DEADLOCK ON THE BOARD*

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Abstract

We develop a dynamic model of board decision making. We show that directors may knowingly retain the policy they all think is the worst just because they fear they may disagree about what policy is best in the future—the fear of deadlock begets deadlock. Board diversity can exacerbate deadlock. Hence, shareholders may optimally appoint a biased director to avoid deadlock. On the other hand, the CEO may appoint unbiased directors, or even directors biased against him, to create deadlock and thereby entrench himself. Still, shareholders may optimally give the CEO some power to appoint directors. Our theory thus gives a new explanation for CEO entrenchment. It also gives a new perspective on director tenure, staggered boards, and short-termism.

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1 Introduction

The board of directors is the highest decision-making authority in a corporation. But sometimes boards struggle to make decisions. In surveys, 67% of directors report the inability to decide about some issues in the boardroom. Moreover, 37% say they have encountered a boardroom dispute threatening the very survival of the corporation (IFC (2014), p. 2).[1] Such a "division among the directors" that "may render the board unable to take effective management action"—such deadlock on the board—can even lead directors to "vote wholly in disregard of the interests of the corporation" (Kim (2003), pp. 113, 120).[2] Deadlock on the board can be so costly to US corporations that most states have adopted deadlock statutes, which often give courts the power to dissolve a deadlocked corporation, a power they rarely have otherwise, except in the event of default or fraud. A substantial legal literature studies how corporations can resolve deadlock ex post.[3] In this paper, we ask how deadlock can be avoided ex ante. Can the right mix of directors ensure a board makes efficient decisions? And, if so, how should director elections be structured to help achieve the right board composition? Should director elections be staggered or should all directors be chosen at once? Should director tenure be limited? And should shareholders have all the power to choose directors or should the CEO have some power as well?

To address these questions, we develop a dynamic model of board decision making in which deadlock on the board is the result of the fear of future deadlock: directors

[1] Further, from 2004–2006, 166 directors experienced disputes so severe that they publicly resigned from their boards at US public corporations, accepting potential damage to their careers (Marshall (2013); see also Agrawal and Chen (2017)).

[2] Last summer, deadlock on the board made it hard for Uber to appoint a CEO. According to the *New York Times*, "Uber's C.E.O. selection...illustrates the high-wire act of herding eight board members...toward consensus." Moreover, deadlock on the board led one frontrunner for the job, Meg Whitman, to withdraw her name from consideration, saying "it was becoming clear that the board was still too fractured to make progress on the issues that were important to me" ("Inside Uber's Wild Ride in a Search of a New C.E.O." *New York Times*, August 29, 2017). Whitman's description of Uber's board mirrors the dictionary definition of deadlock: "a situation, typically one involving opposing parties, in which no progress can be made" (*New Oxford American Dictionary*).

[3] See, e.g., Duke (1972), Howe (1967), Kim (2003), Landeo and Spier (2014a, 2014b), Lew (1979–1980), McDonald (1979).

refuse to replace a current policy with a new one because they fear that other directors will refuse to replace the new policy in the future. Shareholders suffer, since a deadlocked board struggles to remove low-quality policies or executives—a deadlocked board leads to an entrenched CEO. We find that boardroom diversity can exacerbate deadlock (its benefits notwithstanding).[4] So can long director tenures, another hotly debated policy issue.[5] Moreover, the anticipation of deadlock can affect board composition via director elections. Shareholders elect directors to avoid deadlock, possibly voting for a director who does not represent their interest but will get along with the rest of the board. In contrast, a CEO may aim to create deadlock, possibly favoring a director who does not get along with the rest of the board, since a deadlocked board will struggle to fire him.

Model preview. In the model, a board made up of multiple directors decides on a corporate policy at each date. The model is based on three key assumptions, reflecting how real-world boards operate. (i) Directors have different preferences over policies. We refer to these different preferences as "biases," as they could reflect misspecified beliefs or anticipated perks. However, they could also reflect reasonable diversity of opinion (as we formalize in Subsection 7.1). For example, in the context of CEO turnover decisions, an activist's representative on the board could be biased toward an outside candidate with a history of asset divestitures, and an executive director could be biased toward an internal candidate with experience at the firm. (ii) The set of feasible policies changes over time. For example, different candidates are available to replace the CEO at each date. (iii) The incumbent stays in place whenever the board does not come to a decision. For example, if the board cannot agree on a replacement, the current CEO keeps the job.

Results preview. First, we ask when the board will replace an existing policy with a new one. We find that deadlock on the board can lead directors to knowingly retain a Pareto-dominated policy. In the context of CEO turnover, this implies that

[4]See Ferreira (2010) for a survey of the literature on boardroom diversity.

[5]See, e.g., "Big Investors Question Corporate Board Tenures" (*Wall Street Journal*, March 23, 2016) and Katz and McIntosh (2014).

a CEO can be so severely entrenched that he is not fired even if all directors prefer a replacement. To see why, consider a firm with a bad incumbent CEO, whom the board is considering replacing with an alternative. Suppose all directors agree that the alternative is better than the incumbent, but some directors are especially biased toward him. For example, activist representatives could be biased toward an alternative with a history of divestment, as touched on above. Then, if the alternative becomes the new CEO, the biased directors will try to keep him in place, voting down alternatives in the future, no matter how much other directors prefer them—the new CEO will become entrenched. To prevent this, other (sufficiently patient) directors block the alternative today, keeping the bad incumbent CEO in place to retain the option to get their way in the future—the incumbent CEO becomes entrenched. The fear of entrenchment begets entrenchment.

This mechanism resonates with practice. For example, when Uber's recent search for a new CEO was hindered by disagreement among its directors, one director was pushing for a weak CEO who would be easy to replace in the future. According to *Bloomberg*:

> The company hopes to lock in a CEO by early September. The big question is whether the board can get on the same page. Getting a majority of the eight-person group to support a single candidate is looking to be difficult.... Some...have argued...that Kalanick [a current director and former CEO of Uber] would *prefer a weak CEO just to increase his chance of making a comeback* ("Behind Uber's Messy CEO Search Is a Divided Boardroom," *Bloomberg Technology*, July 28, 2017, emphasis added).[6]

Second, we ask how director tenure affects deadlock. In the current debate (e.g.,

[6]See also "Investor Benchmark Capital Sues Uber Ex-CEO Travis Kalanick" (*Wall Street Journal*, August 10, 2017), according to which "some investors have alleged that Mr. Kalanick...[was] impeding the search, including by rejecting qualified candidates." Cf. footnote 2.

Katz and McIntosh (2014)),[7] arguments against long director tenure focus on concerns about independence and the lack of fresh ideas. Our analysis suggests a distinct yet complementary argument for shorter tenures: in anticipation of a long tenure, directors behave strategically, blocking good candidates, creating deadlock. This provides a counterpoint to the broadly negative view of corporate short-termism.

Third, we ask how board composition affects deadlock. We find that board diversity has a downside: it can exacerbate deadlock. For example, the deadlock caused by an activist's bias toward divestiture is not resolved by adding some executive directors biased toward investment. These directors will block divestiture-oriented policies, even if they agree that they are optimal today, just to preserve a strong bargaining position for the future. More generally, heterogeneous director biases do not cancel out—they do not yield a board that implements policies in shareholders' interests. Rather, they can yield a board that does not implement any policies at all. This is in line with the empirical findings in Goodstein, Gautam, and Boeker (1994) and Knyazeva, Knyazeva, and Raheja (2013) that diversity of directors' skill and experience is negatively associated with strategic change and firm value, respectively. Our results thus offer a counterpoint to the blanket view that a "board should reflect a diversity of thought, backgrounds, skills, experiences and expertise" (Business Roundtable (2016), p. 11). However, that is not to say that a diverse board is all bad in our model. The short-term deadlock created by opposing biases can also benefit shareholders—by blocking policies that some directors are biased toward, a diverse board can prevent permanent tyranny of a biased board.

Continuing the analysis of board composition, we ask how adding unbiased directors affects deadlock. We find that even if unbiased directors act purely in shareholders' interest, adding them to the board can make shareholders strictly worse off.

[7]While many countries, such as the UK, Hong Kong, Singapore, and several EU countries, have adopted some form of term limits for independent directors, the US and Canada do not yet have any specific regulatory guidelines on director tenure. However, many institutional investors, such as BlackRock and State Street, deem director tenures in the US as too long and are voting against reappointments, leading commentators to suggest that director tenure is "the next boardroom battle" (Libit and Freier (2016), p. 5; see also Francis and Lublin (2016)).

To see why, observe that if all directors are biased the same way, they are never deadlocked (although sometimes they act against the interest of shareholders). If some directors are replaced with unbiased directors, the biased directors will respond strategically. They have extra incentive to block shareholder-friendly policies to improve their future bargaining positions. That said, unbiased directors can also benefit shareholders. Like a diverse board, they block policies that other directors are biased toward to prevent them from becoming entrenched. In so doing, unbiased directors can appear passive or even biased in the short-term: they may block policies that enhance short-term value so that they can implement policies that maximize long-term value in the future.

This mechanism was recently manifested at railroad company CSX. There, activist investor Paul Hilal demanded that CSX replace the incumbent CEO by veteran railroad executive Hunter Harrison and, in addition, give Hilal and Harrison six seats on the board. Although Harrison was widely considered to be the perfect candidate to lead CSX, directors were reluctant to agree to the activist's demands: they probably worried that, given support from the new directors, the new CEO would be hard to replace in the future. Hence, they seemed biased, blocking an alternative that was good in the short term, to prevent entrenchment, which could be bad for the firm in the long term.[8]

Fourth, we ask how deadlock affects director appointments. As an immediate result of our board-composition analysis, we find that shareholders may choose to appoint a biased director, since adding an unbiased director to a board with biased incumbent directors may create deadlock. This points to a downside of staggered boards. If only a subset of directors is replaced at a time, today's newly appointed biased directors become tomorrow's incumbent directors. Hence, if shareholders still want to avoid deadlock tomorrow, they will appoint biased directors again, and so on ad infinitum. The board may remain biased forever, even after shareholders have

[8]See, e.g., "The $10 Billion Battle for CSX Stock Will Be Decided Shortly" (*Fortune*, February 15, 2017). Eventually, after gathering the opinion of the company's investors, the board agreed to the activist's demands.

replaced all the directors.

In practice, shareholders do not have full control over director appointments. The CEO often exerts influence over the appointment of new board members (e.g., Hermalin and Weisbach (1998), Shivdasani and Yermack (1999)). We find that even if the CEO's only goal is to retain his position, he will not always appoint directors who are biased towards him. He may prefer directors who are unbiased, or even biased against him. The reason is that they may exacerbate deadlock on the board. Since deadlock makes it hard to fire the CEO, such strategic director appointments can help the CEO entrench himself. Colloquially, deadlock on the board can be better for the CEO than buddies on the board.

Fifth, we ask whether shareholders should give the CEO power over director appointments. We find that by ceding power to the CEO, shareholders can commit not to block his preferred policies in the future, and hence prevent deadlock today. But they should not give the CEO full power over board appointments, so his bias does not take over the board. Typically, they should give the CEO an interior amount of a power, sometimes letting him choose directors and sometimes choosing them themselves.

