geq 24$, and for $n = 12$ we use the 6×6 rate. After obtaining these predicted forecast errors, we calculate a decay parameter for each date's forecast errors, $\phi_t^{(n,m)}$, as the ratio of estimated $\mathbb{E}_t[\varepsilon_{t+n}^{(m)}]$ to $\mathbb{E}_t[\varepsilon_{t+12}^{(12)}]$ for each available $m, n > 12$. This decay specification builds on the one used by De la O and Myers (2021, eq. (13)). The entries of Table A8 report the median decay parameter over all t for each combination of m and n. In all cases the estimates are very close to 1. Assuming that predictable forecast errors are permanent at all horizons might be thought of as providing an estimate of their maximal possible effect. That said, when we estimate the decay parameter in the U.S. (at shorter horizons, unreported), we in fact generally obtain estimates greater than 1, suggesting that setting $\phi^{(n,m)} = 1$ may, if anything, be slightly conservative in the U.S. sample.

D Additional Model Discussion: A Trilemma for Expectation Errors

This appendix continues the discussion in Section 5.3 on how different moments of the data are tied together by the cyclicality of forecast errors. We begin with the Campbell-Shiller price-dividend decomposition in (8). Assume that the expectations $\mathbb{E}_t[\cdot]$ in that decomposition refer to agents' subjective beliefs, and $p_t - d_t$ is the observed log price-dividend ratio. Now consider an alternative economy in which all agents have rational expectations. For arbitrary equilibrium variable x_t in the observed data, denote the corresponding variable in the alternative RE economy by x_t^{RE}. Define the wedge between these two variables to be $\widetilde{x}_t = x_t - x_t^{RE}$. For example, $\widetilde{p_t - d_t}$ is the wedge between the observed price-dividend ratio and the one that would be observed in the alternative economy with RE. Up to a constant, it satisfies

$$\widetilde{p_t - d_t} = \widetilde{CF}_t - \widetilde{\mathcal{F}}_t - \widetilde{RF}_t. \tag{A16}$$

Assume for simplicity that $\widetilde{RF}_t = 0$. The following variance decomposition for the price-dividend wedge therefore holds:

$$\operatorname{var}\left(\widetilde{p_t - d_t}\right) = \operatorname{var}\left(\widetilde{CF}_t\right) + \operatorname{var}\left(\widetilde{\mathcal{F}}_t\right) - 2\operatorname{cov}\left(\widetilde{CF}_t, \widetilde{\mathcal{F}}_t\right). \tag{A17}$$

Alternatively, one can also use the following decomposition given (A16):

$$\text{var}\left(\widetilde{p_t - d_t}\right) = \text{cov}\left(\widetilde{p_t - d_t}, \widetilde{CF}_t\right) - \text{cov}\left(\widetilde{p_t - d_t}, \widetilde{\mathcal{F}}_t\right). \tag{A18}$$

The wedges $\widetilde{CF}_t$ and $\widetilde{\mathcal{F}}_t$ can be understood as expectation errors along the lines considered in Section 5: if subjective expectations are too high relative to RE, then the wedge will be positive (and forecast errors, defined as realized − forecast, are likely to be negative). According to either of the decompositions in (A17)–(A18), therefore, one must choose from at most two of the following three features of any model of expectation errors:

1. Volatile expectation errors for returns (and/or fundamentals)
2. Volatile price-dividend ratio relative to a rational benchmark
3. Countercyclical return expectation errors (positive return expectation errors in bad times)

For example, if excessively positive cash-flow and return forecast revisions occur in good times (after positive news), then $\text{cov}\left(\widetilde{CF}_t, \widetilde{\mathcal{F}}_t\right) > 0$ in (A17). Alternatively, in the version expressed in (A18), positive comovement between price-dividend and forward-rate wedges similarly detracts from a model's ability to generate volatile $\widetilde{p_t - d_t}$. This form of overreaction to *realized outcomes* (cash flows and/or returns) may be intuitively appealing, but it limits a model's ability to speak to variation in the price-dividend ratio through expectation errors alone.[1]

Our empirical results, and our model of expectation errors, instead suggest overreaction of forward rates to *spot rates*, rather than realized returns. Unlike realized returns, we find that spot and forward rates *increase* in bad times. The negative covariance between fundamental news and return expectation errors in principle allows for a volatile price-dividend ratio.

[1]For example, Nagel and Xu (2022) obtain a price-dividend ratio volatility about 50% lower than that observed in the data (see their Table 5). Similarly, De la O and Myers (2021) report that in the model of Barberis et al. (2015), "movements in dividend change expectations are almost completely negated by movements in price change expectations. This leads to low variation in the price-dividend difference" (p. 1370); Campbell (2017) provides a related discussion of the Barberis et al. (2015) results.

Appendix Tables and Figures

Table A1
Option Sample

This table reports the region, the abbreviation, the underlying index, the sample period, and the sample length in months for each exchange. The last column indicates whether the exchange is in the main sample. See Appendix B.1 for more details.

Region	Abbrv	Index	Start	End	Length	Main
Panel A. North America						
United States	USA	S&P 500	199001	202112	384	Y
Panel B. Europe						
Belgium	BEL	BEL 20	200201	202109	228	
Switzerland	CHE	SMI	200201	202109	237	Y
Germany	DEU	DAX	200201	202109	237	Y
Spain	ESP	IBEX 35	200610	202109	180	Y
Finland	FIN	OMXH25	200201	202109	237	
France	FRA	CAC 40	200304	202109	222	Y
United Kingdom	GBR	FTSE 100	200201	202109	237	Y
Italy	ITA	FTSE MIB	200610	202109	180	Y
Netherlands	NLD	AEX	200201	202109	219	
Sweden	SWE	OMXS30	200705	202109	173	
Panel C. Pan-Europe						
Euro Stoxx 50	SX5E	SX5E	200201	202109	237	Y
Stoxx Europe 50	SX5P	SX5P	200201	202109	237	
Stoxx Europe 600	SXXP	SXXP	200509	202109	193	
Panel D. Asia						
Australia	AUS	ASX 200	200401	202104	208	Y
China	CHN	HSCEI	200601	202104	184	
Hong Kong	HKG	Hang Seng	200601	202104	184	Y
Japan	JPN	Nikkei 225	200405	202104	204	
Korea	KOR	KOSPI 200	200407	202104	202	
Taiwan	TWN	TAIEX	200510	202104	187	

Table A2
Summary Statistics

This table reports summary statistics for the 6-month spot rate $\mu_{t+6}^{(6)}$ (left panel) and the 6×6-month forward rate $f_t^{(6,6)}$ (right panel). The units are annualized percentage points. The sample is the longest available for each exchange in the full sample.