Related literature. A relatively small number of theory papers studies strategic decision making by multiple directors on a corporate board.[9] We contribute to this literature by including dynamic interactions, which none of these papers study.[10] Indeed, none of our results would obtain with a one-shot decision since deadlock would not arise.

We also add to the broader theory literature on boards.[11] Our finding that board diversity can exacerbate deadlock complements existing work on the downsides of director independence, since independent directors are likely to have different

[9] See Baranchuk and Dybvig (2009), Chemmanur and Fedaseyeu (2017), Harris and Raviv (2008), Levit and Malenko (2016), Malenko (2014), and Warther (1998).

[10] One paper that features directors' dynamic interactions, but not their strategic decision making, is Garlappi, Giammarino, and Lazrak (2017)—in their model, the board maximizes a weighted average of directors' utilities. Cf. footnote 14.

[11] See Adams, Hermalin, and Weisbach (2010) for a survey.

views than insiders on the board.[12] And our finding that a CEO may prefer to appoint unbiased directors, even when they may fire him in the future, contrasts with Hermalin and Weisbach (1998), another paper in which a CEO appoints directors with the power to fire him.

At an abstract level, our model of board decisions falls within the class of dynamic collective choice models with endogenous status quo explored in the political economy literature, notably in Dziuda and Loeper (2016).[13] We embed this literature's notion of deadlock in a corporate finance framework to apply it to corporate boards.[14] This allows us to study board/committee composition, director appointments/elections, and the role of the CEO. This leads to our main results, none of which have parallels in that literature.

Our explanation of entrenchment, which is based only on directors' strategic behavior, contrasts with those in the finance literature, which are based largely on a CEO's actively entrenching himself (e.g., "invest[ing] in businesses related to their own background and experience")[15] or directors' direct utility costs of firing a CEO (e.g., because he is a friend).[16]

Layout. In Section 2, we present the model. In Section 3, we describe the baseline mechanism of deadlock on the board and entrenched policies. In Section 4, we analyze board composition. In Section 5 and Section 6, we study director appointments and who should appoint directors. In Section 7, we discuss robustness and analyze extensions. In Section 8, we discuss our model's empirical implications

[12]See Adams and Ferreira (2007), Kumar and Sivaramakrishnan (2008), Laux (2008), and Malenko (2014).

[13]See also Austen-Smith, Dziuda, Harstad, and Leoper (2016), Duggan and Kalandrakis (2012), Dziuda and Leoper (2017), and Zápal (2012).

[14]Deadlock in our Proposition 2 is a feature of the equilibrium in Dziuda and Loeper's (2016) Corollary 2. Garlappi, Giammarino, and Lazrak (2017) also find a version of this result: a board passes up an investment all directors believe is good, knowing they will disagree about how to manage it later. This results in underinvestment, but not full entrenchment because directors in their model do not act strategically (see Appendix Subsection A.2.2).

[15]Shleifer and Vishny (1989), p. 125. See also, e.g., Zwiebel (1996).

[16]See, e.g., Chemmanur and Fedaseyeu (2017), Coles, Daniel, and Naveen (2014), Taylor (2010), and Warther (1998).

and how to test them. Section 9 concludes.

2 Model

There is a board comprising two directors, $i \in \{1, 2\}$, who decide on a policy at each of two dates, $t \in \{1, 2\}$.[17] (See Section 7 for N-director and infinite-horizon extensions.) At date t, the board can replace the current "incumbent" policy x_{t-1} with an alternative policy y_t. Decisions are made by strict majority voting: if both directors vote for the alternative y_t, then y_t becomes the incumbent policy, $x_t = y_t$; otherwise, the incumbent policy stays in place, $x_t = x_{t-1}$. The policy in place creates value $v(x_t)$ at date t, so shareholders get $v(x_1) + \delta v(x_2)$, where δ is the rate of time preference. (We allow for $\delta > 1$, since date 2 may represent more calendar time than date 1.) Directors care about firm value, but they can be biased. Each director i maximizes the sum $v(x_1) + b_i(x_1) + \delta\big(v(x_2) + b_i(x_2)\big)$, where b_i is her bias.[18] We discuss different interpretations of directors' biases in Section 7.1.

Policies differ in two dimensions: in how much value they create for shareholders and in how much they appeal to biased directors. We capture shareholder value with "quality" $q \in \{h, \ell\}$. If the date-t policy x_t is of high quality h, then $v(x_t) = v_h$; if x_t is of low(er) quality ℓ, then $v(x_t) = v_\ell < v_h$. We capture the appeal to biased directors by adding a "bias" type $\tau \in \{\alpha, \beta\}$ to each policy and allowing directors to be either α- or β-biased, where a τ-biased director gets $b_i(x_t) = b_\tau$ if the policy x_t is type τ and $b_i(x_t) = 0$ otherwise. We also allow for unbiased directors, for whom $b_i(x_t) = 0$ for all policies x_t.

We assume that the qualities and bias types are i.i.d. at date 1 and date 2. p_q

[17] By restricting attention to two-director boards, we circumvent the concern that different decision-making protocols lead to different results. E.g., unanimity and majority voting are equivalent. That said, this restriction does not drive the results. Proposition 2, which underlies most of our analysis, holds in an N-director version of our model; see the proof and footnote 23.

[18] $v(x_t)$ need not represent the common value of all shareholders, but could rather represent the average value of shareholders with heterogenous biases, e.g., half of the shareholders could value x_t above $v(x_t)$ and half below. Thus, directors' biases could also reflect the heterogenous biases/preferences of individual shareholders.

and p_τ denote the probabilities that an alternative y_t is of quality $q \in \{h, \ell\}$ and of bias type $\tau \in \{\alpha, \beta\}$, respectively. $\bar{v} := p_h v_h + p_\ell v_\ell$ denotes the average quality of y_t and $v_0 := v(x_0)$ denotes the quality of the initial incumbent policy x_0.[19]

As touched on in the Introduction, disagreement among directors is common on corporate boards.[20] To capture this, we assume that the directors' biases are relatively large.

Assumption 1 *Biased directors are sufficiently biased: for $\tau \in \{\alpha, \beta\}$,*

$$b_\tau > \max\left\{ \frac{v_h - v_0 + \delta p_\ell (v_h - v_\ell)}{\delta p_\tau p_\ell}\,,\; v_h - v_\ell \right\}. \tag{1}$$

Solution concept. We solve for subgame perfect equilibria—sequentially rational strategies for each director $i \in \{1, 2\}$ to vote for/against y_t for $t \in \{1, 2\}$ given consistent beliefs—such that directors use the following tie-breaking rules if they are indifferent.

Assumption 2 *Directors do not vote against strictly Pareto-dominant policies at the final date.*[21] *If both directors are indifferent, the incumbent stays in place.*

Board composition. If a director is unbiased we indicate her type with ν. If a director is biased toward τ-policies, we refer to her as τ-biased and indicate her type with τ (so τ can represent a director type as well as a policy type). We use primes to denote the opposite director or policy: if $\tau = \alpha$, then $\tau' = \beta$, and vice versa. Hence,

[19] An alternative policy y_t is one of four types $h\alpha$, $h\beta$, $\ell\alpha$, and $\ell\beta$. However, the initial policy x_0 is not necessarily one of these types. We allow for this because we are interested in the case in which a policy x_0 is entrenched even though it is "worse" than *any* alternative y_t (see Section 3).

[20] For example, a recent survey of global directors emphasizes the importance of disagreement on boards as follows: "In the boardroom, disagreements are often unavoidable—especially when the board is composed of independent-minded, skilled, and outspoken directors. This is not a bad thing. There should be a debate in the boardroom" (IFC (2014), p. 2).

[21] In particular, if both directors weakly prefer the alternative y_2 to the incumbent x_1 and one director strictly prefers y_2 to x_1, then (i) if one director is indifferent between y_2 and x_1, she votes in the interest of the director with strict preference and (ii) if the director with strict preference is indifferent between voting for and against (because she is not pivotal), she votes sincerely.

a ν-ν board is an "unbiased" board in which both directors are unbiased; a τ-τ board is a "fully biased" board in which directors have the same bias; a τ-ν board is a "partially biased" board in which one director is τ-biased and the other is unbiased; and a τ-τ' board is a "diverse" board in which directors have opposing biases.

3 Entrenchment

Until stated otherwise (cf. Section 5), suppose that the initial policy x_0 is "very bad," in that it is worse for shareholders than low-quality alternatives, $v_0 < v_\ell$, and no director is biased toward it, $b_i(x_0) = 0$ for $i \in \{1, 2\}$. Thus, directors prefer *any* alternative y_t to x_0. In a one-shot model, they always vote to replace it.

Proposition 1 (One-shot benchmark.) *Suppose the board votes only once.*[22] *The incumbent policy x_0 is always replaced, regardless of board composition.*

Do directors also vote to replace x_0 in our dynamic model? Not if the board is diverse, since in a dynamic model directors with opposing biases vote strategically. In particular, with a diverse board, the α-biased director votes against all β-alternatives and the β-biased director votes against all α-alternatives. This leaves x_0 in place at date 1, even though both directors would be strictly better off with any other policy.

Proposition 2 (Entrenchment.) *Given a diverse (τ-τ') board, the incumbent policy x_0 is entrenched: no replacement is ever appointed at date 1.*[23]

Intuitively, the τ-biased director knows that if a τ'-alternative is chosen, the τ'-biased director will vote against replacing it with any τ-alternative at date 2 (given $b_{\tau'} > v_h - v_\ell$ by Assumption 1). In contrast, if the incumbent bad policy x_0 stays in

[22] This is tantamount to supposing there is no second date in our model.

[23] As we spell out formally in the proof, this result does not rely on there being only two directors. If there are $N > 2$ directors and N possible alternative policies, then the same intuition leads to the same result: directors block Pareto-dominating policies at date 1 to preserve the option to implement their preferred policy at date 2.

place, the τ'-biased director will vote in favor of any τ-alternative at date 2. Because the τ-biased director's bias towards τ-policies is sufficiently large (by Assumption 1), she blocks any τ'-policy at date 1. Even though retaining the very bad incumbent is costly in the short term, she wants to preserve the option to get her way in the long term. There is complete deadlock: each director votes against policies that would make her better off today to preserve the option of implementing a policy that would make her even better off in the future.[24]

Perhaps the most important function of real-world boards is appointing CEOs. If the incumbent policy x_0 represents the incumbent CEO, and the alternatives y_t represent potential replacement CEOs, our model generates CEO entrenchment, which seems to be a major source of corporate inefficiency (Taylor (2010)). In our model, unlike in others, entrenchment arises without any opportunistic behavior by the CEO or director disutility of firing. Rather, it arises only due to the constraints imposed by the dynamic consistency of multiple strategic directors.

Deadlock in our model results from directors' concern about board negotiations that will occur in the future—directors vote strategically to increase their chances of implementing their preferred policies later in their tenure on the board. The rate of time preference δ in our model can be viewed as a measure of directors' remaining tenure: if a director has a short tenure, she does not care about future policies, so δ is low; in contrast, if she has a long tenure, she cares a lot about them, so δ is high. This interpretation yields the next corollary.