	6-Month Spot Rate $\mu_{t+6}^{(6)}$				6×6-Month Forward Rate $f_t^{(6,6)}$			
	Mean	St. Dev.	p10	p90	Mean	St. Dev.	p10	p90
Panel A. North America								
USA	2.17	1.40	0.97	3.81	2.15	0.99	1.11	3.45
Panel B. Europe								
BEL	2.42	2.07	0.95	3.83	2.56	2.22	1.07	4.24
CHE	1.96	1.34	0.98	3.25	1.88	0.80	1.19	2.71
DEU	2.85	1.86	1.38	4.75	2.63	1.07	1.57	3.97
ESP	3.37	1.83	1.50	5.92	2.89	1.17	1.60	4.45
FIN	2.79	2.21	0.91	6.00	2.38	1.85	0.82	4.42
FRA	2.57	1.56	1.23	4.36	2.44	1.04	1.43	3.87
GBR	2.21	1.51	0.97	3.95	2.12	1.00	1.19	3.42
ITA	3.60	1.67	2.03	5.86	3.17	1.35	1.85	5.21
NLD	2.85	2.18	1.10	5.11	2.43	1.26	1.24	4.07
SWE	2.79	1.80	1.36	4.58	2.44	1.41	1.41	3.77
Panel C. Pan-Europe								
SX5E	2.89	1.83	1.31	5.15	2.65	1.08	1.52	4.12
SX5P	2.23	1.62	1.04	3.86	2.11	1.48	1.12	3.40
SXXP	2.14	1.63	0.73	4.17	2.05	1.53	0.74	4.03
Panel D. Asia								
AUS	1.97	1.39	0.86	3.52	1.84	1.20	0.87	3.18
CHN	4.24	4.01	2.06	7.42	3.90	3.29	2.20	6.37
HKG	2.98	2.56	1.33	5.63	2.85	1.93	1.63	4.53
JPN	2.93	2.20	1.49	4.15	2.60	1.57	1.55	3.48
KOR	2.23	2.16	0.85	3.91	2.14	1.85	0.88	3.77
TWN	2.39	1.90	0.98	4.50	2.36	1.45	1.17	4.22

Table A3
Alternative Measures

See Appendix B.3 for more details. Panels A to C report estimates from panel regressions in the main sample. The sample is the longest available for each exchange. Panel D reports estimates from time-series regressions in the United States. The sample is from January 1990 to June 2021. Mincer-Zarnowitz regressions test $H_0 : \beta_1 = 1$, as in Table 1. The average forecast error tests $H_0 : \bar{\varepsilon}_t = 0$, as in Table 3. Error-predictability regressions test $H_0 : \beta_1 = 0$, as in Table 4. Coibion-Gorodnichenko regressions test $H_0 : \beta_1 = 0$, as in Table 5. The risk premium is the 6-month spot rate in 6 months for Mincer-Zarnowitz, average error, and error-predictability regressions. The risk premium is the 3-month spot rate in 6 months for Coibion-Gorodnichenko regressions. The units are annualized percentage points. All regressions include exchange fixed effects and report a within R^2. Panel regressions, in the main sample, report standard errors clustered by exchange and date. Time-series regressions, in the United States, report Newey-West standard errors with lags selected following Lazarus et al. (2018) and fixed-b p-values.

	$\mu^{(6)}_{t+6}$												$\mu^{(3)}_{t+6}$				
	Mincer-Zarnowitz				Average Error			Error Predictability					Coibion-Gorodnichenko				
	β_1	$se(\beta_1)$	p-val	R^2	$\bar{\varepsilon}_t$	$se(\bar{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	N	β_1	$se(\beta_1)$	p-val	R^2	N
Panel A. Alternative Integration Bounds																	
Truncation: 35 Moneyness	0.78	0.091	**	0.18	0.34	0.094	***	-0.019	0.039		0.00	2139	-0.16	0.16		0.00	2104
Truncation: 45 Moneyness	0.68	0.074	***	0.17	0.26	0.097	**	-0.082	0.041	*	0.01	2137	-0.22	0.15		0.01	2103
Truncation: 55 Moneyness	0.63	0.065	***	0.16	0.22	0.099	*	-0.12	0.042	**	0.02	2140	-0.24	0.14		0.01	2106
Truncation: 65 Moneyness	0.59	0.059	***	0.15	0.20	0.10	*	-0.15	0.044	***	0.03	2138	-0.24	0.13	*	0.01	2104
Truncation: Observable Moneyness	0.56	0.053	***	0.15	0.19	0.10	*	-0.17	0.043	***	0.04	2140	-0.22	0.10	*	0.01	2106
Extrapolation: Static Moneyness	0.56	0.056	***	0.15	0.14	0.10		-0.15	0.041	***	0.04	2244	-0.16	0.12		0.01	2204
Extrapolation: Dynamic Moneyness	0.56	0.055	***	0.15	0.16	0.11		-0.17	0.046	***	0.03	2241	-0.25	0.12	*	0.01	2199
Extrapolation: Baseline	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227	-0.24	0.12	*	0.01	2186
Panel B. Alternative Liquidity Filters																	
Outliers	0.60	0.055	***	0.17	0.18	0.098		-0.20	0.050	***	0.05	2241	-0.26	0.16		0.01	2208
Open Interest: After 01/1996	0.52	0.057	***	0.14	0.16	0.11		-0.18	0.045	***	0.04	2033	-0.26	0.13	*	0.02	1966
Outliers and Open Interest: After 01/1996	0.56	0.051	***	0.16	0.19	0.097	*	-0.21	0.050	***	0.06	2040	-0.27	0.17		0.01	1975
Panel C. Alternative Surfaces																	
Volatility Surface: After 01/1996	0.57	0.051	***	0.17	0.087	0.095		-0.21	0.053	***	0.06	2163	-0.22	0.16		0.01	2131
SVI Surface: USA and SX5E	0.59	0.047	*	0.17	0.051	0.11		-0.19	0.039		0.04	609	-0.077	0.14		0.00	597
Panel D. Alternative Prices																	
Bid-Ask Midpoint: USA	0.67	0.096	***	0.22	0.021	0.15		-0.17	0.066	**	0.03	378	-0.29	0.12	**	0.01	375
Bid Prices: USA	0.64	0.099	***	0.21	0.034	0.14		-0.15	0.077	**	0.03	378	-0.34	0.12	***	0.02	374
Ask Prices: USA	0.66	0.089	***	0.23	0.0051	0.15		-0.18	0.060	***	0.04	378	-0.15	0.12		0.00	375