Corollary 1 (Tenure.) *Suppose (instead of Assumption 1) that $b_\tau > (v_h - v_\ell)/p_\tau$ for each τ. Given a diverse board, increasing director tenure leads to entrenchment in the sense that x_0 is always replaced at date 1 for δ sufficiently small but never replaced for δ sufficiently large.*

[24] This extends the standard real options intuition that it is optimal to delay irreversible decisions (see, e.g., Dixit and Pindyck (1994)). Here, if a director exercises her option to replace the incumbent today, her choice is endogenously irreversible, since the other director will refuse to exercise her option to replace the new incumbent in the future.

Deeming director tenures too long, a number of institutional investors, such as BlackRock and State Street, are now voting against reappointments, leading commentators to suggest that director tenure is "the next boardroom battle" (Libit and Freier (2016), p. 5; see also Francis and Lublin (2016)). The argument for shorter tenures has centered around the idea that after a long tenure, a director may become too close to management and may also lack fresh ideas about the business. Our analysis offers a new, complementary perspective on the downside of long tenures: in anticipation of a long tenure, directors behave strategically, creating deadlock.

More generally, our analysis uncovers a cost of long-termism: long-termism can incentivize strategic voting, exacerbating deadlock. This provides a counterpoint to the broadly negative view of corporate short-termism; see, e.g., the former Vice President Joe Biden's opinion that short-termism "saps the economy" (Biden (2016)).

4 Board Composition

Our results so far show a downside of diverse boards: directors with opposing biases create deadlock. Now we ask how board composition can mitigate/aggravate deadlock. Does an unbiased director on the board resolve deadlock? No. The unbiased director votes in the interest of shareholders at each date. But, anticipating as much, the biased director responds strategically. Just as in the case of a diverse board, she strategically blocks high-quality policies not of her preferred type today, anticipating that the unbiased director will make them hard to replace in the future.

Lemma 1 (Cost of director heterogeneity.) *Consider a τ-ν-partially biased board. The τ-biased director votes against the high-quality τ'-alternative and votes in favor of all other alternatives.*

Although an unbiased director does not completely resolve deadlock, she prevents x_0 from becoming fully entrenched. But perhaps a biased director can resolve deadlock even further? Yes, in fact. If the other director is biased the same way, she

does not strategically block alternatives today, knowing she will always be able to implement her preferred policies in the future. With a fully biased board, one director does not have to make sure that the other director is dissatisfied with the incumbent to preserve the option to replace it. Hence, director heterogeneity can be bad for shareholders, since deadlock prevents some high-quality policies from getting through.

Proposition 3 (Shareholder optimal board composition.) *Define*

$$\Delta_\tau := \delta(1-p_\tau)p_h\left(v_h - v_\ell\right) - \left(v_\ell - v_0\right). \tag{2}$$

Shareholders are better off with a fully τ-biased board than a τ-ν-partially biased board if and only if

$$p_\ell p_\tau \Delta_\tau < p_{\tau'} p_h \Big(v_h - v_0 + \delta p_{\tau'} p_\ell (v_h - v_\ell) \Big), \tag{3}$$

and are always better off with a fully biased or partially biased board than a diverse board.

Although this result stresses a cost of director heterogeneity, it also suggests a benefit: given a τ-director on the board, a ν-director on the board can prevent low-quality τ-policies from becoming entrenched. Namely, she can strategically block low quality τ-policies that the τ-biased director would make hard to replace.

Corollary 2 (Benefit of director heterogeneity.) *Consider a τ-ν-partially biased board. The unbiased director votes against the low-quality τ-alternative if and only if $\Delta_\tau > 0$ and votes in favor of all other alternatives.*

Observe that the unbiased director may appear passive, or even biased, in the short-term, voting against some alternatives even though the incumbent policy is even worse ($v_0 < v_\ell$). This is because she wants to avoid being stuck with a low-quality policy in the long-term: by blocking the low-quality alternative that the other director is biased toward, she increases her chances of implementing a high-quality policy in the future.

In summary, an unbiased director acts in shareholders' interest, strategically blocking alternatives as long as the long-term benefit of implementing a high-quality τ'-policy outweighs the short-term cost of keeping the incumbent policy x_0 in place (this benefit and cost correspond to the two terms in Δ_τ). However, the biased director responds strategically, which can make shareholders worse off with a partially biased board than with a fully biased board. This is the case whenever the benefit from the unbiased director strategically blocking low-quality τ-alternatives is less than the cost from the τ-biased director strategically blocking high-quality τ'-alternatives (this benefit and cost correspond to the left-hand side and right-hand side of equation (3)).

Finally, note that, like a partially biased board, a diverse board has the benefit of preventing some low-quality policies from being implemented. However, in this baseline specification, this benefit is less valuable than that of the fully biased board, i.e. than preventing x_0 from being entrenched at date 1 (and hence getting a better set of policies to choose from at date 2). So shareholders always prefer the fully biased board to the diverse board. This is no longer the case if we relax the assumption that the alternative quality is identically distributed, as we show in Appendix A.5.3, to stress this potential benefit of diversity.

5 Appointing Directors

In this section, we study how deadlock affects director appointments. Suppose, first, that shareholders have full control over director appointments and consider a board with a τ-biased director in place and an empty seat to be filled at date 0. Will shareholders necessarily appoint an unbiased director who will act in their interest? Or a τ'-biased director who will counteract the τ-biased director? Not necessarily. Diverse and partially biased boards are not always good for shareholders, since they are prone to deadlock (Proposition 3). Hence, shareholders may appoint a τ-biased director, creating a fully biased board that makes some bad decisions but avoids

deadlock.

Corollary 3 (Shareholders' director appointments.) *Suppose there is a τ-biased director in place and an empty board seat. Shareholders appoint a τ-biased director if condition (3) holds. Otherwise they appoint an unbiased director.*

In our setup, shareholders would like to replace all directors at once with unbiased directors, since an unbiased board always acts exactly in their interest. But practical concerns could make this unattractive, because, e.g., some incumbent directors have indispensable expertise. Hence, shareholders' director appointments must account for the biases of incumbent directors. Given the costs of deadlock, the best response may be to exacerbate these biases, rather than to attenuate them.

Another source of shareholders' inability to replace all directors at once is a staggered board, which "prevents shareholders from replacing a majority of the board of directors without the passage of at least two annual elections" (Bebchuk and Cohen (2005), p. 410). The literature emphasizes that this can prevent efficient takeovers and proxy fights by forcing bidders and activists to win two far-apart elections (Bebchuk, Coates, and Subramanian (2002)). Our analysis suggests it may be even worse than we thought. If shareholders want to avoid deadlock today, they appoint new directors with biases in line with the current incumbent directors. But with staggered elections, today's new biased directors become tomorrow's incumbent directors. And if shareholders want to avoid deadlock tomorrow, they will appoint biased directors again, and so on ad infinitum. In other words, our analysis suggests that staggered elections may lead the board to stay biased forever, even after shareholders have replaced every director with a new director.

CEO appoints directors. In practice, shareholders do not always have full control over director appointments: the CEO can appoint some directors to the board as well (Hermalin and Weisbach (1998), Shivdasani and Yermack (1999)). Hence, we ask which directors the CEO will appoint. If his sole objective is to keep his position,[25] will he always appoint directors who are biased toward him? No. In

[25] I.e., the CEO's objective function is $U = \mathbb{P}\big[\text{employed at date 1}\big]w_1 + \mathbb{P}\big[\text{employed at date 2}\big]w_2$

fact, he may prefer to appoint directors biased against him, since this may exacerbate deadlock on the board and make it hard to fire him (Proposition 2).

Here, we return to the interpretation of the incumbent policy x_0 as the incumbent CEO and of the alternatives y_t as potential replacement CEOs (cf. the Introduction and Section 3). It follows from Proposition 2 that a "very bad" CEO chooses a diverse board to entrench himself.

Corollary 4 ("Very bad" CEO's board ranking.) *Given $v_0 < v_\ell$ and $b_i(x_0) = 0$, the incumbent CEO's preference over boards is as follows:*

$$\text{diverse} \succ \text{partially biased} \succeq \text{unbiased} \sim \text{fully biased}. \tag{4}$$

So far, we assumed that no director was biased toward the incumbent policy, to explore how a "very bad" policy/CEO could become entrenched. Now, we assume that the CEO is of type τ, to explore how the CEO appoints directors biased toward/against him. A high-quality CEO is only at risk of being fired if a director is biased against him and hence always prefers directors biased towards him:

Proposition 4 (High-quality CEO's board ranking.) *Given $v_0 = v_h$ and $b_i(x_0) = b_\tau$, the incumbent τ-CEO's preference over boards is as follows:*

$$\begin{aligned} \text{fully } \tau\text{-biased} \sim \text{diverse} \sim \tau\text{-}\nu\text{-partially biased} \sim \text{unbiased} \\ \succ \tau'\text{-}\nu\text{-partially biased} \succ \text{fully } \tau'\text{-biased}. \end{aligned} \tag{5}$$

In contrast to a high-quality CEO, a low-quality CEO is at risk of being fired even by directors biased toward him, since they prefer a high-quality CEO of the same bias type. Thus, like the very bad CEO above, a low-quality CEO wants to exploit deadlock on the board to avoid being fired. In fact, deadlock on the board can be more valuable for him than favoritism from the board.

for some weights or "wages" w_1 and w_2. Only the proof of Proposition 5 depends on the form of the CEO's objective.

Proposition 5 (Low-quality CEO's board ranking.) *Given $v_0 = v_\ell$ and $b_i(x_0) = b_\tau$, as long as p_τ is sufficiently large,*[26] *the incumbent τ-CEO's preference over boards is as follows:*

$$\begin{aligned} &\tau'\text{-}\nu\text{-partially biased} \succ \text{fully } \tau\text{-biased} \sim \text{diverse} \sim \\ &\tau\text{-}\nu\text{-partially biased} \succ \text{unbiased} \succ \text{fully } \tau'\text{-biased.} \end{aligned} \tag{7}$$

The low-quality τ-CEO benefits from having a τ-biased director on the board to prevent him from being replaced by any τ'-CEO. However, with a τ-biased director on the board, he is always replaced when a high-quality τ-CEO is available. There is no deadlock: even if the other director is τ'-biased, she will not vote strategically at date 1 because she knows that the τ-biased director will prevent her from getting her way at date 2 anyway. Thus, the CEO may be better off with a τ'-ν-partially biased board, because there is deadlock: the τ'-biased director votes against the high-quality τ-CEO (to preserve her option of appointing a τ'-CEO tomorrow) and the unbiased director votes against the low-quality τ'-CEO (to prevent his entrenchment). Hence, given an empty board seat, the CEO may appoint a director biased against him.

Corollary 5 (Low-quality τ-CEO's director appointments.) *Suppose there is an unbiased director in place and an empty board seat. A τ-CEO appoints a τ'-biased director for some parameters (specified in the proof).*

6 Who Should Appoint Directors?

As we touched on in Section 5, CEOs often have the power to appoint new directors. Could it be optimal for shareholders to give the CEO this power? In our baseline

[26] Specifically, we require that

$$(p_\tau - p_{\tau'})p_h w_1 > \Big(p_{\tau'} - p_\tau p_h(p_{\tau'}p_h + p_\tau p_\ell)\Big)w_2, \tag{6}$$

where w_1 and w_2 are as in footnote 25. In the proof we also give the low-quality τ-CEO's rankings for other parameters.

setup, the answer is no. Since directors are appointed at date 0, shareholders appoint the best director(s) for them, taking into account potential deadlock in the future. However, when directors are appointed at date 1, this is no longer necessarily the case, as we show in a modified setup here.