Table A4
Full Sample Regressions

This table reports estimates from panel regressions in the full sample. Columns (1)–(2) are Mincer-Zarnowitz regressions of future realized spot rates on forward rates, as in Table 1. Columns (3)–(4) are instrumented Mincer-Zarnowitz regressions, as in Table 2. Column (5) is the average forecast error, as in Table 3. Columns (6)–(7) are error-predictability regressions, as in Table 4. Columns (8)–(9) are Coibion-Gorodnichenko regressions, as in Table 5. The risk premium is the 6-month spot rate in 6 months for Mincer-Zarnowitz, average error, and error-predictability regressions. The risk premium is the 3-month spot rate in 6 months for Coibion-Gorodnichenko regressions. The units are annualized percentage points. Standard errors are clustered by exchange and date. The sample is the longest available for each exchange in the full sample.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\mu^{(6)}_{t+6}$							$\mu^{(3)}_{t+6}$	
	Mincer-Zarnowitz		Mincer-Zarnowitz (IV)		Average Error	Error Predictability		Coibion-Gorodnichenko	
	$\mu^{(6)}_{t+6}$	$\mu^{(6)}_{t+6}$	$\mu^{(6)}_{t+6}$	$\mu^{(6)}_{t+6}$	$\varepsilon^{(6)}_{t+6}$	$\varepsilon^{(6)}_{t+6}$	$\varepsilon^{(6)}_{t+6}$	$\varepsilon^{(3)}_{t+6}$	$\varepsilon^{(3)}_{t+6}$
$f^{(6,6)}_t$	0.53***	0.48***	0.63***	0.58***					
	(0.058)	(0.053)	(0.062)	(0.061)					
$f^{(2,1)}_t$						-0.23***	-0.26***		
						(0.049)	(0.049)		
$f^{(6,3)}_t - f^{(9,3)}_{t-3}$								-0.35***	-0.36***
								(0.11)	(0.11)
Intercept	1.33***		1.09***		0.19	0.81***		0.28*	
	(0.16)		(0.17)		(0.12)	(0.16)		(0.14)	
p-value: $\beta_1 = 1$	0.0000	0.0000	0.0000	0.0000	-	-	-	-	-
Observations	4199	4199	4199	4199	4199	4199	4199	4111	4111
Fixed Effects	None	Ex	None	Ex	None	None	Ex	None	Ex
Adjusted R^2	0.17	0.19	0.17	-	-	0.06	0.08	0.05	0.05
Within R^2	-	0.14	-	0.13	-	-	0.08	-	0.05

Table A5
Additional Robustness Checks

See Appendix C for more details. Mincer-Zarnowitz regressions test $H_0 : \beta_1 = 1$, as in Table 1. The average forecast error tests $H_0 : \bar{\varepsilon}_t = 0$, as in Table 3. Error-predictability regressions test $H_0 : \beta_1 = 0$, as in Table 4. Coibion-Gorodnichenko regressions test $H_0 : \beta_1 = 0$, as in Table 5. The risk premium is the 6-month spot rate in 6 months for Mincer-Zarnowitz, average error, and error-predictability regressions. The risk premium is the 3-month spot rate in 6 months for Coibion-Gorodnichenko regressions. The units are annualized percentage points. All regressions include exchange fixed effects and report a within R^2. Standard errors are clustered by exchange and date.

	$\mu^{(6)}_{t+6}$												$\mu^{(3)}_{t+6}$				
	Mincer-Zarnowitz				Average Error			Error Predictability					Coibion-Gorodnichenko				
	β_1	$se(\beta_1)$	p-val	R^2	$\bar{\varepsilon}_t$	$se(\bar{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	N	β_1	$se(\beta_1)$	p-val	R^2	N
Panel A. Baseline																	
Main Sample: 01/1990 to 06/2021	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227	-0.24	0.12	*	0.01	2186
Full Sample: 01/1990 to 06/2021	0.48	0.053	***	0.14	0.19	0.12		-0.26	0.049	***	0.08	4199	-0.36	0.11	***	0.05	4111
Panel B. Winsorization																	
Main Sample: 01/1990 to 06/2021	0.60	0.059	***	0.19	0.15	0.093		-0.15	0.048	**	0.03	2227	-0.16	0.10		0.01	2186
Full Sample: 01/1990 to 06/2021	0.56	0.073	***	0.20	0.16	0.095		-0.21	0.051	***	0.05	4199	-0.15	0.084	*	0.01	4111
Panel C. Trimming																	
Main Sample: 01/1990 to 06/2021	0.57	0.056	***	0.18	0.091	0.078		-0.10	0.049	*	0.01	2029	0.066	0.087		0.00	2008
Full Sample: 01/1990 to 06/2021	0.55	0.061	***	0.20	0.11	0.074		-0.15	0.047	***	0.03	3828	0.029	0.079		0.00	3791
Panel D. Balanced Panel																	
Main Sample: 10/2006 to 06/2019	0.49	0.058	***	0.12	0.22	0.13		-0.20	0.050	***	0.05	1674	-0.29	0.12	**	0.02	1658
Full Sample: 05/2007 to 06/2019	0.43	0.052	***	0.12	0.19	0.15		-0.31	0.041	***	0.10	3214	-0.39	0.11	***	0.06	3194
Panel E. Subsamples																	
Main Sample: 01/1990 to 10/2011	0.49	0.078	***	0.11	0.37	0.19	*	-0.25	0.064	***	0.07	1113	-0.41	0.14	**	0.03	1076
Main Sample: 11/2011 to 06/2021	0.30	0.12	***	0.06	-0.024	0.099		-0.11	0.081		0.01	1114	0.016	0.077		0.00	1110

Table A6
Alternative Horizons

See Appendix C for more details. Mincer-Zarnowitz regressions test $H_0 : \beta_1 = 1$, as in Table 1. The average forecast error tests $H_0 : \bar{\varepsilon}_t = 0$, as in Table 3. Error-predictability regressions test $H_0 : \beta_1 = 0$, as in Table 4. The risk premium is the m-month spot rate in n months. The units are annualized percentage points. All regressions include exchange fixed effects and report a within R^2. Standard errors are clustered by exchange and date. The sample is the longest available for each exchange in the main sample.