Given that in most firms CEOs have board seats and some power to appoint directors, we assume now that one director represents the CEO. We assume that she is τ-biased, reflecting, e.g., her private benefits of control or concerns about future employment. The other director can be of any bias type. But, unlike above, we assume she retires after date 1 (but before y_2 is realized). How her replacement is chosen depends on the CEO's power, denoted by π: with probability π, the CEO chooses the replacement and with probability $1-\pi$ shareholders do.

Ceding power to the CEO can help prevent deadlock. When the CEO controls the board at date 2, she does not block policies at date 1, since she does not need to improve her future bargaining position. By ceding power to the CEO, shareholders are able to commit not to block her preferred policies at date 2, and hence to improve date 1 outcomes. The next result summarizes how much power shareholders optimally give to the CEO to manage the tradeoff between avoiding deadlock at date 1 and not getting their preferred policy at date 2.

Proposition 6 *Define*

$$\bar{\pi} := 1 - \frac{v_h - v_0 + \delta p_l p_{\tau'} (v_h - v_l)}{\delta p_l p_\tau [b_\tau - (v_h - v_l)]} \in (0,1) . \tag{8}$$

Given a very bad incumbent x_0, the shareholder optimal CEO power π^* is

$$\pi^* = \begin{cases} \bar{\pi} & \text{if} \quad v_0 + \delta\bar{v} < v_h + \delta(1-\bar{\pi})v_h + \delta\bar{\pi}\big(v_h(1-p_\tau p_l) + v_l p_\tau p_l\big) \\ \pi \in [0,\bar{\pi}) & \text{otherwise.} \end{cases} \tag{9}$$

Intuitively, shareholders optimally give some power over director elections to the

CEO if the costs of deadlock are sufficiently high: observe $\pi^* > 0$ when v_0 is much lower than v_h, by the condition in equation (9). The cutoff $\bar{\pi}$ represents the least power the CEO must have not to block high-quality τ'-alternatives, i.e. to avoid deadlock.

7 Robustness and Extensions

7.1 Interpretation of Biases

Heterogenous biases. Heterogeneous director biases are the key driver of our results. These biases capture realistic heterogeneity among directors. For example, in start-ups, founding entrepreneurs often sit on boards beside capital providers like VCs, which have different objectives for the corporation. Indeed, early this year at Applied Cleantech, a technology start-up, deadlock on the board was so severe that the investors on the board sued the founder for control. In mature firms, equity blockholders typically sit on the board. The blockholder could be an heir to a family firm, with an interest in preserving her legacy. Or an activist investor, with an interest in preserving her reputation for fast value-enhancement. Other kinds of director heterogeneity are common. For example, in Germany it is common for directors to represent stakeholders such as bank creditors or employees/unions.

Director heterogeneity can also reflect heterogeneity among shareholders themselves, who have different preferences, e.g., due to different beliefs and portfolio positions. In close corporations, diverse shareholders sit directly on the board. But even in public corporations, diverse shareholders appoint directors to represent their diverse interests.

Preferences vs. beliefs. We have described directors' biases as reflecting differences in their preferences (i.e. tastes) over policies. But they can reflect differences in beliefs. To see why, consider the following setup, which is equivalent to ours. At the end of each date the policy x_t either "succeeds," generating value V or "fails," generating zero. Directors agree to disagree about the success probability. An unbi-

ased director believes the policy succeeds with probability $\pi_\nu(x_t)$, so that her value of the policy coincides with shareholders', i.e. $\pi_\nu(x_t)V = v(x_t)$, or $\pi_\nu(x_t) = v(x_t)/V$. A τ-biased director believes the success probability of a τ-policy is $\pi_\tau(x_t)$, so that her value of the policy is $v(x_t) + b_\tau$, i.e. $\pi_\tau(x_t)V = v(x_t) + b_\tau$ or

$$\pi_\tau(x_t) = \frac{v(x_t) + b_\tau}{V} = \pi_\nu(x_t) + \frac{b_\tau}{V}. \tag{10}$$

Note that, by our definition, although unbiased directors have the same beliefs as shareholders, these are not necessarily the "true" beliefs. "Biased" directors may be able to assess success probabilities better than shareholders.

7.2 $N > 2$ Directors and Uncertain Biases

Here we show that our results are not specific to boards with just two directors. Suppose now that there are N directors, but still just two alternatives, and decisions are made by majority voting. All directors are either τ-biased or τ'-biased. Each director knows her bias, but not the biases of other directors. Define q_τ as the probability that most directors are τ-biased, i.e.,

$$q_\tau := \mathbb{P}\left[\text{at least } \frac{N+1}{2} \text{ directors are } \tau\text{-biased}\right]. \tag{11}$$

Here, we ask whether a very bad incumbent policy x_0 can still be entrenched in this setup. Will a τ-biased director prefer to vote against a high-quality τ' alternative to retain the very bad incumbent x_0 at date 1?

At date 2, all directors vote sincerely (it is a weakly dominant strategy). Thus, if the high-quality alternative is in place, it is retained unless the date-2 alternative is type-τ and the majority of directors are τ-biased. Hence, given a high-quality τ' incumbent, a τ-biased director's expected date-2 payoff is

$$\tau\text{-biased director's date-2 payoff}\Big|_{x_1 \text{is } h\tau'} = q_{\tau'} v_h + q_\tau\Big(p_\tau\big(\bar{v} + b_\tau\big) + p_{\tau'} v_h\Big). \tag{12}$$

Whereas her payoff given the incumbent is x_0 is as in the two-director model, since the alternative is always implemented at date 2:

$$\tau\text{-biased director's date-2 payoff}\Big|_{x_1=x_0} = \bar{v} + p_\tau b_\tau. \tag{13}$$

Adding the date-1 payoffs to the expressions above, we get the following condition for when a τ-biased director prefers to retain the incumbent x_0 than to implement a high-quality τ'-alternative:

$$v_0 + \delta\Big(\bar{v} + p_\tau b_\tau\Big) > v_h + \delta\Big(q_{\tau'} v_h + q_\tau\Big(p_\tau\big(\bar{v} + b_\tau\big) + p_{\tau'} v_h\Big)\Big), \tag{14}$$

which yields the following proposition.

Proposition 7 *With N directors and uncertain biases, a τ-biased director votes against a high-quality τ'-alternative as long as her bias is sufficiently large, i.e.,*

$$b_\tau > \frac{v_h - v_0 + \delta\Big(p_\ell\big(1 - q_\tau p_\tau\big)\big(v_h - v_\ell\big)\Big)}{\delta p_\tau q_{\tau'}}. \tag{15}$$

This implies that a version of deadlock can arise even if the majority of the board is biased the same way. As long as directors are not certain that most other directors are biased the same way, they vote to keep the very bad incumbent policy in place, blocking high-quality alternatives. Observe, however, if $q_{\tau'} \to 0$ the condition in the proposition (equation (15)) is never satisfied. In words, if τ-directors know they are in the majority, they never block high-quality alternatives.

7.3 Infinite Horizon

Here we show that our results are not specific to our two-date setup. To do so, we consider an infinite horizon version of our baseline model and show that a very bad policy x_0 will still be entrenched with a diverse board. Here, we define x_0 as entrenched if all ℓ-quality policies are blocked. This definition is stronger than in the

baseline model, since it applies to all dates (not just date 1), but weaker in that it applies only to ℓ-quality policies (not to h-quality policies).[27] Assume that $v_0 = 0$ (a normalization), that $p_\alpha = p_\beta = 1/2$ (for simplicity), and that $\delta \in (0, 1)$, so that the value functions are well defined.

Proposition 8 (Infinite-horizon Entrenchment.) *Suppose that $v_0 = 0$, $p_\alpha = p_\beta = 1/2$, and $\delta \in (0, 1)$. Given a diverse board, there is entrenchment in the infinite horizon version of the model as long as directors' biases are neither too high nor too low, i.e., as long as*

$$\max\left\{\frac{2\Big(2v_\ell - \delta\big(p_h v_h + 2(1-p_h)v_\ell\big)\Big)}{\delta p_h\,(2-2\delta+\delta p_h)},\ \frac{2\,(v_h - v_\ell)}{2-2\delta+\delta p_h}\right\} \leq \frac{b_\tau}{1-\delta} \leq \frac{2v_h}{\delta p_h}. \tag{16}$$

Observe that this result requires not only that directors' biases are not too small, as in the corresponding result in the baseline model (Proposition 2), but also that they are not too large (relative to v_h). This ensures that the τ-director does not block the h-quality τ'-alternative.

8 Empirical Implications

Turning to our model's empirical content, we discuss empirical proxies for our model's key quantities and empirical predictions corresponding to its main results.

Proxies. Boards meet in the privacy of the boardroom without disclosing their minutes. Hence, deadlock is unlikely to be revealed publicly except in the most extreme cases, such as those that wind up in court, that result in director resignations, or that record directors voting in dissent.[28] This lack of data makes it hard

[27] Even in the baseline model, the outcome in which x_0 stays in place forever is not an equilibrium. At date 2, an alternative is always implemented. Analogously, in the infinite horizon version, this extreme form of entrenchment is not an equilibrium (given the tie-breaking rule in Assumption 2). Both directors would be better off with any alternative, and would have a profitable one-shot deviation to implement it.

[28] Translation company Transperfect and startup Applied Cleantech are recent examples of dead-

to test for deadlock directly. But our model suggests a way to test for deadlock indirectly: deadlock is manifested in boards' retaining incumbent policies, even when superior alternatives are available (Proposition 2). Applied to boards' key decisions, CEO turnover and corporate strategy, deadlock can be measured/proxied for by the following:

1. longer CEO tenure and, conditional on CEO termination, longer periods to appoint a new CEO (as with Uber's deadlocked board);

2. slow changes in strategy in response to a changing environment, even at the expense of the firm's competitiveness (as is common in corporations, Hannan and Freeman (1984), Hopkins, Mallette, and Hopkins (2013)).

A number of our predictions require proxies not only for deadlock, but also for directors' "biases" b_τ representing their preferences/private benefits or beliefs (Subsection 7.1). Proxies for directors' preferences include the stakeholders they represent—directors could represent VC investors, activists, founding families, employee unions, outside creditors, and corporate executives, all of which are likely to have different preferences over/private benefits from different company policies. Proxies for directors' beliefs include diversity in directors' experience, expertise, backgrounds, or skills, all of which are likely to lead to different views on the best policy for a company.

Predictions. Our main results correspond to testable predictions on the determinants of deadlock.