	$\mu_{t+n}^{(m)}$											
	Mincer-Zarnowitz				Average Error			Error Predictability				
	β_1	$se(\beta_1)$	p-val	R^2	$\bar{\varepsilon}_t$	$se(\bar{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	N
Panel A. 4-Month Equity Premium												
1-Month Ahead: $n = 1$, $m = 3$	0.90	0.048	*	0.62	0.044	0.073		-0.12	0.040	**	0.03	2269
2-Months Ahead: $n = 2$, $m = 2$	0.77	0.058	***	0.35	0.055	0.11		-0.20	0.053	***	0.04	2268
3-Months Ahead: $n = 3$, $m = 1$	0.68	0.065	***	0.20	0.094	0.14		-0.24	0.055	***	0.04	2235
Panel B. 5-Month Equity Premium												
1-Month Ahead: $n = 1$, $m = 4$	0.94	0.043		0.67	0.067	0.064		-0.075	0.033	*	0.02	2269
2-Months Ahead: $n = 2$, $m = 3$	0.83	0.053	**	0.42	0.12	0.10		-0.13	0.043	**	0.02	2268
3-Months Ahead: $n = 3$, $m = 2$	0.74	0.063	***	0.25	0.16	0.13		-0.15	0.049	**	0.02	2241
4-Months Ahead: $n = 4$, $m = 1$	0.62	0.078	***	0.13	0.27	0.15		-0.19	0.053	***	0.02	2236
Panel C. 6-Month Equity Premium												
1-Month Ahead: $n = 1$, $m = 5$	0.95	0.037		0.70	0.067	0.058		-0.054	0.029	*	0.01	2269
2-Months Ahead: $n = 2$, $m = 4$	0.86	0.047	**	0.46	0.14	0.091		-0.094	0.036	**	0.01	2268
3-Months Ahead: $n = 3$, $m = 3$	0.78	0.058	***	0.30	0.21	0.12		-0.11	0.044	**	0.01	2249
4-Months Ahead: $n = 4$, $m = 2$	0.67	0.072	***	0.17	0.27	0.14	*	-0.15	0.050	**	0.02	2238
5-Months Ahead: $n = 5$, $m = 1$	0.57	0.076	***	0.09	0.32	0.17	*	-0.20	0.057	***	0.02	2228
Panel D. 9-Month Equity Premium												
3-Months Ahead: $n = 3$, $m = 6$	0.81	0.055	***	0.39	0.15	0.087		-0.065	0.036		0.01	2257
4-Months Ahead: $n = 4$, $m = 5$	0.73	0.066	***	0.26	0.21	0.10	*	-0.10	0.044	**	0.01	2242
5-Months Ahead: $n = 5$, $m = 4$	0.65	0.071	***	0.17	0.26	0.12	*	-0.14	0.049	**	0.02	2232
6-Months Ahead: $n = 6$, $m = 3$	0.55	0.071	***	0.10	0.28	0.14	*	-0.17	0.051	***	0.02	2224
Panel E. 12-Month Equity Premium												
3-Months Ahead: $n = 3$, $m = 9$	0.81	0.049	***	0.44	0.094	0.070		-0.059	0.036		0.01	2258
6-Months Ahead: $n = 6$, $m = 6$	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227
9-Months Ahead: $n = 9$, $m = 3$	0.39	0.097	***	0.06	0.25	0.15		-0.24	0.052	***	0.04	2191

Table A7
Alternative Horizons: Coibion-Gorodnichenko Regressions

This table reports Coibion-Gorodnichenko regressions of future realized forecast errors on current 1-month forecast revisions:

$$\varepsilon^{(1)}_{i,t+4} = \beta_0 + \beta_1 \left(f^{(4,1)}_{i,t} - f^{(5,1)}_{i,t-1} \right) + e_{i,t+4}$$

The realized spot rate is the future expectation of the 1-month equity premium, the forward rate is the current expectation of the same risk premium, and the forecast error is the realized spot rate minus the forward rate. The units are annualized percentage points. Panel regressions, in columns (1)–(4), report standard errors clustered by exchange and date. Time-series regressions, in columns (5)–(6), report Newey-West standard errors with lags selected following Lazarus et al. (2018). The sample is the longest available for each exchange in the main sample.

	(1) Main	(2) Main	(3) Main	(4) Main excl U.S.	(5) U.S. Only	(6) SX5E Only
$f^{(4,1)}_t - f^{(5,1)}_{t-1}$	-0.26**	-0.27**	-0.36***	-0.30**	-0.0063	-0.24*
	(0.11)	(0.11)	(0.065)	(0.11)	(0.21)	(0.14)
Intercept	0.28				-0.051	0.42
	(0.16)				(0.16)	(0.32)
Observations	2217	2217	2070	1838	379	232
Fixed Effects	None	Ex	Ex/Date	Ex	None	None
Standard Errors	Cluster	Cluster	Cluster	Cluster	Newey-West	Newey-West
Cluster	Ex/Date	Ex/Date	Ex/Date	Ex/Date	-	-
Adjusted R^2	0.01	0.01	0.83	0.01	-0.00	-0.00
Within R^2	-	0.01	0.03	0.01	-	-

Table A8
Long-Horizon Forecast Error Decay

This table reports estimates of the predicted forecast error decay for the Euro Stoxx 50. The decay $\phi_t^{(n,m)}$ parameter follows the expected decay specification from De la O and Myers (2021):

$$\mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] = \phi_t^{(n,m)}\,\mathbb{E}_t\left[\varepsilon_{t+12}^{(12)}\right]$$

The estimate is the median by horizon:

$$\phi^{(n,m)} = median\left\{\left|\phi_t^{(n,m)}\right|\right\}$$

The predicted forecast error is from a time-series regression of future realized forecast errors on current forward rates: the predictor is the $n-12\times 12$-month forward rate for $n \geq 24$ and the 6×6-month forward rate for $n = 12$. The sample is from September 2005 to September 2014.

	n-months						
m-months	12	24	36	48	60	72	84
12	1.00	1.05	1.13	1.08	1.04	0.99	0.99
24	1.00	1.06	1.04	1.10	1.05	0.97	
36	1.00	1.04	1.06	1.01	0.90		
48	1.00	0.96	0.95	1.07			
60	1.00	1.01	0.97				
72	1.00	1.02					

Table A9
Model Calibration: Objective Parameters

This table reports AR(3) time-series regressions for 3-month spot rates:

$$\mu_t^{(3)} = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \phi_1 \, \mu_{t-1}^{(3)} + \phi_2 \, \mu_{t-2}^{(3)} + \phi_3 \, \mu_{t-3}^{(3)} + e_t$$