Prediction 1 *All else equal, deadlock is more likely on more diverse boards (cf. Proposition 2).*

lock cases that have gone all the way to court. Agrawal and Chen (2017) and Marshall (2013) analyze director resignations resulting from board disputes, which US companies must disclose by a 2004 SEC law. Jiang, Wan, and Zhao (2016) study independent directors voting in dissent, which Chinese firms must disclose by law. Of course, companies typically want to keep such disagreements private, so boards on which directors resign or vote in dissent should make up only a fraction of deadlocked boards.

This is consistent with Goodstein, Gautam, and Boeker's (1994) finding that diversity in directors' occupational or professional backgrounds is associated with less strategic change, such as fewer divestitures and reorganizations. Likewise, it is consistent with Knyazeva, Knyazeva, and Raheja's (2013) finding that diversity in directors' expertise and incentives leads to lower investment and lower firm value.

Prediction 2 *All else equal, deadlock is more likely when directors' remaining tenures are longer (cf. Corollary 1).*

In contrast to much of the literature, which focuses on directors' past tenure, this prediction underscores the costs and benefits of directors' future tenure. Strategic voting and deadlock on the board result from directors' incentive to improve their bargaining positions in anticipation of future negotiations. Hence, our model suggests that deadlock is less likely to arise if many directors are likely to leave a board soon, e.g., because they are nearing retirement or they are reaching the legal maximum tenure (in jurisdiction where such a maximum exists, such as the UK, Hong Kong, Singapore, and several EU countries).

In our model, a director strategically blocks an alternative because she wants to prevent other directors from blocking other alternatives in the future. Hence, given data on individual director voting, we have the following testable predictions:

Prediction 3 *All else equal, a director is more likely to vote against a policy if*

(a) there are other directors on the board who especially favor this alternative;

(b) these other directors have long expected remaining tenure;

(c) the director himself has longer expected remaining tenure.

This suggests a director is relatively likely to vote against a CEO candidate nominated by an influential blockholder on the board, since the blockholder is likely to nominate someone she is biased toward. For example, hedge fund activist campaigns are increasingly including the demand to replace the incumbent CEO. Our model suggests that directors on the board are relatively likely to vote against the activist's

candidate if the activist has (or will get) board representation. Indeed, as discussed in the Introduction, this is exactly what happened during Paul Hilal's activist campaign at CSX. Likewise, directors at Uber blocked candidates during its CEO search last summer. Some directors were opposed to Meg Whitman because they viewed her as "potentially compromised by her strong affiliation with Benchmark," the VC blockholder that had a seat on the board.[29]

That said, we acknowledge that our model is stylized, and we have abstracted away from at least one force pushing in the opposite direction: Fear of future alienation could make a director reluctant to vote against a powerful director's proposal. We hope future work will study the theoretical interaction between and empirical relevance of these two mechanisms.

Finally, our analysis of director appointments (Section 5) speaks to how CEO power affects deadlock.

Prediction 4 *Among companies with poor quality CEOs, deadlock is more likely if the CEO has more power to appoint directors (cf. Corollary 4).*

9 Conclusion

We argue that deadlock on the board can cause pervasive entrenchment, and hence explain why corporations are often too slow to turn over their top management and to adapt their strategies to a changing competitive environment. Our results hinge on the dynamic interaction between multiple directors' decisions, something new to the literature on corporate boards. Indeed, deadlock in our model is entirely a consequence of dynamic consistency: the board is deadlocked because it fears it will become deadlocked in the future.

This dynamic model gives a new take on board composition, director appointments, and director tenure. It suggests board diversity has a downside: it can exacerbate deadlock. As such, even adding unbiased directors to the board can cre-

[29] "Inside Uber's Wild Ride in a Search of a New C.E.O." *New York Times*, August 29, 2017.

ate deadlock. Hence, shareholders may optimally appoint a biased director to avoid deadlock. On the other hand, the CEO may appoint unbiased directors, or even directors biased against him, to create deadlock and thereby entrench himself. Still, shareholders may optimally give the CEO some power to appoint directors. We also uncover a cost of long director tenure: the more directors focus on the future, the more they vote strategically; they block policies today to preserve a strong bargaining position in the future, creating deadlock in the process.

A Proofs

A.1 Proof of Proposition 1

Given x_0 is very bad, voting for the alternative is a strict best response if the other director votes for. Hence, replacing the incumbent is always an equilibrium, and there is no equilibrium in which one director votes for and the other votes against. The tie-breaking rule in Assumption 2 rules out an equilibrium in which either director votes against.[30]

A.2 Proof of Proposition 2

To prove the proposition, we solve the model backward. The key observation is that if the "very bad" incumbent policy x_0 is in place at date 2, no alternative is blocked. This means that directors have incentive to keep x_0 in place at date 1 to preserve the option to implement their preferred alternatives at date 2. Thus, at date 1, the τ-biased director blocks all τ' alternatives and, symmetrically, the τ'-biased director blocks all τ alternatives.

We now proceed to characterize a τ-biased director's payoffs at date 2 and then to show that she blocks all τ' alternatives at date 1. (The argument for the τ'-biased director is identical.)

Date 2. Since $b_\tau > v_h - v_\ell$ by Assumption 1, a τ-biased director prefers a low-quality τ-policy to a high-quality τ'-policy. Thus, she blocks any τ'-policy if any τ-policy is in place. A high-quality τ-alternative gets through at date 2 if x_0 or a low-quality τ-policy is in place. Thus, the τ-biased director's payoffs as a function

[30]Note, however, that without this assumption, both directors voting against would be an equilibrium, since if one director votes against, the incumbent always stays in place, making voting against a weak best response to voting against.

of the date-1 policy x_1 are as follows:

$$\tau \text{ director's payoff} = \begin{cases} v_0 + \delta\big(\bar{v} + p_\tau b_\tau\big) & \text{if } x_1 = x_0, \\ v_\ell + b_\tau + \delta\Big(p_\tau p_h v_h + (1 - p_\tau p_h)v_\ell + b_\tau\Big) & \text{if } x_1 \text{ is type } \ell\tau, \\ v_\ell + \delta\Big(p_{\tau'} p_h v_h + (1 - p_{\tau'} p_h)v_\ell\Big) & \text{if } x_1 \text{ is type } \ell\tau', \\ v_h + b_\tau + \delta\big(v_h + b_\tau\big) & \text{if } x_1 \text{ is type } h\tau, \\ v_h + \delta v_h & \text{if } x_1 \text{ is type } h\tau'. \end{cases} \tag{17}$$

Date 1. Observe immediately that the τ-biased director prefers high-quality τ' policies to low-quality τ' policies at date 1. Now observe further that she prefers x_0 to high-quality τ'-policies, since

$$v_0 + \delta\Big(\bar{v} + p_\tau b_\tau\Big) > v_h + \delta v_h \tag{18}$$

if and only if

$$b_\tau > \frac{v_h - v_0 + \delta(v_h - \bar{v})}{\delta p_\tau} \tag{19}$$

$$= \frac{v_h - v_0 + \delta p_\ell (v_h - v_\ell)}{\delta p_\tau}, \tag{20}$$

which is implied by Assumption 1. Thus, she blocks any τ'-alternative policy.

A.2.1 What if there are N directors?

The result above does not depend on the number of directors. To see this, suppose, instead, that there are N directors on the board and N (or more) policies $\tau_1, ..., \tau_N$, where each policy τ_n is the date-2 alternative with probability p_{τ_n}. Now consider a diverse board, with one director of each bias type. Observe that the condition for a τ_n-biased director to prefer the incumbent policy x_0 to a high-quality policy of

a different type (i.e. not her preferred type τ_n) is the same as in the two-director two policy case. It is given by equation (18) above (with τ replaced by τ_n). The intuition is also unchanged. Each director knows that she will be able to implement her preferred policy at date 2 only if the incumbent x_0 stays in place, so she votes against all alternatives not of her preferred type.[31]

A.2.2 What if there is no strategic interaction?

Here we illustrate that entrenchment is the result of strategic blocking. It does not obtain if the board follows a non-strategic group decision protocol, as in Garlappi, Giammarino, and Lazrak (2017) (although a kind of inertia/underinvestment exists, in line with Garlappi, Giammarino, and Lazrak's findings). Consider a diverse board that maximizes the weighted average of the payoffs of the α-biased director and the β-biased director, as in Garlappi, Giammarino, and Lazrak (2017). Call λ the weight on the α-biased director's payoff. Thus, the payoff from the very bad policy x_0 is

$$\text{payoff}\Big|_{x_0} = \lambda v_0 + (1-\lambda)v_0 + \delta\Bigg(\lambda\big(\bar{v} + p_\alpha b\big) + (1-\lambda)\big(\bar{v} + p_\beta b\big)\Bigg) \tag{21}$$

$$= v_0 + \delta\Big(\bar{v} + \big(\lambda p_\alpha + (1-\lambda)p_\beta\big)b\Big) \tag{22}$$

where we have used the fact that the alternative policy y_2 is always implemented at date 2, no matter what it is. For entrenchment to occur with this specification, this payoff has to be bigger than the payoff given any alternative, in particular it must be that

$$\text{payoff}\Big|_{x_0} > \text{payoff}\Big|_{x_1 \text{ is } h\alpha} \quad \text{and} \quad \text{payoff}\Big|_{x_0} > \text{payoff}\Big|_{x_1 \text{ is } h\beta}. \tag{23}$$

[31] Unlike in many group–decision making environments with $N > 2$, this argument is not sensitive to the decision rule. Since $N-1$ directors want to keep the incumbent policy in place at date 1, it does not matter if they block alternatives via a veto rule, majority voting, or supermajority. Implementing a high-quality τ_n-policy requires either that the τ_n-biased director is a dictator or, similarly, that unanimity is required not to replace the incumbent.

Consider these two inequalities in turn. When x_1 is $h\alpha$, we require that

$$v_0 + \delta\Big(\bar{v} + \big(\lambda p_\alpha + (1-\lambda)p_\beta\big)b\Big) > v_h + \lambda b + \delta\Big(v_h + \lambda b\Big). \tag{24}$$

Given all the v-terms are bigger on the right, the above implies that $\lambda p_\alpha + (1-\lambda)p_\beta > \lambda$. Or

$$\lambda < \frac{p_\beta}{1 - p_\alpha + p_\beta} = \frac{1}{2}, \tag{25}$$

where we have used the fact that $p_\alpha + p_\beta = 1$. And, likewise,

$$v_0 + \delta\Big(\bar{v} + \big(\lambda p_\alpha + (1-\lambda)p_\beta\big)b\Big) > v_h + (1-\lambda)b + \delta\Big(v_h + (1-\lambda)b\Big), \tag{26}$$

which implies that $\lambda p_\alpha + (1-\lambda)p_\beta > 1 - \lambda$, or

$$\lambda > \frac{1 - p_\beta}{1 + p_\alpha - p_\beta} = \frac{1}{2}. \tag{27}$$

Clearly the inequalities in (25) and (27) are inconsistent. Hence, preference aggregation without strategic interaction does not generate entrenchment.

Note that for fixed λ, you can get that the board does not implement one of the policies, either $h\alpha$ or $h\beta$. This is analogous to Garlappi, Giammarino, and Lazrak's underinvestment. But with strategic directors, the board implements neither of the policies, neither $h\alpha$ nor $h\beta$. This is our entrenchment.

A.3 Proof of Corollary 1

The result follows from two observations. (i) For $\delta = 0$, directors care only about today's policy. Hence, they implement any alternative at date 1 (see the benchmark in Proposition 1). There is no entrenchment. (ii) For $\delta \to \infty$, the condition in the corollary implies Assumption 1 (recalling that we allow for $\delta > 1$ since date 2 can represent more calendar time than date 1). Hence, there is entrenchment by Proposition 2.

A.4 Proof of Lemma 1

First observe that, on a τ-ν partially biased board, the unbiased director votes for τ-policies over τ'-policies of the same quality, given the tie-breaking rule in Assumption 2. Thus, the only state in which the unbiased director votes against the τ-biased director at date 2 is when x_1 is $h\tau'$ and y_2 is $\ell\tau$; in words, when the incumbent is an h-quality τ'-policy and the alternative is an ℓ-quality τ-policy. In anticipation of this, the τ-biased director blocks the $h\tau'$-alternative at date 1 whenever

$$v_0 + \delta\Big(\bar{v} + p_\tau b_\tau\Big) > v_h + \delta v_h + \delta p_\tau p_h b_\tau, \tag{28}$$

or

$$b_\tau > \frac{v_h - v_0 + \delta p_\ell (v_h - v_\ell)}{\delta p_\tau p_\ell}, \tag{29}$$

which holds by Assumption 1.

The τ-biased director votes for all other date-1 alternatives since they all increase her date-1 payoff and do not decrease her date-2 payoff.

A.5 Proof of Corollary 2 and Proposition 3

Here, we prove Corollary 2 first and Proposition 3 second.

A.5.1 Proof of Corollary 2

Consider the unbiased director on the τ-ν board. And suppose the date-1 alternative y_1 is $\ell\tau$. If it becomes the incumbent, i.e. if $x_1 = y_1$, then the τ-director will block the $h\tau'$-alternative at date 2, since $v_h - v_\ell < b_\tau$ by Assumption 1. Thus, the unbiased director's payoffs as a function of the date-1 policy x_1 are:

$$\nu\text{-director's payoff}\Big|_{y_1 \text{ is } \ell\tau} = \begin{cases} v_0 + \delta\bar{v} & \text{if } x_1 = x_0, \\ v_\ell + \delta\Big(p_\tau p_h v_h + (1 - p_\tau p_h)\, v_\ell\Big) & \text{if } x_1 \text{ is } \ell\tau. \end{cases} \tag{30}$$

Comparing these payoffs, we find that the independent director blocks the $\ell\tau$-alternative if and only if

$$v_\ell - v_0 < \delta(1-p_\tau)p_h\,(v_h - v_\ell) \tag{31}$$

or $\Delta_\tau > 0$, which is the condition in the proposition.

The unbiased director votes for all other policies: he votes for any high-quality policy, and he does not block the $l\tau'$-policy because the other director will always agree to replace it by a high-quality alternative in the future.

A.5.2 Proof of Proposition 3

τ-τ board vs. τ-ν board. On a fully τ-biased board, directors always agree at date 2. Hence, there is no strategic blocking at date 1. Since $v_0 < v_\ell$, directors will always replace the inferior manager at date 1. Shareholders' expected payoff is

$$\begin{aligned} V_{\tau\text{-}\tau} = p_\tau p_h\Big(v_h + \delta v_h\Big) + p_{\tau'} p_h\Big(v_h + \delta p_\tau p_\ell v_\ell + \delta\,(1 - p_\tau p_\ell)\,v_h\Big) + \\ + p_\tau p_\ell\Big(v_\ell + \delta p_\tau p_h v_h + \delta(1-p_\tau p_h)v_\ell\Big) + p_{\tau'} p_\ell\Big(v_\ell + \delta\bar{v}\Big). \end{aligned} \tag{32}$$

Note that the second and third term follow from the fact that $v_h - v_\ell < b_\tau$ by Assumption 1: at date 2, τ-directors will replace an $h\tau'$-policy with an $\ell\tau$-policy but not an $\ell\tau$ policy with an $h\tau'$-policy.

On a τ-ν board, the analysis follows from Lemma 1 and Corollary 2. Recall that the ν-director's strategy depends on whether $\Delta_\tau \lesseqgtr 0$. Hence, we consider these cases in turn.

Case 1: $\Delta_\tau < 0$. Shareholders' expected payoff $V_{\tau\text{-}\nu}^{\Delta_\tau<0}$ is

$$\begin{aligned} V_{\tau\text{-}\nu}^{\Delta_\tau<0} =& p_\tau p_h\Big(v_h + \delta v_h\Big) + p_{\tau'} p_h\Big(v_0 + \delta\bar{v}\Big) + \\ &+ p_\tau p_\ell\Big(v_\ell + \delta p_\tau p_h v_h + \delta\,(1 - p_\tau p_h)\,v_\ell\Big) + p_{\tau'} p_\ell\Big(v_\ell + \delta\bar{v}\Big). \end{aligned} \tag{33}$$

Hence

$$V_{\tau\text{-}\tau} - V_{\tau\text{-}\nu}^{\Delta_\tau<0} = p_{\tau'}p_h\Big(v_h + \delta p_\tau p_\ell v_\ell + \delta\left(1 - p_\tau p_\ell\right)v_h - v_0 - \delta\bar{v}\Big) \tag{34}$$

$$= p_{\tau'}p_h\Big(v_h - v_0 + \delta p_{\tau'}p_\ell(v_h - v_\ell)\Big) > 0. \tag{35}$$

Case 2: $\Delta_\tau > 0$. Here, shareholders' expected payoff $V_{\tau\text{-}\nu}^{\Delta_\tau>0}$ is

$$V_{\tau\text{-}\nu}^{\Delta_\tau>0} = p_\tau p_h\left(v_h + \delta v_h\right) + p_{\tau'}p_h\left(v_0 + \delta\bar{v}\right) + p_\tau p_\ell\left(v_0 + \delta\bar{v}\right) + p_{\tau'}p_\ell\left(v_\ell + \delta\bar{v}\right). \tag{36}$$

Hence

$$V_{\tau\text{-}\tau} - V_{\tau\text{-}\nu}^{\Delta>0} = p_{\tau'}p_h\Big(v_h + \delta p_\tau p_\ell v_\ell + \delta\left(1 - p_\tau p_\ell\right)v_h - v_0 - \delta\bar{v}\Big) + \tag{37}$$

$$+ p_\tau p_\ell\Big(v_\ell + \delta p_\tau p_h v_h + \delta\left(1 - p_\tau p_h\right)v_\ell - v_0 - \delta\bar{v}\Big) \tag{38}$$

$$= p_{\tau'}p_h\Big(v_h - v_0 + \delta p_{\tau'}p_\ell(v_h - v_\ell)\Big) + p_\tau p_\ell\Big(v_\ell - v_0 - \delta p_{\tau'}p_h(v_h - v_\ell)\Big) \tag{39}$$

$$= p_{\tau'}p_h\Big(v_h - v_0 + \delta p_{\tau'}p_\ell(v_h - v_\ell)\Big) - p_\tau p_\ell\Delta_\tau. \tag{40}$$

This is positive exactly when condition (3) in the statement of the proposition is satisfied.

τ-ν board vs. τ-τ' board. Here, we show that shareholders always prefer a τ-ν board to a τ-τ' board, i.e.

$$V_{\tau\text{-}\nu} - V_{\tau\text{-}\tau'} > 0, \tag{41}$$

where

$$V_{\tau\text{-}\tau'} = v_0 + \delta\bar{v}, \tag{42}$$

by Proposition 2, and $V_{\tau\text{-}\nu}$ is given by equation (33) if $\Delta_\tau \leq 0$ and by equation (36) if $\Delta_\tau \geq 0$.

Again, consider the two cases for $\Delta_\tau \lessgtr 0$.

Case 1: $\Delta_\tau < 0$. Substituting equations (42) and (33) into inequality (41) and

simplifying, we see that the partially biased board is better than the diverse board if

$$p_\tau p_h(v_h - v_0) + p_\ell(v_\ell - v_0) + \delta p_\tau^2 p_\ell p_h(v_h - v_\ell) > 0. \tag{43}$$

This is always satisfied since $v_h > v_\ell > v_0$.

Case 2: $\Delta_\tau > 0$. Substituting equations (42) and (36) into inequality (41) and simplifying, we see that the independent board is better than the diverse board if

$$p_\tau p_h\Big(v_h - v_0 + \delta(v_h - \bar{v})\Big) + p_{\tau'} p_\ell(v_\ell - v_0) > 0. \tag{44}$$

This is always satisfied since $v_h > v_\ell > v_0$.

τ-τ board vs. τ-τ' board. Here, we show that a τ-biased board is always preferred to a diverse board, i.e.

$$V_{\tau\text{-}\tau} > V_{\tau\text{-}\tau'} \tag{45}$$

where $V_{\tau\text{-}\tau}$ and $V_{\tau\text{-}\tau'}$ are given by equations (32) and (42) respectively. Substituting, a τ-biased board is preferred to a diverse board if and only if

$$\begin{aligned} &p_\tau p_h\Big(v_h + \delta v_h\Big) + p_{\tau'} p_h\Big(v_h + \delta p_\tau p_\ell v_\ell + \delta\left(1 - p_\tau p_\ell\right) v_h\Big) + \\ &+ p_\tau p_\ell\Big(v_\ell + p_\tau \delta p_h v_h + \delta(1 - p_\tau p_h) v_\ell\Big) + p_{\tau'} p_\ell\Big(v_\ell + \delta\bar{v}\Big) > v_0 + \delta\bar{v}. \end{aligned}$$

Simplifying and rearranging, we get that a τ-biased board is preferred to a diverse board if and only if

$$\bar{v} - v_0 + \delta p_\tau^2 p_\ell p_h\left(v_h - v_\ell\right) + \delta p_{\tau'}^2 p_h p_\ell(v_h - v_\ell) > 0, \tag{46}$$

which is always satisfied.

A.5.3 Non-stationary Qualities

Here, we relax the assumption that alternative qualities are identically distributed. In this setup, a diverse board can be preferred to a biased board. Hence, we can highlight that a diverse board has the benefit of preventing some low-quality policies from being implemented and becoming entrenched (as a partially biased board does (Corollary 2)). It has this benefit in the baseline model too, but it is always outweighed by another benefit of the biased board: by preventing the date-1 entrenchment of x_0, the biased board gets a better set of policies to choose from at date 2.

We use the following notation. As above, p_h denotes the probability that the alternative is of type h at date 1, but now let $\hat{p}_h \neq p_h$ denote the probability that the alternative is of type h at date 2. Analogously, as above, $\bar{v} = p_h v_h + p_\ell v_\ell$ denotes average value of date-1 alternatives, but let $\hat{v} := \hat{p}_h v_h + \hat{p}_\ell v_\ell$ denote the average value of date-2 alternatives (where $\hat{p}_\ell := 1 - \hat{p}_h$).