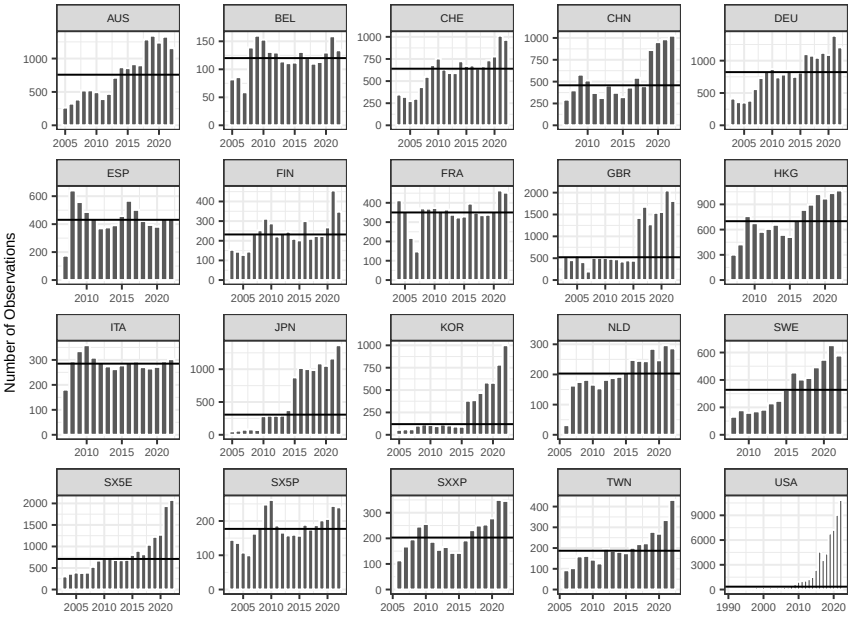
The units are annualized percentage points. Standard errors are from 10,000 Monte Carlo simulations of length T months. The sample is the longest available for each exchange in the main sample.

	AUS	CHE	DEU	ESP	FRA	GBR	HKG	ITA	SX5E	USA
ϕ_1	0.59	0.78	0.79	0.76	0.87	0.90	0.74	0.66	0.77	0.94
	(0.07)	(0.07)	(0.07)	(0.08)	(0.07)	(0.07)	(0.07)	(0.08)	(0.07)	(0.05)
ϕ_2	0.02	-0.01	-0.06	-0.07	-0.14	-0.19	0.16	-0.02	-0.03	-0.26
	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)	(0.08)	(0.09)	(0.09)	(0.08)	(0.07)
ϕ_3	0.22	0.05	0.11	0.09	0.12	0.14	-0.05	0.11	0.11	0.18
	(0.07)	(0.06)	(0.06)	(0.08)	(0.07)	(0.06)	(0.07)	(0.07)	(0.06)	(0.05)
$\bar{\mu}$	6.19	6.13	8.89	10.49	7.69	6.73	9.30	11.63	8.99	6.40
	(1.36)	(1.00)	(1.62)	(1.48)	(1.23)	(1.21)	(2.44)	(1.27)	(1.59)	(0.89)
σ_e	3.23	2.86	3.82	4.41	2.89	2.89	5.04	4.31	3.69	2.56
	(0.16)	(0.13)	(0.18)	(0.23)	(0.14)	(0.13)	(0.27)	(0.23)	(0.17)	(0.09)
T	208	237	237	180	222	237	184	180	237	384

Figure A1
Filtered Option Prices

Panel A plots the number of options after filters. Panel B plots the share of filtered options with positive open interest. Each bar is the annual median from daily data. The black line is the full sample median from daily data. The sample is the longest available for each exchange. Option prices have maturity $30 \leq \tau \leq 365$ days. See Appendix B.1 for more details.

Panel A. Number of Observations



Panel B. Positive Open Interest

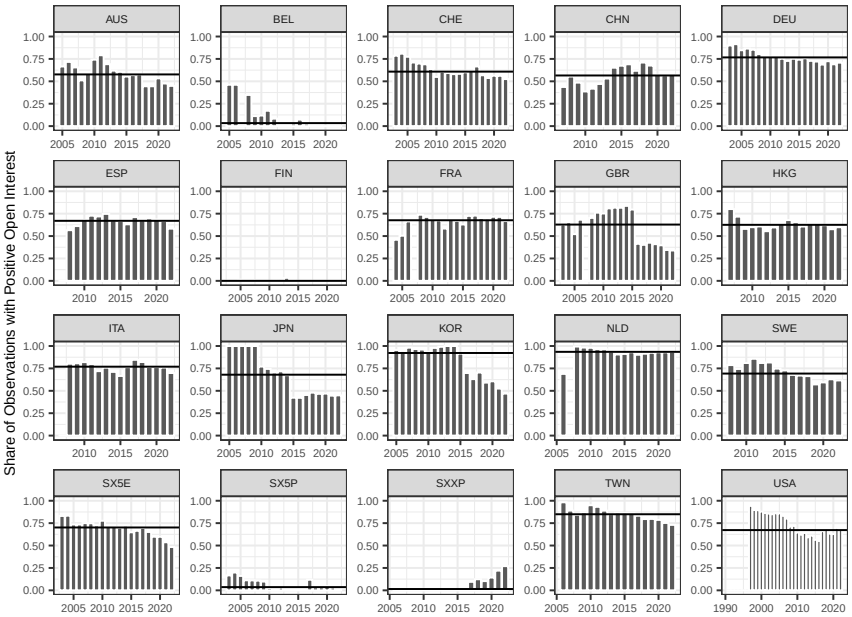
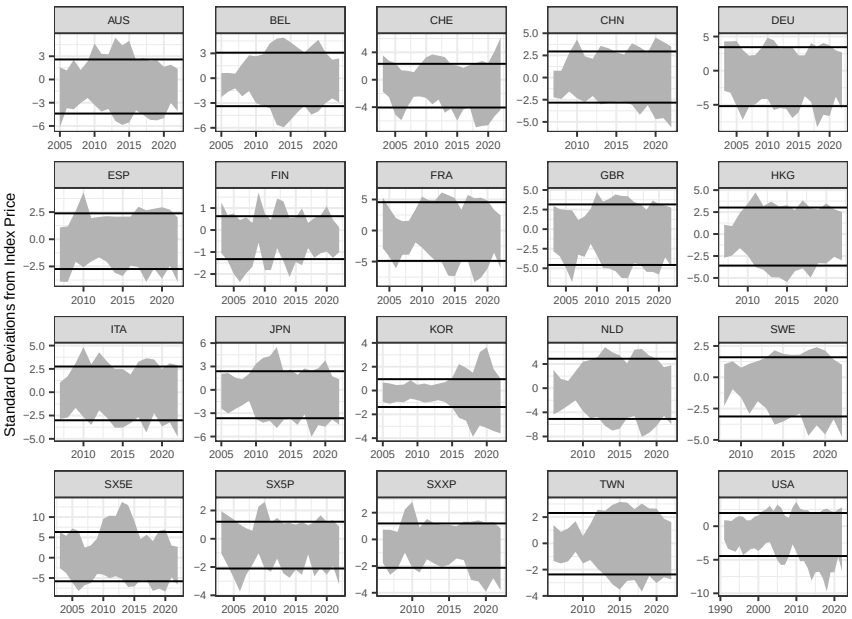