We now compare the value $V_{\tau\text{-}\tau}$ of a τ-τ board with the value $V_{\tau\text{-}\tau'}$ of a τ-τ' board: $V_{\tau,\tau} \geq V_{\tau,\tau'}$ if and only if

$$\begin{aligned} &p_\tau p_h\Big(v_h + \delta v_h\Big) + p_{\tau'} p_h\Big(v_h + \delta p_\tau \hat{p}_\ell v_\ell + \delta\left(1 - p_\tau \hat{p}_\ell\right) v_h\Big) + \\ &+ p_\tau p_\ell\Big(v_\ell + \delta p_\tau \hat{p}_h v_h + \delta\left(1 - p_\tau \hat{p}_h\right) v_\ell\Big) + p_{\tau'} p_\ell\Big(v_\ell + \delta \hat{v}\Big) - \Big(v_0 + \delta \hat{v}\Big) \geq 0 \end{aligned} \tag{47}$$

Simplifying this expression is lengthy (although elementary), so we divide it into a few steps.

- *Date-1 value.* We can group the terms not multiplied by δ as follows,

$$p_h v_h + p_\ell v_l - v_0 = \bar{v} - v_0. \tag{48}$$

  This is always positive, implying a fully biased board always increases the date-1 value.

- *Date-2 value.* We can group the terms multiplied by δ as follows (omitting δ):

$$p_\tau \Big[p_h v_h \ + p_\ell \left(p_\tau \hat{p}_h v_h + (1 - p_\tau \hat{p}_h) \, v_\ell \right) - \hat{v} \Big] + \tag{49}$$

$$+ p_{\tau'} \Big[p_h \left(p_\tau \hat{p}_\ell v_\ell + (1 - p_\tau \hat{p}_\ell) \, v_h \right) + p_\ell \hat{v} - \hat{v} \Big] \tag{50}$$

The first term in square brackets above can be rewritten as

$$\begin{aligned} & p_h v_h + p_\tau p_\ell \hat{p}_h (v_h - v_\ell) + p_\ell v_\ell - \hat{v} \\ = \ & p_\tau p_\ell \hat{p}_h \left(v_h - v_\ell \right) + \bar{v} - \hat{v} \end{aligned}$$

The second term in square brackets above can be rewritten as

$$-p_\tau p_h \hat{p}_\ell (v_h - v_\ell) + p_h v_h + p_\ell \hat{v} - \hat{v} \tag{51}$$

$$= \ -p_\tau p_h \hat{p}_\ell (v_h - v_\ell) + p_h v_h + (1 - p_\ell) \hat{v} \tag{52}$$

$$= \ -p_\tau p_h \hat{p}_\ell (v_h - v_\ell) + p_h \big(v_h - \hat{v} \big) \tag{53}$$

$$= \ -p_\tau p_h \hat{p}_\ell (v_h - v_\ell) + p_h \big[v_h - (1 - \hat{p}_\ell) v_h - \hat{p}_\ell v_\ell \big] \tag{54}$$

$$= \ -p_\tau p_h \hat{p}_\ell (v_h - v_\ell) + p_h \hat{p}_\ell (v_h - v_\ell) \tag{55}$$

$$= \ p_{\tau'} p_h \hat{p}_\ell (v_h - v_\ell). \tag{56}$$

In summary, the fully τ-biased board is better than the diverse board if and only if

$$p_\tau \left[p_\tau p_\ell \hat{p}_h \left(v_h - v_\ell \right) + \bar{v} - \hat{v} \right] + p_{\tau'}^2 p_h \hat{p}_\ell (v_h - v_\ell) \geq 0.$$

To see that this may be violated, set $v_\ell = 0$ and $\hat{p}_h = 1$, so $\hat{p}_\ell = 0$, $\hat{v} = v_h$, and $\bar{v} = p_h v_h$. The condition becomes

$$p_\tau \Big(p_\tau p_\ell v_h + p_h v_h - v_h \Big) \geq 0 \tag{57}$$

which is never satisfied since $p_\ell p_\tau + p_h = 1 - p_\ell (1 - p_\tau) < 1$.

A.6 Proof of Corollary 3

The result follows immediately from Proposition 3.

A.7 Proof of Corollary 4

First observe that, since $v_0 < v_l$ (the incumbent CEO is "very bad"), he is always fired at date 2. Hence, he just wants to minimize the probability he is fired at date 1, which varies with board composition as follows.

- With a τ-τ or ν-ν board, he is always fired at date 1, since there is no strategic blocking at date 1.
- With a τ-τ' board, on the other hand, he is never fired at date 1—he is entrenched by Proposition 2.
- With a τ-ν board, he is retained when y_1 is type $h\tau'$ (by Lemma 1) and, for some parameters, when y_1 is type $\ell\tau$ (Corollary 2) and fired otherwise.

This yields the rainking stated in the corollary.

A.8 Proof of Proposition 4

- With a τ-τ, τ-τ', τ-ν, or ν-ν board, the CEO is never fired: the $h\tau$-CEO is the best policy for both τ-biased directors and unbiased directors. Hence he is never fired because they always block less-preferred alternatives (and keep him given equally-preferred alternatives by Assumption 2).
- With a τ'-ν board, he gets fired the first time there is an $h\tau'$-alternative, i.e. y_t is type $h\tau'$.
- With a τ'-τ' board he is fired the first time there is a τ'-alternative, i.e. y_t is type $h\tau'$ or $\ell\tau'$.

For a given y_t the τ'-ν board fires the CEO only if the τ'-τ' board does so (and the τ'-τ' board also fires the CEO for other realizations of y_t). Hence, the CEO (strictly) prefers the τ'-ν board to the τ'-τ' board.

In summary, the CEO's ranking is as stated in the proposition.

A.9 Proof of Proposition 5

CEO's objective. Recall that the CEO maximizes his expected tenure (see footnote 25). Since we want to allow date 1 and date 2 to represent different amounts of calendar time, we assume his objective is given by

$$U = \mathbb{P}\Big[\text{employed at date 1}\Big]w_1 + \mathbb{P}\Big[\text{employed at date 2}\Big]w_2, \tag{58}$$

where the weights w_t could represent his wage at date t or, alternatively, the ratio w_2/w_1 could represent his rate of time preference if he just values being employed.

CEO payoff given board compositions. Consider each of the six possible board compositions.

1. τ-τ *board.* Here, the CEO is fired the first time there is an $h\tau$-alternative. (Recall that the τ-biased director prefers the $\ell\tau$-CEO to the $h\tau'$-CEO by Assumption 1.) Hence,

$$U_{\tau\text{-}\tau} = (1 - p_\tau p_h)w_1 + (1 - p_\tau p_h)^2 w_2. \tag{59}$$

2. τ-ν *board.* Here, the board's decision rule coincides with that of the τ-τ board. Hence,

$$U_{\tau\text{-}\nu} = (1 - p_\tau p_h)w_1 + (1 - p_\tau p_h)^2 w_2. \tag{60}$$

3. τ-τ' *board.* Here, as in the simpler cases above, there is no strategic blocking. The reason is that the τ-director blocks *any* τ'-alternative (since she prefers the $\ell\tau$-CEO to an $h\tau'$-CEO by Assumption 1). As a result, the τ'-director knows

she can never hire a τ'-CEO, and hence wants to hire a high-quality τ-CEO as soon as possible. The CEO is fired the first time there is a $h\tau$-alternative, as with the τ-τ and τ-ν boards. Hence,

$$U_{\tau\text{-}\tau'} = (1 - p_\tau p_h)w_1 + (1 - p_\tau p_h)^2 w_2. \tag{61}$$

4. *ν-ν board.* Here, the CEO is fired the first time there is a high-quality alternative. Hence,

$$U_{\nu\text{-}\nu} = (1 - p_h)w_1 + (1 - p_h)^2 w_2 = p_\ell w_1 + p_\ell^2 w_2. \tag{62}$$

5. *τ'-τ' board.* Here, the CEO is fired the first time there is a τ'- or high-quality alternative. Hence,

$$U_{\tau'\text{-}\tau'} = p_\tau p_\ell w_1 + (p_\tau p_\ell)^2 w_2.$$

6. *τ'-ν board.* Here, there is strategic blocking. Specifically, by an argument analogous to that of Lemma 1, the τ'-biased director strategically blocks $h\tau$-alternatives, since

$$v_h + \delta\Big(v_h + p_{\tau'} p_h b_{\tau'}\Big) < v_\ell + \delta\Big(\bar{v} + p_{\tau'} b_{\tau'}\Big) \tag{63}$$

by Assumption 1.[32] And, by an argument analogous to Corollary 2, the independent director blocks $\ell\tau'$: she is indifferent between the incumbent $\ell\tau$ and the alternative $\ell\tau'$ today, but if $\ell\tau'$ is appointed today, the τ'-biased director will prevent her from appointing an $h\tau$-alternative in the future.

[32] To see this, observe equation (63) can be rewritten as

$$b_{\tau'} > \frac{v_h - v_\ell + \delta(v_h - \bar{v})}{\delta p_\ell p_{\tau'}} = \frac{v_h - v_\ell + \delta p_\ell (v_h - v_\ell)}{\delta p_\ell p_{\tau'}}, \tag{64}$$

which is implied by Assumption 1 given $v_0 < v_\ell$.

Hence,

$$U_{\tau'\text{-}\nu} = (1 - p_{\tau'}p_h)w_1 + (1 - p_{\tau'}p_h)p_\tau p_\ell w_2. \tag{65}$$

CEO's ranking. From the computations above, we can observe immediately that

$$U_{\tau\text{-}\tau} = U_{\tau\text{-}\nu} = U_{\tau\text{-}\tau'} > U_{\nu\text{-}\nu} > U_{\tau'\text{-}\tau'}. \tag{66}$$

The question is how $U_{\tau'\text{-}\nu}$ compares with the above.

- $U_{\tau'\text{-}\nu} > U_{\tau\text{-}\tau}$ if

$$(1 - p_{\tau'}p_h)w_1 + (1 - p_{\tau'}p_h)p_\tau p_\ell w_2 > (1 - p_\tau p_h)w_1 + (1 - p_\tau p_h)^2 w_2 \tag{67}$$

  or

$$(p_\tau - p_{\tau'})p_h w_1 + (-p_{\tau'} - p_{\tau'}p_\tau p_h p_\ell - p_\tau^2 p_h^2 + p_\tau p_h)w_2 > 0 \tag{68}$$

  or

$$(p_\tau - p_{\tau'})p_h w_1 > \Big(p_{\tau'} - p_\tau p_h(p_{\tau'}p_h + p_\tau p_\ell)\Big)w_2. \tag{69}$$

  This is always satisfied for p_τ sufficiently large (i.e. $p_{\tau'}$ sufficiently small), giving the ranking in the proposition.

- $U_{\tau'\text{-}\nu} > U_{\nu\text{-}\nu}$ if

$$(p_\ell p_\tau + p_h p_{\tau'})w_1 + (p_\ell p_\tau + p_h p_{\tau'})(1 - p_\ell p_\tau)w_2 > p_\ell w_1 + p_\ell^2 w_2. \tag{70}$$

- $U_{\tau'\text{-}\nu} > U_{\tau\text{-}\tau'}$ if

$$(p_\ell p_\tau + p_h p_{\tau'})w_1 + (p_\ell p_\tau + p_h p_{\tau'})(1 - p_\ell p_\tau)w_2 > (1 - p_\tau p_h)w_1 + (1 - p_\tau p_h)^2 w_2. \tag{71}$$

In summary, $\tau\text{-}\tau \sim \tau\text{-}\nu \sim \tau\text{-}\tau' \succ \nu\text{-}\nu \succ \tau'\text{-}\tau'$ and the ranking of $\tau'\text{-}\nu$ depends on the inequalities (67), (70), and (71) above, as stated in the proposition.