Figure A2
Minimum/Maximum Strike Price by Maturity

This figure plots the minimum/maximum strike price by maturity bin. The minimum/maximum is the annual median from daily data. The black line is the full sample minimum/maximum from daily data. The units are risk-neutral standard deviations from the index price. The sample is the longest available for each exchange. See Appendix B.1 for more details.

Panel A. 92 to 182 Days-to-Maturity



Panel B. 274 to 365 Days-to-Maturity

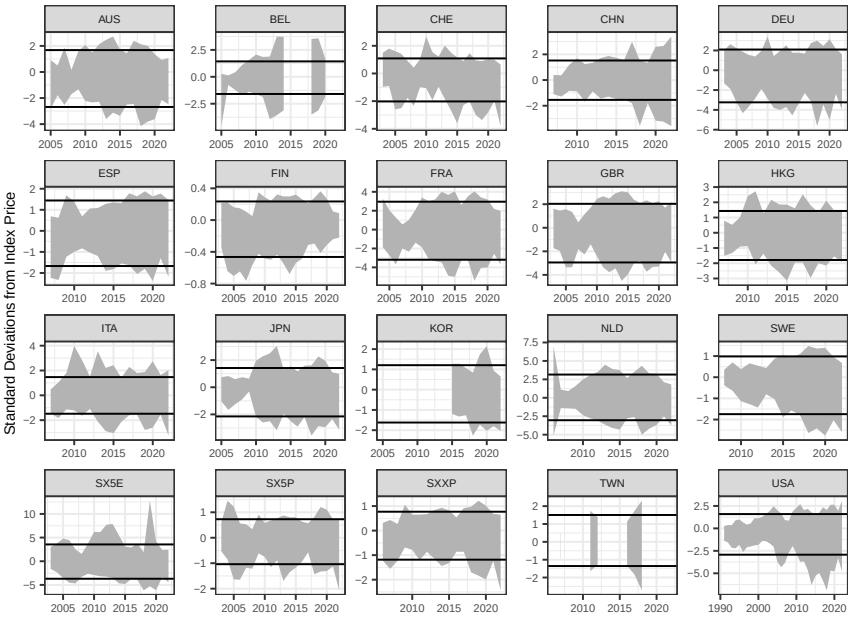
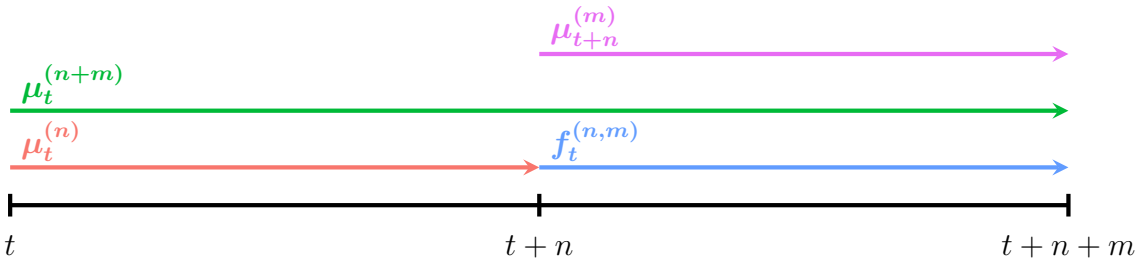


Figure A3
Timeline: Current/Realized Spot Rates and Forward Rates

See Section 2 for more details.



- $\mu_t^{(n)} = \mathcal{L}_t^{(n)}$ is the n-month spot rate at time $= t$
- $\mu_t^{(n+m)} = \mathcal{L}_t^{(n+m)}$ is the $n+m$-month spot rate at time $= t$
- $f_t^{(n,m)} = \mu_t^{(n+m)} - \mu_t^{(n)}$ is the m-month forward rate n months from time $= t$
- $\mu_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)}$ is the m-month spot rate at time $= t+n$

Figure A4
Current Spot and Forward Rates in the Full Sample

This figure plots the current 6-month spot rate $\mu_t^{(6)}$ (blue) and the 6×6-month forward rate $f_t^{(6,6)}$ (red) in the full sample. The sample is the longest available for each exchange.