A.10 Proof of Proposition 6

Appointments. Consider appointment decision after date 1. Since this is the last date, the new board will make a one-shot decision. By Proposition 1, directors vote sincerely, for their preferred policy. Hence, whoever makes the appointment chooses the director that represents its interests: shareholders appoint an unbiased director; the CEO appoints a τ-biased director. (This is in contrast to our analysis in Section 5. There, appointments took into account strategic voting and deadlock.)

Date 2. At date 2, the board is fully biased with probability π and partially τ-biased with probability $1-\pi$.

First consider the fully biased board. There are four possibilities for the incumbent policy x_1:

- If a $\tau\ell$ policy is in place, it is replaced only with a τh policy (and kept in place otherwise).
- If a τh policy is in place, it is never replaced.
- If a $\tau' l$ policy is in place, the board replaces it with any alternative except $\tau' l$.
- If a $\tau' h$ policy is in place, it is replaced by τl and τh and is not replaced otherwise.
- If x_0 is in place, it is always replaced.

Now consider the partially biased board. There are five possibilities for the incumbent policy x_1:

- If a $\tau\ell$ policy is in place, it is replaced only with a τh policy (the τ-biased CEO blocks all other alternatives)
- If a τh policy is in place, it is never replaced.
- If a $\tau'\ell$ policy is in place, it is replaced by any alternative except $\tau'\ell$.

- If a $\tau' h$ policy is in place, it is replaced only by a τh policy (the unbiased director votes against low-quality alternatives).

- If x_0 is in place, it is always replaced.

Date 1. Since one director retires at the end of date 1, she only maximizes her date-1 payoff. She does not vote strategically, but rather votes for of any alternative regardless of her bias, as in the one-shot benchmark (Proposition 1).

Consider the CEO's voting decision. There are four possible alternatives.

- If y_1 is of type τ ($\tau\ell$ or τh) she votes for it given her bias.

- If y_1 is of type $\tau' l$, she votes for it, since with the policy in place, she will be still able to implement any τ policy at date 2 regardless of the composition of the board.

- If y_1 is of type $\tau' h$, voting for/against comes with a trade-off. If she votes for, and policy becomes the incumbent, her payoff is

$$v_h + \delta(1-\pi)\big(v_h + b_\tau p_\tau p_h\big) + \delta\pi\Big(b_\tau p_\tau + v_l p_\tau p_l + v_h\left(1 - p_\tau p_l\right)\Big). \tag{72}$$

  If she votes against, her payoff is

$$v_0 + \delta\big(\bar{v} + b_\tau p_\tau\big). \tag{73}$$

  Comparing the two payoffs, the CEO votes for the $\tau' h$-alternative if and only if

$$\pi \geq 1 - \frac{v_h - v_0 + \delta p_l p_{\tau'}\left(v_h - v_l\right)}{\delta p_l p_\tau\big(b_\tau - (v_h - v_l)\big)} =: \bar{\pi} \tag{74}$$

  because, by Assumption 1, the denominator is positive. Note also that, by Assumption 1, $\bar{\pi} \in (0,1)$.

Shareholder optimal CEO power. Now we calculate shareholders' expected payoffs in case $\pi \geq \bar{\pi}$ and $\pi < \bar{\pi}$.

Case 1: $\pi \geq \bar{\pi}$. In this case, the CEO does not block the $\tau' h$ alternative. Shareholder value is

$$p_l p_\tau \big((v_l + \delta v_l (1 - p_\tau p_h) + v_h p_\tau p_h \big) \tag{75}$$

$$+ p_h p_\tau (v_h + \delta v_h) + p_l p_{\tau'} (v_l + \delta \bar{v}) \tag{76}$$

$$+ p_h p_{\tau'} \Big(v_h + \delta (1 - \pi) v_h + \delta \pi \big(v_h \left(1 - p_\tau p_l\right) + v_l p_\tau p_l \big) \Big) \tag{77}$$

In this case, $\pi^* = \bar{\pi}$: shareholders optimally choose the lowest CEO power, to minimize probability that a $\tau' h$ incumbent is replaced by a τl alternative at date 2.

Case 2. $\pi < \bar{\pi}$. In this case, the CEO strategically blocks the $\tau' h$ alternative. Shareholder value is

$$p_l p_\tau \big(v_l + \delta v_l (1 - p_\tau p_h) + v_h p_\tau p_h \big) \tag{78}$$

$$+ p_h p_\tau (v_h + \delta v_h) + p_l p_{\tau'} (v_l + \delta \bar{v}) \tag{79}$$

$$+ p_h p_{\tau'} (v_0 + \delta \bar{v}). \tag{80}$$

In this case, π does not affect shareholder value.

Hence, $\pi^* = \bar{\pi}$ if

$$v_0 + \delta \bar{v} < v_h + \delta (1 - \bar{\pi}) v_h + \delta \bar{\pi} \big(v_h \left(1 - p_\tau p_l\right) + v_l p_\tau p_l \big) \tag{81}$$

and $\pi^* \in [0, \bar{\pi})$ otherwise.

It may also be worth pointing out as an aside that if $v_0 + \delta \bar{v} < v_h + \delta \left(v_h \left(1 - p_\tau p_l\right) + v_l p_\tau p_l \right)$, then $\pi = 1$ is better for shareholders than $\pi = 0$: full CEO control over director appointments can be better for shareholders than full shareholder control.

A.11 Proof of Proposition 8

Here, we first consider an outcome with entrenchment. Then, given this outcome, we compute the value functions at each date as a function of the incumbent policy.

Finally, we check the inequalities in equation (92) given the expressions for the value functions.

Entrenchment outcome. Consider the following outcome:

- If x_0 is in place, the τ-biased director votes for τ- and $h\tau'$-policies, but against $\ell\tau'$-policies.
- If an $h\tau$ policy is in place, the τ-biased director votes against all alternatives.
- If an $\ell\tau'$ policy were in place (off equilibrium), the τ'-biased director votes against all τ-policies.

Continuation values. Defining u_τ^x as a τ-director's continuation value at any date t given x is chosen at date t (but before the date-t flow payoffs are realized). We state the value functions as a lemma. For clarity, we compute them for general parameters even though we formulate the proposition only for $v_0 = 0$ and $p_\tau = p_{\tau'} = 1/2$.

Lemma 2 (Value functions.) *The value functions are as follows:*

$$u_\tau^{h\tau} = \frac{v_h + b}{1-\delta}, \tag{82}$$

$$u_\tau^{h\tau'} = \frac{v_h}{1-\delta}, \tag{83}$$

$$u_\tau^{\ell\tau} = \frac{1}{1-\delta(1-p_\tau p_h)}\left(v_\ell + b + \delta p_\tau p_h \frac{v_h + b}{1-\delta}\right), \tag{84}$$

$$u_\tau^{\ell\tau'} = \frac{1}{1-\delta(1-p_{\tau'} p_h)}\left(v_\ell + \delta p_{\tau'} p_h \frac{v_h}{1-\delta}\right), \tag{85}$$

$$u_\tau^{x_0} = \frac{1}{1-\delta p_\ell}\left(v_0 + \delta p_\tau p_h \frac{v_h + b}{1-\delta} + p_{\tau'} p_h \frac{v_h}{1-\delta}\right). \tag{86}$$

These expressions follow from direct computation given the supposed outcome. Indeed:

- $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$. If an h-policy is in place, it stays in place forever. Hence, we can write the value functions $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ recursively as

$$u_\tau^{h\tau} = v_h + b + \delta u_\tau^{h\tau} \tag{87}$$

and

$$u_\tau^{h\tau'} = v_h + \delta u_\tau^{h\tau'}. \tag{88}$$

Solving for $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ gives the expressions in the lemma.

- $u_\tau^{\ell\tau}$ and $u_\tau^{\ell\tau'}$. If an ℓ-policy is in place (off equilibrium), it stays in place until it is replaced with an h-policy of the same bias-type. Hence, we can write the value functions $u_\tau^{\ell\tau}$ and $u_\tau^{\ell\tau'}$ recursively as

$$u_\tau^{\ell\tau} = v_\ell + b + \delta\big(p_\tau p_h u_\tau^{h\tau} + (1 - p_\tau p_h) u_\tau^{\ell\tau}\big) \tag{89}$$

and

$$u_\tau^{\ell\tau'} = v_\ell + \delta\big(p_{\tau'} p_h u_\tau^{h\tau'} + (1 - p_{\tau'} p_h) u_\tau^{\ell\tau'}. \tag{90}$$

Substituting for $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ from above and solving for $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ gives the expressions in the lemma.

- $u_\tau^{x_0}$. If x_0 is in place, it stays in place until it is replaced with an h-policy (of either type). Hence, we can write the value function $u_\tau^{x_0}$ recursively as

$$u_\tau^{x_0} = v_0 + \delta\Big(p_\ell u_\tau^{x_0} + p_\tau p_h u_\tau^{h\tau} + p_{\tau'} p_h u_\tau^{h\tau'}\Big). \tag{91}$$

Substituting for $u_\tau^{h\tau}$ and $u_\tau^{h\tau'}$ from above and solving for $u_\tau^{x_0}$ gives the expression in the lemma.

Equilibrium. The outcome above is a subgame perfect equilibrium [33] as long as

$$u_\tau^{h\tau} \geq u_\tau^{\ell\tau} \geq u_\tau^{h\tau'} \geq u_\tau^{x_0} \geq u_\tau^{\ell\tau'}, \tag{92}$$

where the last inequality reflects deadlock. Now we set $v_0 = 0$ and $p_\tau = p_{\tau'} = 1/2$ and show that these inequalities are satisfied given the expressions above for the value functions.

- $u_\tau^{h\tau} \geq u_\tau^{\ell\tau}$ is immediate.
- $u_\tau^{\ell\tau} \geq u_\tau^{h\tau'}$ reduces to
$$b_\tau \geq \frac{2(1-\delta)\,(v_h - v_\ell)}{2 - 2\delta + \delta p_h} \tag{93}$$
- $u_\tau^{h\tau'} \geq u_\tau^{x_0}$ reduces to
$$b_\tau \leq \frac{(1-\delta)2v_h}{\delta p_h}. \tag{94}$$
- $u_\tau^{x_0} \geq u_\tau^{\ell\tau'}$ reduces to
$$b_\tau \geq \frac{2(1-\delta)\Big(2v_\ell - \delta\big(p_h v_h + 2(1-p_h)v_\ell\big)\Big)}{\delta p_h\,(2 - 2\delta + \delta p_h)}. \tag{95}$$

Together, the inequalities above yield the condition in the proposition.

[33] In line with Assumption 2, this preference ordering implies that the equilibrium is not the result of directors' indifference. Directors are not driven by the fact that if one director votes against, the other director is never pivotal. Note, however, that this ranking is only a sufficient condition, and other equilibria could exist that do not satisfy it.

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