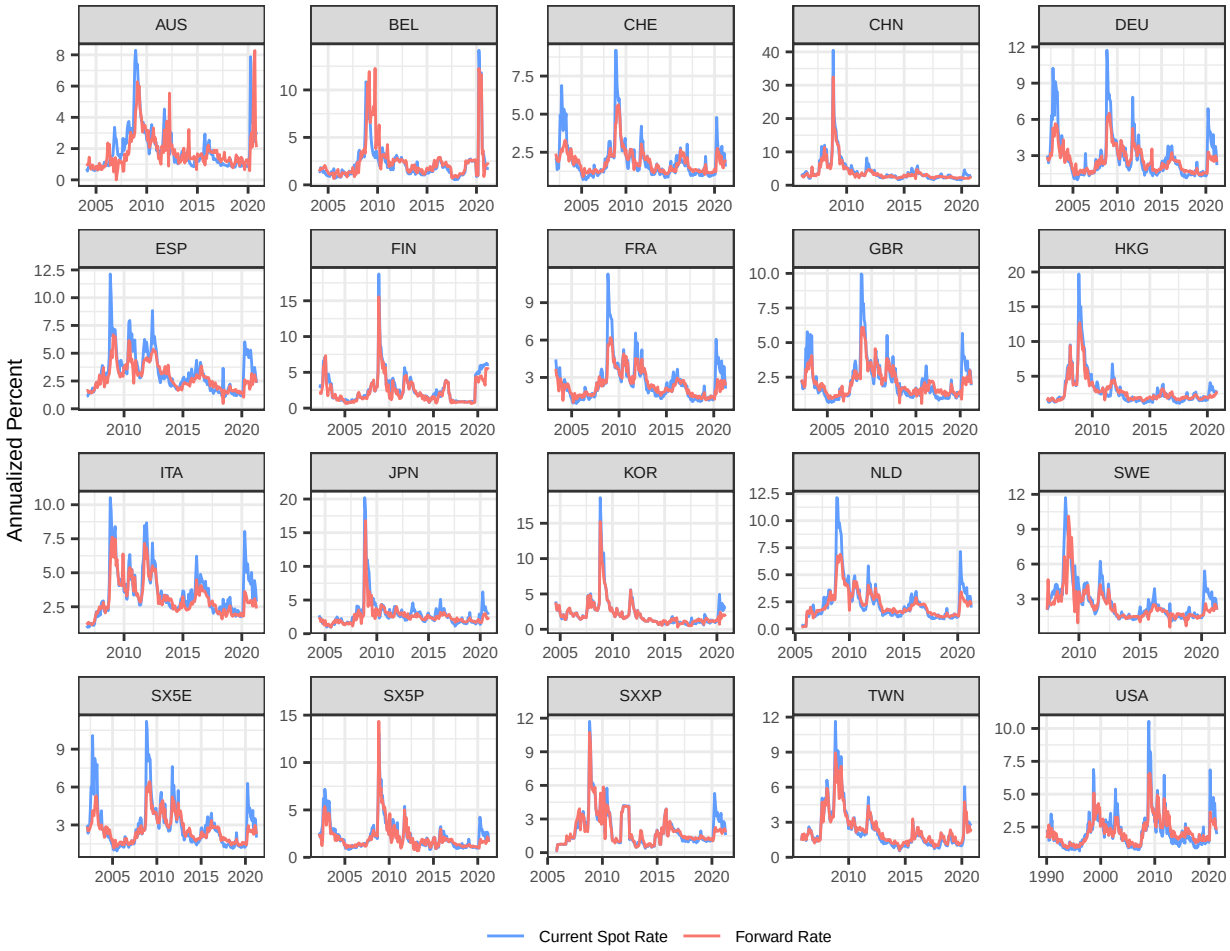


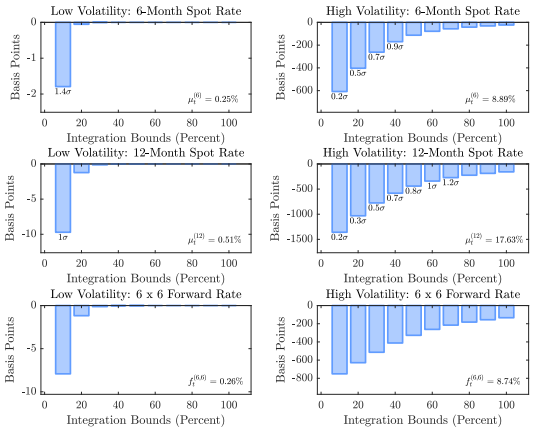
Figure A5
Measurement Error: Spot and Forward Rates

This figure describes truncation bias in the Black-Scholes model (left panel) and the stochastic volatility jump (SVJ) model (right panel). Integration bounds are in moneyness $^{K}/_{P}$ units from the index price. Bar labels are in volatility standard deviations from the index price. Black-Scholes parameters: $P_t = 100$, $r = 0.05$, $q = 0.02$. SVJ parameters under the risk-neutral measure are from Bakshi, Cao, and Chen (1997):

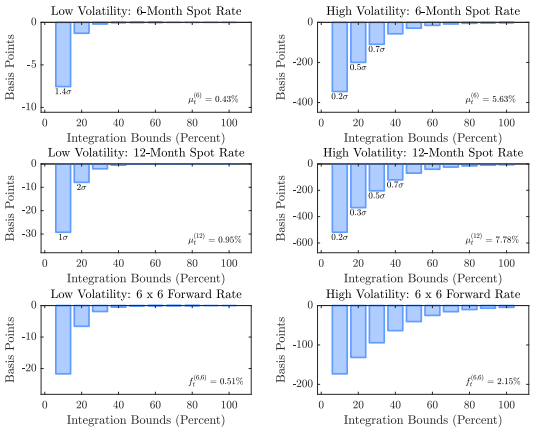
θ_v	κ_v	σ_v	ρ	μ_J	σ_J	λ
0.040	2.030	0.380	-0.570	-0.050	0.070	0.590

Under Black-Scholes (SVJ), low volatility is $IV = 10\%$ ($\sqrt{v_t} = 10\%$) and high volatility is $IV = 60\%$ ($\sqrt{v_t} = 60\%$). The units are non-annualized basis points. See Appendix B.5 for more details.

Panel A. Black-Scholes: Truncation Bias without Extrapolation



Panel B. SVJ: Truncation Bias without Extrapolation



Panel C. SVJ: Truncation Bias with Extrapolation

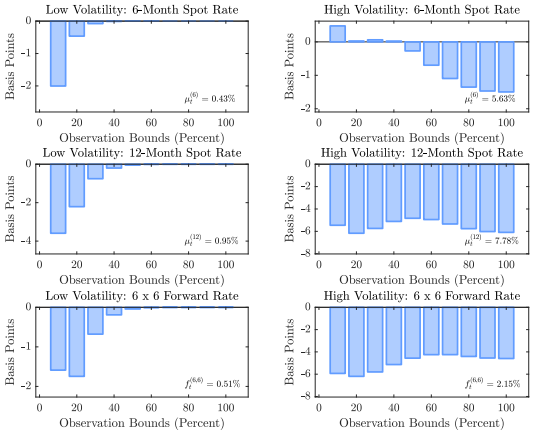


Figure A6
Measurement Error: Coibion-Gorodnichenko Regressions

This figure quantifies how much correlated measurement error is necessary to produce the Coibion-Gorodnichenko regression slopes in the data with monthly forecast revisions (left panel) and quarterly forecast revisions (right panel), as in Table 5 and Table A7, respectively. The solid lines are the slopes in simulations. The shaded regions are 95% confidence bands in 50,000 samples. The blue circles are slopes in the data. The sample is the longest available for each exchange in the main sample. See Appendix B.5 for more details.

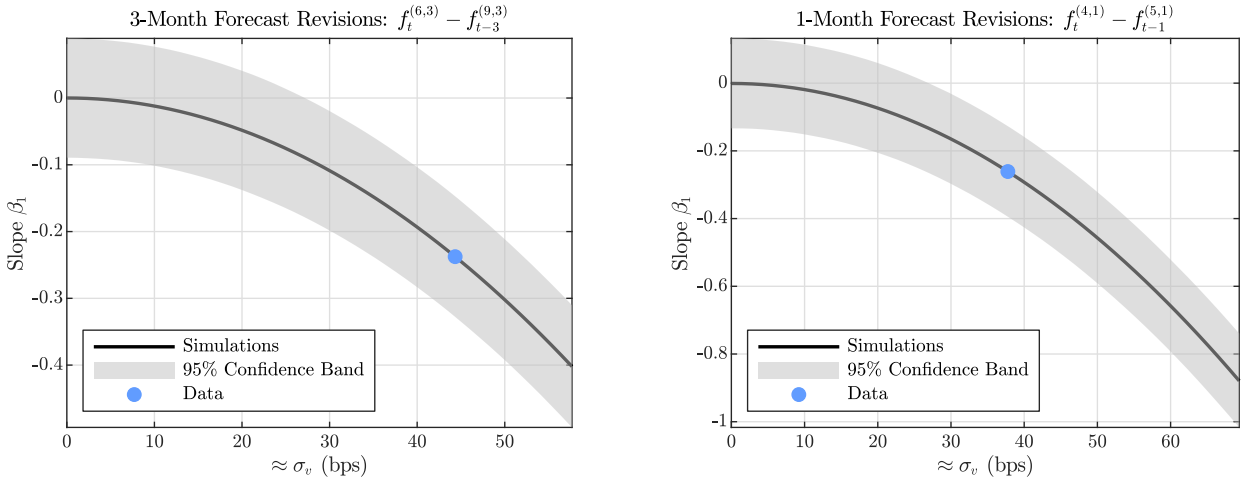
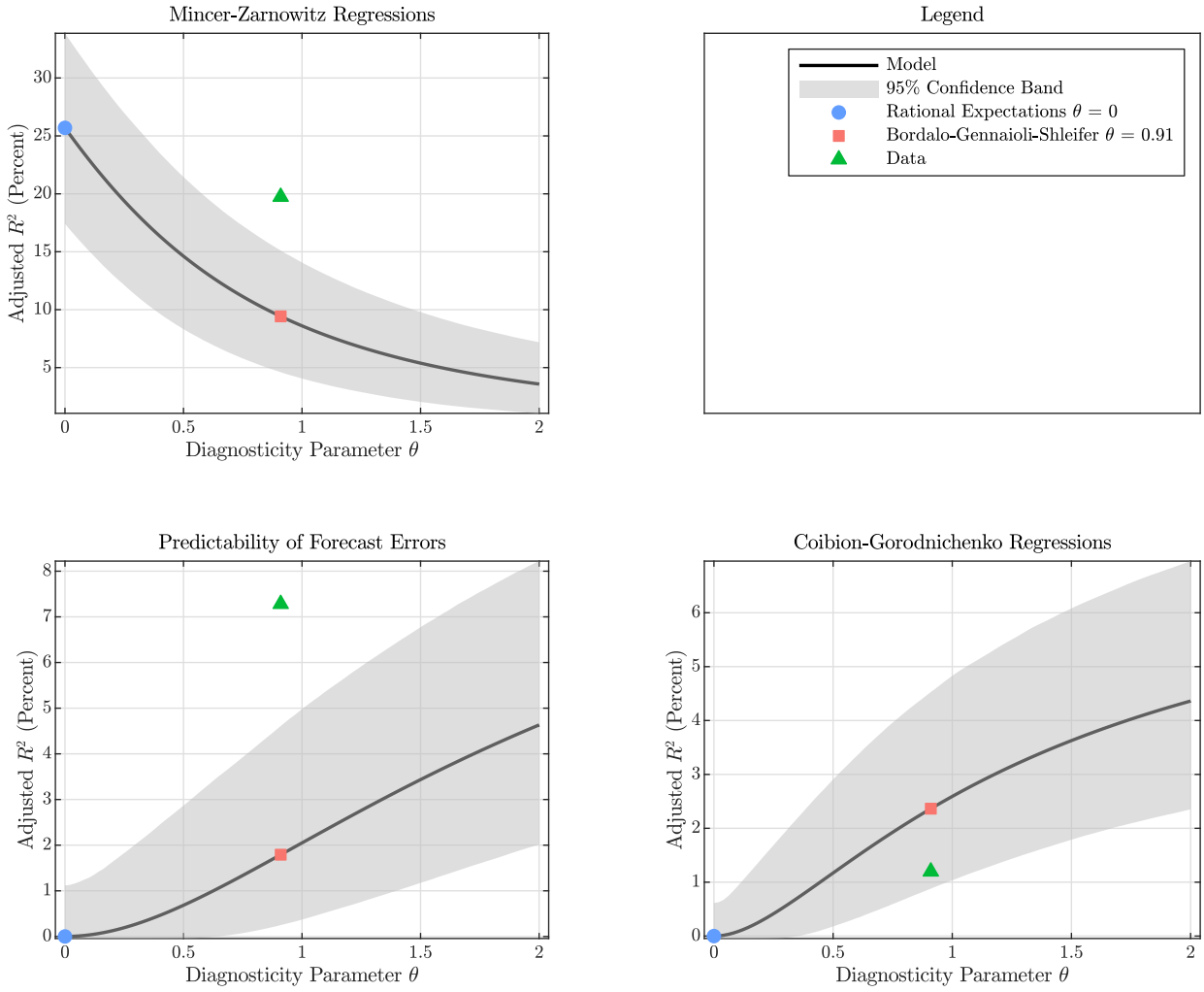


Figure A7
Model Calibration: Regression R^2s

This figure reports regression R^2s in the calibrated model of expectation errors. The model is calibrated from the standpoint of an unconstrained log utility investor fully invested in the market. Table A9 reports the objective parameters. The solid lines are model-implied population R^2s in a single long sample. The shaded regions are model-implied 95% confidence bands in 10,000 short samples. The blue circles are model-implied R^2s under rational expectations with $\theta = 0$. The red squares are model-implied R^2s under diagnostic expectations with $\theta = 0.91$ from Bordalo, Gennaioli, and Shleifer (2018). The green triangles are R^2s in the data. The sample is the longest available for each exchange in the main sample.



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